Latent Gaussian models to decide on spatial closures for bycatch management in the Barents Sea shrimp fishery

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January 4, 2016
Abstract

In the Barents Sea and adjacent water, fishing grounds are closed for shrimp fishing by the Norwegian Directorate of Fisheries Monitoring and Surveillance Service (MSS) if the expected number of juvenile fish caught are predicted to exceed a certain limit per kilogram shrimp (*Pandalus borealis*). Today, a simple ratio estimator, which do not fully utilize all data available, is in use. In this research we construct a Bayesian hierarchical spatio-temporal model for improved prediction of the bycatch ratio in the Barents Sea shrimp fishery. More predictable bycatch will be an advantage for the MSS due to more correct decisions and better resource allocation, and for the fishermen due to more predictable fishing grounds. The model assumes that the occurrence of shrimp and juvenile cod can be modeled by linked regression models containing several covariates (including 0-group abundance estimates) and random effects modeled as Gaussian fields. Integrated Nested Laplace Approximations (INLA) is applied for fast calculation. The method is applied to prediction of the bycatch ratio for Atlantic cod (*Gadus morhua*).

Introduction

Trawling for shrimps in the Barents Sea takes place at the seabed, mostly at around 200-400 meters depths where the shrimp concentration is highest ([Jakobsen and Ozhigin](#)) page 172). To limit the bycatch, and thereby ensure a sustainable ecosystem and fishery in the Barents Sea, rules are made on the amount of bycatch that is allowed. To reduce bycatch, sorting grids were imposed in 1992/1993 ([ICES](#) [1994]). A sorting grid is a device on the trawl that sorts out the fish bigger than shrimps and thereby reduces the bycatch. In 1983, the Joint Soviet-Norway Fisheries Commission imposed a regulation that implies that fishing grounds are closed if the expected number of fish caught exceeds a certain limit per kilogram shrimp ([Veim et al.](#) 1994). Today this ratio limit is 0.8 for cod, and there are similar rules for bycatch of haddock, redfish and Greenland halibut ([Fiskeridirektoratet](#) 2005). These ratio limits are determined by
the biological status of the fish and shrimp stocks, as well as the economical value of the particular fish species (Veim et al., 1994). In this work we investigated bycatch of juvenile cod, but the methodology introduced is general and can be applied to bycatch of other species.

The method used today for regulating the shrimp fishery is as follows: When MSS suspects that there is a high rate of bycatch in a certain area, an inspector from MSS joins or rents a trawler and counts the number of juvenile cod caught as bycatch by new trawl hauls in that area. The bycatch ratio is then predicted by dividing the total number of juvenile cod by the total catch of shrimps. Based on this prediction a decision is made whether to close the area. After an area is held closed for some time (often some months), data from new trawl hauls are collected and a decision whether to open is made.

A statistical modeling approach for prediction of the bycatch ratio of cod in the Barents Sea shrimp fishery has previously been presented in Aldrin et al. (2012). The model considered in this paper is an extension of their model. The main extension from a statistical point of view is that all the parameters in our model are modeled simultaneously and that a Bayesian approach is applied. This results in a more statistical rigorous method to estimate the parameters and to quantify the uncertainty. We use integrated nested Laplace approximations (INLA, Rue et al., 2009; Martins et al., 2013) for performing the calculations involved. The INLA-technique is implemented in the user-friendly R-INLA package in R (R Core Team, 2014, the R-INLA package can be downloaded from http://www.r-inla.org). R-INLA has recently been used to model bycatch of Greenland sharks in the Greenland halibut fishery (Cosandey-Godin et al., 2014). Our paper goes further in constructing methodology for decisions as well as also providing models for catch of targeted species.

The main extension from a biological point of view is that we included several important explanatory variables. New variables considered are, among others, abundance estimates of 0-group cod (juvenile cod less than one year old) in the Barents Sea, the distance trawled and the type of trawling equipment used. In particular, a connection between
the yearly strength of the 0-group of cod in the Barents Sea and the bycatch of juvenile
cod is of biological interest. Before September/October, the 0-group lives in the upper
layers of the sea and grows to around the same size as the shrimps (mean length about
8 cm [Ottersen and Loeng, 2000]. After September/October the 0-group changes to a
demersal life stage, which means that they start living at the seabed of the Barents
Sea [Jakobsen and Ozhigin, 2011 page 228-230]). The trawlers target shrimps at the
seabed and it is therefore reasonable to believe that the amount of bycatch within the
shrimp fishery industry is related to the abundance of 0-group fish within the area. As
far as we know there has been no statistical research on such a connection before.

The results in this paper can help MSS to optimize their resource allocation and improve
their decision making, and make short time future fishing grounds more predictable for
the fishermen. The model proposed can easily be extended to prediction of bycatch for
other species and to other fisheries. The model can also be combined with many types of
random effects as well as observation models (e.g. zero-inflated models as suggested by
Aldrin et al., 2012).

Data

We used 7363 observations of shrimp trawl hauls from 1994 to 2006 provided by the
Institute of Marine Research (IMR) in Bergen, Norway. Originally we were given 7420
observations of shrimp catch and bycatch that were also used to predict bycatch ratios
in Aldrin et al. (2012). But after a thorough study of the data, we discarded 57 observa-
tions and further corrected 14 observations of shrimp catches that were wrongly given in
kilogram instead of ton. See Fig. 1 for an illustration of the locations of the observations
and Table 1 for a short summary of the data. In the data there were 5419 observations
with a single trawl, 1727 with a double trawl and 217 with a triple trawl. Approximately
a fifth of the observations lack information about the circumference of the trawl. For
these observations, which had only simple and double trawls, we fixed the circumference
to the average number of meshes around the opening within the type of trawl, that is
2200 meshes for a single trawl and 2480 meshes for a double trawl. There were 18.6 %
zeroes in the bycatch data, and most of them were in the summer when we should often
expect low bycatch.

Every late summer, around August/September, IMR and the Polar Research Institute
of Marine Fisheries and Oceanography (PINRO) in Murmansk, Russia, cooperate to es-
timate 0-group abundance. These estimates were calculated by a standard procedure:
Short trawls, each 0.5 nautical mile, were taken at three or more depths with head-line at
0, 20 m, 40 m, and so on. The number of cod caught was then corrected with a capture
efficiency function of cod length, and scaled up to make an estimate of the 0-group abun-
dance per square nautical mile (Eriksen et al., 2009). Fig. 2 shows the spatial locations of
the 0-group estimates in four different years. The number of estimates (spatial locations)
varied from 230 to 400 in the period 1993 to 2006 which means we had detailed informa-
tion about where the 0-group individuals were located in August/September.

Models

The Bayesian hierarchical model contains two main sub-models, one for catch of shrimps
(kg) and one for bycatch (counts). Let $C(s, t)$ be kilogram shrimp caught at time $t$ and
location $s$, scaled to be per nautical mile, and set $Z(s, t) = \log(C(s, t))$. The model for
the shrimp catch is defined as

$$Z(s, t) = X_Z(s, t)\beta_Z + \alpha_Z(s) + \nu_Z(t) + \gamma_Z(s, t) + \epsilon_Z(s, t).$$

(1)

Here $X_Z(s, t)$ is a vector of covariates (e.g. seasonal effect and gear equipment) and
$\beta_Z$ is the corresponding vector of regression coefficients. Three random effect terms are
included; the spatial random field, $\alpha_Z(s)$, is intended to capture that the amount of
shrimps might depend on local features, e.g. shrimps are known to be located at frontal
zone areas (Jakobsen and Ozhigin, 2011, page 173). The temporal random field, $v_Z(t)$, is intended to capture that catches change over time. The spatio-temporal random field, $\gamma_Z(s, t)$, is intended to capture that observations close in both space and time are probably more equal. All these random effects are modeled as Gaussian fields with dependence structures defined through covariance functions. Finally, $\epsilon_Z(s, t)$ describes measurement noise or micro-scale variability.

We assume a similar model for the bycatch. Let $B(s, t)$ be the number of juvenile cod caught at time $t$ and location $s$, scaled to be per nautical mile, and set $Y(s, t) = \log(B(s, t) + 1)$. Our model for the bycatch is defined as

$$Y(s, t) = X_Y(s, t) \beta_Y + \alpha_Y(s) + v_Y(t) + \gamma_Y(s, t) + \epsilon_Y(s, t),$$

(2)

where the interpretation of the terms involved are similar to model (1). The covariates that have been considered are given in Table 2. The 0-group abundance and shrimp catch covariate in Table 2 are only used in the bycatch model (2). Alternative models such as Poisson, negative binomial and a zero-inflated negative binomial distribution have also been considered, but they did not perform as well as the log-Gaussian distribution, see further comments on this in the discussion section.

The seasonal effect included requires some further discussion. We used a Fourier series (Lay, 2006, page 456) for the seasonal effect. The Fourier series is given by

$$f(t') = \sum_{i=1}^{r} (c_i \sin(it') + d_i \cos(it')),$$

(3)

were $t' \in [0, 2\pi]$ with $t'(1st January) = 0$, $t'(31st December) = 2\pi$ and linear in time. Here $c_i$ and $d_i$ correspond to regression coefficients within equations (1) and (2). Fourier
series are used since the seasonal effect should be the same at the start and the end of the year, and because seasonal effects typically have a harmonic pattern.

The growth rate of the cod depends on the temperature \cite{Jorgensen1992}, and the time at which a 0-group cod changes to a demersal life phase might depend on its size. We therefore allow the seasonal effect to be a function of latitude since it is typically colder in the north. This is implemented in the model by first assuming two different Fourier series, one at the northernmost location point containing data and another at the southernmost location point. Seasonal effects at other locations are then defined to be convex combinations of the seasonal effects in these two points. The weights in the convex combination are chosen to range from 0 to 1 and to be linear in the vertical distance between the location and the northernmost and southernmost point.

Spatial, temporal, and spatio-temporal Gaussian random fields

We included three correlation structures in our models (1) and (2) via Gaussian random fields, one spatial, one temporal and one spatio-temporal. This section describes the correlation structures for the Gaussian random fields involved in models (1) and (2). For brevity we will not use the subheadings $Z$ and $Y$ when elaborating the correlation functions.

We assume that the spatially correlated Gaussian field, $\alpha(s)$, has zero mean and follows the stationary Matern covariance function \cite{Stein1999} given by:

$$\text{Cov}(\alpha(s_1), \alpha(s_2)) = \frac{\sigma^2_\alpha}{2^{\nu-1}\Gamma(\nu)} (\kappa||s_1 - s_2||)^\nu K_\nu(\kappa||s_1 - s_2||),$$

where $\sigma^2_\alpha$ is the marginal variance, $\nu$ is a smoothing parameter, $\kappa$ is a spatial scale parameter, $||s_1 - s_2||$ is the distance between $s_1$ and $s_2$ in kilometers and $K_\nu(\cdot)$ is the modified Bessel function of the second kind. In this study we fixed $\nu = 1$ since this
value is implemented in the R-INLA package and since the value of $\nu$ is typically poorly identifiable (Blangiardo and Cameletti 2015, page 194).

We assume the time-dependent zero-mean Gaussian random field, $\nu(t)$, to be constant within years while independent between years, with variance $\sigma^2_\nu$. An AR(1)-structure in the yearly effect was also investigated, but this extra structure was not supported by data. It is important to note that we define the first month of the year to be September when we refer to a yearly effect in the bycatch model. This is reasonable because in September/October the 0-group starts entering a demersal life stage, and thereby starts living on depths where shrimp trawling occurs (Jakobsen and Ozhigin 2011, page 230). In the shrimp model, the year starts in January.

For the spatio-temporal interaction term, $\gamma(s, t)$, we assume a stationary zero-mean Gaussian field with a separable covariance function. We test three different, but quite similar covariance functions. The first two are given by

$$\text{cov}\left(\gamma(s_1, t_1), \gamma(s_2, t_2)\right) = \sigma^2_\gamma \exp\left(-\frac{||s_1 - s_2||^q}{\theta_s} - \frac{|t_1 - t_2|}{\theta_t}\right)$$

with $q = 1$ or $2$. Here $||s_1 - s_2||$ is the distance between $s_1$ and $s_2$ in kilometers, $|t_1 - t_1|$ is the time difference in days and $\theta_s$ and $\theta_t$ describe the correlation lengths in space and time. Both $q = 1$ and $q = 2$ give special cases of the Matern covariance function (4) as the spatial contribution to the separable spatio-temporal interaction (5), the first with $\nu = 0.5$ and the second with $\nu = \infty$ (Minasny and McBratney 2005).

The third covariance function considered was introduced within the R-INLA framework by Cameletti et al. (2013) and also tested (but rejected) in Cosandey-Godin et al. (2014). In this case the covariance function is indirectly defined through the introduction of a spatial grid overlapping the area of interest and a dynamic model for changes between
time points:

\[ \xi_r = a \xi_{r-1} + \omega_r, \quad \omega_r \sim N(0, \tilde{\Sigma}) \quad r = 1, \ldots, T. \]  

Here \( \xi_r = (\xi(s_1, r), \ldots, \xi(s_d, r)) \) are the values of the spatio-temporal process at time point \( r \) and grid points \( s_1, \ldots, s_d \). \( a \) is an unknown autoregressive parameter and \( \xi_0 \sim N(0, \tilde{\Sigma}) \). The covariance matrix \( \tilde{\Sigma} \) is specified such that it approximates a Matern covariance matrix in space for the \( d \) spatial grid points with \( \nu = 1 \) (see Cameletti et al., 2013 for further details).

Notice that the covariance structures in (5) and (6) are almost identical, except that in (6) we discretize time and approximate the Matern covariance function \( \nu = 1 \) as the spatial contribution to the separable spatio-temporal interaction. See the appendix for a detailed derivation of this.

Predictions of bycatch ratio for management

The bycatch ratio in an area \( A \) at time \( t \) is defined by (Ye, 2002):

\[ R_{A,t} = \frac{\sum_{s \in A} \text{Bycatch}(s, t)}{\sum_{s \in A} \text{Target catch}(s, t)}, \]  

where \( \text{Bycatch}(s, t) \) is the number of juvenile cod caught in a trawl haul at location \( s \) at time \( t \), and \( \text{Target catch}(s, t) \) is the kilogram of shrimp caught. The bycatch ratio (7) can be interpreted as the total bycatch ratio over a large number of hypothetical trawls taken in area \( A \) at time \( t \).

The bycatch ratio (7) in an area \( A \) at time \( t \) is predicted as in Aldrin et al., (2012) by Monte Carlo estimation.
\[ \hat{R}_{A,t} = \frac{1}{N} \sum_{i=1}^{N} \sum_{g=1}^{G} B_i(s_g,t) C_i(s_g,t). \] (8)

Here the outer sum is the Monte Carlo estimation, the inner sums approximate the sums in (7) where \{s_1, ..., s_G\} is a sufficiently dense set of spatial grid points in A. Here it is important that N must be large to encounter the uncertainty in \( \hat{R}_{A,t} \), and G must be large to ensure that the estimated bycatch ratio can be interpreted as the total bycatch ratio over a large number of hypothetical trawls. We used \( N = 2000 \) and found \( G \approx 200 \) appropriate in our application.

In our research we have seen that the magnitude of the seasonal effect on shrimp catch and the spatio-temporal correlation parameters varies in space and therefore we only used observations relatively close to the center of the area of interest when predicting the bycatch ratio (7). In our application the areas are typically defined by a few vertices, and the center of the area we define as the point with shortest sum of distances to all the vertices defining the area. To obtain the bycatch ratio predictions we only used observations closer than 600 km from the center of the area of interest. We expect that these observations are enough for making a good prediction and that we gain by excluding observations far away in space because of the more accurate estimation of the magnitude of the seasonal effect of shrimp catch and range of the spatio-temporal correlation in the area of interest.

**Inference**

The models for shrimp catch (1) and bycatch (2) are general additive latent Gaussian, and efficient computation can thereby be performed through the R-package R-INLA (http://www.r-inla.org, Rue et al., 2009, Martins et al., 2013). We have always used default priors (who are reasonably non-informative, see details in the appendix), and thereby let
the 7363 observations inform the posterior distributions.

For computationally efficiency we approximate the spatial Gaussian fields, \( \alpha^Z(s) \) and \( \alpha^Y(s) \) in equation (1) and (2), with Markov random fields. The approximation method used is explained in [Lindgren et al. (2011)] and is based on that the Matern covariance function (4) is a solution of a stochastic partial differential equation (SPDE). This solution can be approximately represented by a Markov random field with a sparse precision matrix which makes it possible to apply fast Laplace approximations [Rue et al. 2009].

Since we approximate the spatial Gaussian field with a Markov random field we need to define a spatial grid, this grid is shown in Fig. 3. Such triangulation based grids are easy to create in the R-INLA package and have several clear advantages compared to regular square grids. To make the Markov random field approximation continuous we further let the value at each point in the domain (that is not a vertex) be a convex combination of the estimated values at the three vertices defining a triangle around it [Lindgren et al. 2011]. Many of the observations are very close in space. In order not to make the triangulation very dense, we have chosen the triangulation such that no edges are closer than 20 km from each other. This has negligible effect on the results and it speeds up the calculations compared to letting each observation location be a vertex.

The covariance structure for the spatio-temporal effects defined in (5) is currently not directly available in R-INLA. However, a generic class is available where the precision matrix is given by \( Q = \tau C \) where \( \tau^{-1} \) is the marginal variance and \( C \) is fully specified. In our case \( C \) is a function of the parameters \( \theta_s \) and \( \theta_t \) in (5) resulting in that R-INLA can only be applied for prespecified values of these parameters. By running R-INLA several times and maximizing the marginal likelihood, posterior modes for \( \theta_s \) and \( \theta_t \) are obtained. In this research we only used the posterior mode of \( \theta_s \) and \( \theta_t \) and thereby neglected the uncertainty in these two parameters. To do fast approximation, R-INLA further requires sparse precision matrices. We made the precision matrix sparse by truncating to zero all elements in \( C \) that are less than 0.01 and are referring to locations more than one range unit away from each other. The range is here defined as the distance in time and space.
with correlation equal to 0.1. We tried different small thresholds for setting the elements $C$ to zero, and the differences of the results were negligible.

Consider now the spatio-temporal correlation structure introduced in [Cameletti et al. (2013)](#), see equation (6). A problem in using this correlation structure for our data is that the observations are unstructured in space and time. To use this approach we need to discretize time and define a spatial grid approximation also for this part of the model (6). For computational reasons, a very coarse spatial as well as temporal discretization is needed. We chose to discretize time in intervals of 30 days, and used a spatial mesh with 346 edges and with no edges closer than 50 km from each other.

**Model selection**

For model selection, we used the procedure recommended in [Zuur (2009, page 121)](#) where first the correlation structures are specified (through selection of which of the three random effects that should be included), using all relevant covariates, followed by a selection of significant covariates using the selected correlation structure.

We used four methods when evaluating correlation structures: Bayes factor ([Gelfand, 1996](#)), pseudo-Bayes factor ([Gelfand, 1996](#)), the DIC-value ([Spiegelhalter et al., 2002](#)) and mean square error (MSE) of the observed values compared with the expected value of (1) and (2), respectively. The Bayes factor is the ratio of the marginal likelihoods (ML) from a pair of models. The pseudo-Bayes factor is the ratio of the cross-validation densities (CVD) given by $\text{CVD} = \prod_{i=1}^{n} P(y_i | y_{-i}, M)$, where $y_{-i}$ are all the observations except $y_i$ and $M$ represents the model. See [Rue et al. (2009)](#) on how the ML and CVD are calculated within R-INLA. When calculating the MSE we remove every tenth observation and predict these, this we repeat ten times until we have predictions for all the observations. We used the Bayes factor for backward elimination of covariates.
Computational features

The research was done on a computer with Intel Core i5-2500 CPU 3.30GHz × 4 processor, and R-INLA utilizes all the four cores. With the 7363 observations the calculations took about 16 minutes for the final bycatch model and five minutes for the final shrimp catch model after the posterior mode of the spatio-temporal parameters $\theta_s$ and $\theta_t$ (eq (5)) was found.

Results

The results section is divided into three parts: 1) covariates, 2) covariance structure, and 3) model performance with regards to decision making on time/area closures compared to previous models in this fishery (Aldrin et al., 2012).

Covariates

Table 3 lists the covariates that were selected for the prediction of shrimp and bycatch. For the description of the seasonal effects we included one harmonic term in the shrimp model, and three harmonic terms in the bycatch model. The seasonal effect of bycatch varied in space, the further north the later the seasonal effect will increase in late fall/early winter. See Fig. 4 for illustration of the seasonal effects.

By looking at credibility intervals, we found a clear effect of the strength of the 0-group of cod in the Barents Sea on the bycatch when aggregating the 0-group estimates over space, see Table 3. Our model predicts that if the 0-group abundance doubles, the bycatch increases by approximately 29% with 95% credibility interval (13%, 47%). The Bayesian factor was indifferent to the inclusion of the 0-group when the yearly effect was included, but the inclusion of the 0-group halved the variance of the year effect, giving better predictive power when included. We therefore decided to include this effect into
The more shrimp that is caught, the more bycatch we can expect. If we double the shrimp catch the bycatch increases with approximately 18% (16%, 21%). In times of the year when there is neither midnight sun nor polar nights the model predicts that it is much harder to catch shrimp and we get less bycatch in the night. The size of the coefficients implies that the shrimp catch reduces with 34% (27%, 41%), and the bycatch reduces with 23% (11%, 33%). Since both the bycatch and the shrimp catch decrease during night time trawling, this variable has lesser effect on the bycatch ratio. In time of the year when there is midnight sun or polar nights we found no night effect.

The model found that larger equipment often leads to larger catch. Compared to using a single trawl, the model predicts that the shrimp catch increases by 80% (67%, 95%) if we use a double trawl and 222% (153%, 306%) if we use a triple trawl. We have few observations with triple trawls, which might explain the large uncertainty of this coefficient. The bycatch is predicted to increase by 32% (17%, 48%) if we use a double trawl while we did not find any increase by using a triple trawl. That triple trawls have no effect on the bycatch we think is intuitively surprising, the reason might be that the shape of the trawl differs when several trawls are used or that we do not have enough observations with triple trawls.

Covariance structure

When considering model selection with respect to the covariance structure (random effects), both the shrimp and bycatch models, including spatio-temporal correlation structure given by \( q = 1 \), were clearly favored, see Table 4. The optimal shrimp catch model contains only a spatial and a spatio-temporal interaction term in (1). The optimal bycatch model includes a spatial, a temporal as well as a spatio-temporal interaction term in (2).
Table 5 shows the values of the parameters in the correlation structure in the final model while Fig. 5 shows the spatial effects of the bycatch and the shrimp catch. The ranges in space and time in the spatio-temporal Gaussian fields are estimated to be approximately 160 days and 150 km for the shrimp catch and 90 days and 310 km for the bycatch.

From the estimated mean of the marginal variances in Table 5 we can interpret how the variation in the observations are distributed among the random terms in (1) and (2). We see that most of the variation was in the spatial part, secondly in the spatio-temporal part, thirdly in the unstructured part and least in the temporal part. Note that, as stated above, the latter part is only included in the optimal bycatch model.

**Decision making**

In this section we illustrate how the model performs with respect to the important decision of whether to open or close an area for shrimp fishing. Remember that an area should be closed if the bycatch ratio is expected to exceed 0.8 cod per kilogram shrimp. We predict the bycatch ratio through (8). In this section we first investigate how well the model performs in a certain area where there is much shrimp catch activity. Then we investigate more generally how good the model predicts bycatch ratios when using parts of the observations from MSS as test sets.

As in Aldrin et al. (2012), we predicted the bycatch ratio at 1st of December 2005 in the Hopen area. See Fig. 1 for an illustration of the Hopen area. At that time an inspector from MSS was investigating 21 trawl hauls in the Hopen area on a boat with a single trawl with 3600 meshes around the opening. Our predictions of bycatch are done by taking the fishing gear equipment into account, while Aldrin et al. (2012) did not consider such an effect. We first predicted the bycatch ratio at 1st of December 2005 based only on observations previous to that date. Thereafter we updated the prediction while sequentially including 1, 3, 5, 10, 15 and 21 additional observations sorted in the order
they were taken in the period 3rd to 6th of December 2005. The predictions and credibility intervals of the bycatch ratio are given in Table 6, the predictions by the model currently in use is referred to as the simple model. Confidence intervals of the simple bycatch ratio estimates are calculated by using nonparametric bootstrapping (Efron and Tibshirani, 1994).

We used $G = 203$ in equation (8) when estimating the bycatch ratio in this area, and we observed that using a larger $G$ changed the estimates negligibly. Furthermore, the restriction to only use observations closer than 600 km from the center of the area of interest resulted in that we used 4784 observations before 1st of December 2005.

With the regulation method used today, predictions without any recent observations are not possible and MSS needs to take many new (expensive) observations to obtain reliable results. From Table 6 we see that both our model and the model introduced in Aldrin et al. (2012) are able to do reasonable predictions even with very few recent observations within the area of interest. Furthermore, our predictions are quite close to the predictions given in Aldrin et al. (2012). This is not surprising since we concluded to use a quite similar model. The new model is however able to detect a bycatch ratio that is significantly higher than 0.8 in more cases with few observations compared to the model in Aldrin et al. (2012).

The results clearly indicate that the Hopen area should be closed in the beginning of December 2005. The next decision problem then is when to open again. Our model can, even without extra samples, predict bycatch ratios at any time. Fig. 6 illustrates the predicted bycatch ratios after December 2005 given only the observations up to December 6. These results indicate that the area could be opened for shrimp fishing in April 2006.

We predicted the bycatch ratio in several other periods and locations, with promising results. We illustrate one such set of predictions for bycatch ratios. In 2005 and 2006 there were 18 months with trawl haul observations in the Hopen area. Fig. 7 shows the
bycatch ratio predictions of the trawl hauls for each month in that period using only observations previous to the beginning of the month. From Fig. 7 we see that the model is able to give realistic predictions of the bycatch ratios compared to the observed bycatch ratios. Notice that the predicted bycatch ratio in December 2005 and November 2006 clearly differed from the observed bycatch ratio. This was because of very low shrimp catches that resulted in a high bycatch ratio. The reason for a slightly difference between ratio prediction in June 2006 and the observed bycatch ratio is discussed in the discussion section.

We also investigated how well the bycatch ratio estimation performed when using parts of the observations from MSS as test sets. We defined a test set by sequentially selecting every tenth trawl haul in the data. For these hauls, point predictions together with 90% and 99% prediction intervals for bycatch ratios were calculated. By comparing the prediction intervals with the true observations we were able to investigate the coverage. From Table 7 we see that the prediction intervals have roughly the right coverage. The 90% prediction intervals seemed to have the right coverage for bycatch and shrimp catch, but when looking at the extremes, the 99% prediction intervals seemed to have slightly less coverage. The largest difference is that the model too often failed to predict small shrimp catches, but in a regulation perspective this is not a very important error since low shrimp catches lead to small commercial shrimp catch activity.

Discussion

The objective of this paper was to construct statistical rigid models for shrimp catch and bycatch that can be used to regulate the shrimp fishery with respect to bycatch. This discussion is divided in four parts: The first part is about the covariates and the covariance structures. The second part is about alternative observation models. The third part is elaborating comparisons with earlier research (Aldrin et al., 2012). The fourth and final part is about how the methodology introduced in this paper can be used.
by the MSS and in other biological applications.

**Covariates and correlation**

Fig. 5 shows the spatial effects of the bycatch and the shrimp catch. The spatial structure for bycatch looks very intuitive since the cod spawn mainly in the north of Norway and the larvae drift passively in the upper layers with the currents into the Barents Sea. In August/September the juvenile cod are distributed at most places at the warm side of the Polar Front with typically largest concentration in the central Barents Sea (Jakobsen and Ozhigin 2011, page 230).

Fig. 4a and 4b illustrate the seasonal effects for the bycatch. The increase in September/October is caused by the 0-group entering a demersal phase. The model predicts that a cod changes to a demersal phase later in the north (Hopen) compared to the south (Lyngen). This is reasonable since the cod grows slower in the cold water far north (Jørgensen 1992).

From Fig. 4c we also see that the model predicts higher shrimp catches in late spring compared to the winter. This is probably due to the shrimps vertical migration pattern which is dependent on light conditions (Hopkins et al. 1993). By estimating the seasonal effect of shrimp catch at different areas (not shown), we noticed that the shape of the seasonal effect is the same but the magnitude seems to depend on the location. We tried to account for this interaction between space and seasonal effect, similar to what we did for bycatch, but there was no support in the data for including this into the model.

We tried to utilize the spatial locations of the estimates of the 0-group as a possible spatial predictor for bycatch by using estimates of the number of cod per square nautical mile in areas around the bycatch observations, but data did not support to include this into the model. We therefore concluded only to use the estimated total number of 0-group of cod present in the Barents Sea. These estimates can be found in Jakobsen and Ozhigin (2011).
page 565) and are calculated by the same 0-group data as used in this work. We believe
that there are two main reasons for not being able to utilize the spatial locations of the
estimates of the 0-group. One reason is that the cod can drift a long distance with the
currents before it changes to a demersal life phase later that year. The other reason is that
the amount of cod per nautical mile estimated as in Eriksen et al. (2009) at each location
has a very large, and difficult to quantify, variance. Therefore few observations might give
little information, while spatial aggregation of the 0-group gives more reliable covariates.
To better encounter that the 0-group changes from year to year, we have in addition tried
to include a Gaussian field with a correlation structure given as in Cameletti et al. (2013),
see equation (5), with time discretized as yearly intervals lasting from 1st of September to
31th of August. By visually inspecting the yearly spatial-temporal contributions we have
seen no clear correspondence with the yearly spatially distribution of the 0-group given
in Jakobsen and Ozhigin (2011, page 564). Adding such a correlation structure was neither
supported by data based on our validation methods.

Because of our noninformative priors, and the confounding between the yearly effect and
the 0-group, the Bayes factor equally favored bycatch models with and without the 0-
group (when the yearly effect was included). However, including the 0-group covariate
resulted in a large decrease (from 0.44 to 0.2) in the variability of the year effect, resulting
in higher predictive power from a management perceptive. Because of this we included
this covariate as well.

The amount of shrimp catch was clearly important for the bycatch, even when scaled
by distance. This might be because the shrimp and cod feed on the same prey and
thereby might be concentrated at the same locations. The night effect was clearly an
important covariate for both the bycatch and the shrimp catch. This might be explained
by the shrimps being known to feed on pelagic prey species especially at night and hence
stay semi pelagic above the trawl gear during night (Jakobsen and Ozhigin 2011, page
176).

We both included a pure spatial field and a spatio-temporal random field in the models.
The spatial Gaussian field is intended to capture that some places are expected to have small or large catches due to biological or geophysical features. Inclusion of a pure spatial field resulted in a spatio-temporal field with a much smaller spatial and temporal range than a model without a spatial field. Our model is aimed for predicting sudden changes in the bycatch, and thereby be able to help the MSS to open or close areas. Therefore we need a correlation structure that can detect sudden changes. We believe we managed this in a satisfactory manner. A reason for this is that when including a pure spatial Gaussian field we are able to include a spatio-temporal structure that can only focus on the local changes in time and space. As opposed to previous research (Cosandey-Godin et al., 2014), we concluded to use the continuous correlation structure (5) for the spatio-temporal field in order to take into account the structure in the observations.

Observation models

In addition to the log-Gaussian distribution for the observations in the bycatch model, we also considered Poisson, Negative Binomial and zero-inflated Negative Binomial distribution. We considered a zero-inflated distribution in R-INLA (http://www.r-inla.org) which allows the zero-probability to decrease when the expectation increases because we believe this is reasonable in our application. Of the alternative distributions, the Negative Binomial distribution gave the best fit to the data. When comparing the Negative Binomial distribution with the log-Gaussian distribution, the log-Gaussian distribution gave more accurate predictions (when comparing the sum of absolute errors of the number of cod taken as bycatch). The histogram of Bayesian p-values (Gelman et al., 2003) looked more uniform when using a log-Gaussian distribution. The pseudo-Bayes factor also gave preference to the log-Gaussian distribution. Because of this we chose the log-Gaussian distribution. In Cosandey-Godin et al. (2014) the authors used a latent Gaussian model with zero-inflated negative binomial distribution to estimate bycatch of Greenland shark in the Greenland Halibut fishery. In their application the bycatch values were low (mostly zero). In our application, with many large bycatch-numbers, the log-Gaussian distribu-
There are cases of extremely large observations of bycatch in the summer when the model predicts little bycatch. This is probably because marine resources often are highly patchy (Seber, 1986) and the trawler has trawled through a large school of juvenile cod. One example of this we see in July 2006 (Fig. 7) were one haul contained 616 juvenile cod per distance (compared to (2, 92) cod per distance in the other hauls). Such a large bycatch in one trawl is not normal in the summer and gave, with our model, a Bayesian p-value (Gelman et al., 2003, page 162) approximately equal to 0.0005. This one trawl haul then resulted in the Bayesian p-value of the total bycatch ratio that month (consisting of 9 trawl hauls) became approximately 0.02. We tried to use a $t$-distribution for the observations within the bycatch model to partly encounter for this scenario, but then the degree of freedom was estimated to be 18, and thereby there was little difference compared to the Gaussian distribution.

**Comparison with Aldrin et al. (2012)**

The model introduced in this research is an extension of the model introduced in Aldrin et al. (2012). In that paper the authors introduced an additive regression model for shrimp catch and bycatch and first estimated the regression parameters with the least square method. Then they estimated the hyperparameters in the correlation structure given the regression parameters with a maximum likelihood method (they only used three parameters in the correlation structure since they did not use the very important yearly effect on bycatch and used a spline method for the spatial effect). To estimate the correlation structure in reasonable time, Aldrin et al. (2012) further divided the observations into 24 segments in time and space and assumed independence between segments. In our approach we find the posterior distribution for all the parameters simultaneously and thereby make the method more rigid. We are able to do this because the R-INLA package effectively calculates the posterior distributions.
Aldrin et al. (2012) stated that considering the Negative Binomial distribution would be interesting in further work. R-INLA allows to easily consider different distributions for the observations. In the R-code we only need to change a few lines to let the R-INLA package run with another distribution (http://www.r-inla.org).

A problem encountered and efficiently dealt with in Aldrin et al. (2012) was that the variance of the residuals depended on the expectation. We did not encounter this problem within our model. This may be because we scaled the response by the distance trawled, and thereby accommodate for external factors that might explain the heterogeneity that was present in the Aldrin et al. (2012) model.

Practical implications of the model

The MSS has limited resources and needs to optimize the choice of locations to collect observations for predicting the bycatch ratio. The model introduced in Aldrin et al. (2012) and further extended in this research can help MSS to optimize the use of observations and thereby the collection of data within their resource limits. Our method predicted the bycatch ratio to be high enough to close the Hopen area for shrimp fishing in early December 2005. Without the need for further observations, our method also predicted that the area could be opened in April 2006 (see Fig. 6), thereby saving the cost of collecting expensive new observations.

The model introduced can also be used to predict the historical total bycatch in shrimp fishery. Historical total bycatch prediction has previously often been performed by scaling up the observed bycatch ratio in areas with the commercial shrimp catch (Ye, 2002, Ye et al., 2000, Davies et al., 2009, Amandè et al., 2010). We expect that the model introduced in this research will give more reliable estimates of the total bycatch, including uncertainty. There is ongoing work to predict the historical bycatch of cod, redfish and haddock in the Barents Sea shrimp fishery by using the model introduced here.
Acknowledgments

The authors want to thank Frank Kristoffersen (inspector at MSS) and Erik-Andre Brose Krag (captain on a shrimp trawler) for valuable discussions lasting four days while the first author was on an expedition in Lyngen trawling for shrimps. The authors also want to thank Sondre Aanes, Jørund Gåsemyr, Bent Natvig and Ida Scheel for valuable discussions of the model and comments, statistics for innovation (SFI²) for funding and PINRO for allowing the authors to use the observations of the 0-group of cod. The authors are also very thankful for comments from two anonymous reviewers which led to a much improved article.

Appendix

Night effect

The night effect has been observed by fishermen to be particularly strong in the time of the year when there is neither midnight sun nor polar nights. To accommodate for the night effect we thereby distinguished between two periods in the year named the transient period and the stationary period. The stationary period was defined as the period where there was either midnight sun or polar night, and the transient period was defined as the complement of the stationary period. We then introduced two indicator variables, one defines the transient/stationary period, and another defines day/night. To define the stationary period (and thereby also the transient period) we defined five reference points in the Barents Sea and adjunct waters were we know the stationary period (http://www.yr.no). The five reference points are: Rossoya (80.8°N), Hopen (76.5°N), Bjørnøya (74.5°N), Nordkapp (71.2°N) and Tromsø (69.6°N). We then approximated the stationary period in a location to be the same as in the closest reference point in latitude direction. Furthermore, we define that the trawl was done at night if the trawler started after 9 pm or ended in the period between midnight and 9 am.
Correlation

We now illustrate the similarities stated between the spatio-temporal interaction correlation structures (5) and (6). Let \( k > 0 \) be an integer. We have from (6) that:

\[
\text{cov}\left(\xi(s_1, r), \xi(s_2, r + k)\right) = \text{cov}\left(\xi(s_1, r), a\xi(s_2, r + k - 1) + \omega_{r+k}(s_2)\right)
\]

\[
= a\text{cov}\left(\xi(s_1, r), \xi(s_2, r + k - 1)\right)
\]

\[
= a^k\text{cov}\left(\xi(s_1, r), \xi(s_2, r)\right)
\]

\[
= \sigma_\omega^2 \exp\left(\ln(C(||s_1 - s_2||)) + k\ln(a)\right),
\]

(9)

where \( C(\cdot) \) is the Matern correlation function with \( \nu = 1 \), \( \sigma_\omega^2 \) is the marginal variance of the corresponding Matern covariance function and \( ||s_1 - s_2|| \) is the distance between \( s_1 \) and \( s_2 \) in kilometers. The similarities stated between (5) and (6) are now easily seen.

Priors

The noninformative priors for the hyperparameters used in this research are given in Table 8. The gamma distribution used has the parametrization:

\[
\pi(x|\alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} \exp(-\beta x).
\]

(10)

INLA by default uses an improper prior for the intersect regression coefficient and a \( N(0, 1000) \) prior for the other regression coefficients.
References


Fiskeridirektoratet (2005), ‘Regulations concerning fishing in the sea (in Norwegian)’. **URL:** https://lovdata.no/dokument/SF/forskrift/2004-12-22-1878


URL: http://www.R-project.org


Table 1: *Data used, numbers in parentheses are 90% coverage intervals.*

<table>
<thead>
<tr>
<th>Data</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target catch</td>
<td>Shrimp catch varies between 2.4 kilogram and 17.7 tons (20,3190)</td>
</tr>
<tr>
<td>Bycatch</td>
<td>The number of cod varies between 0 and 35775 cod (0,1008)</td>
</tr>
<tr>
<td>Time</td>
<td>Information about time of catch down to minutes scale</td>
</tr>
<tr>
<td>Location</td>
<td>Information of catch location (single point) given in longitude and latitude</td>
</tr>
<tr>
<td>Distance trawled</td>
<td>The distance trawled in nautical miles (2.5, 15)</td>
</tr>
<tr>
<td>Number of trawls</td>
<td>The number of trawls varies between one, two or three.</td>
</tr>
<tr>
<td>Circumference</td>
<td>The number of meshes around the opening of each trawl (1400, 3000)</td>
</tr>
<tr>
<td>Temperature</td>
<td>Bottom sea temperature (0.17, 9.3)</td>
</tr>
<tr>
<td>Depth</td>
<td>Ocean depth at catch location (227, 410)</td>
</tr>
<tr>
<td>0-group</td>
<td>Abundance predictions of 0-group cod per square nautical mile (0, 465408)</td>
</tr>
</tbody>
</table>

Table 2: *Covariates in the model*

<table>
<thead>
<tr>
<th>Covariates</th>
<th>Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seasonal effect</td>
<td>Continuous</td>
<td>Fourier series (<a href="#">3</a>)</td>
</tr>
<tr>
<td>0-group</td>
<td>Continuous</td>
<td>Logarithm of 0-group abundance of cod</td>
</tr>
<tr>
<td>Temperature</td>
<td>Continuous</td>
<td>Bottom sea temperature</td>
</tr>
<tr>
<td>Depth</td>
<td>Continuous</td>
<td>Ocean depth at catch location</td>
</tr>
<tr>
<td>Time</td>
<td>Continuous</td>
<td>Linear covariate of time</td>
</tr>
<tr>
<td>$Z(s,t)$</td>
<td>Continuous</td>
<td>Logarithm of shrimp catch per nautical mile</td>
</tr>
<tr>
<td>Area of trawl</td>
<td>Continuous</td>
<td>The square of the circumference</td>
</tr>
<tr>
<td>Number of trawls</td>
<td>Categorical</td>
<td>The number of trawls used</td>
</tr>
<tr>
<td>Night effect</td>
<td>Categorical</td>
<td>See appendix</td>
</tr>
</tbody>
</table>

Table 3: *Estimates and 95% credibility intervals of the significant regression coefficients.*

<table>
<thead>
<tr>
<th>Covariate</th>
<th>Shrimp catch Mean</th>
<th>Shrimp catch 95% C.I.</th>
<th>Bycatch of cod Mean</th>
<th>Bycatch of cod 95% C.I.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>3.01</td>
<td>(2.49, 3.51)</td>
<td>$\mu$</td>
<td>0.52</td>
</tr>
<tr>
<td>night effect</td>
<td>-0.41</td>
<td>(-0.52, -0.31)</td>
<td>night effect</td>
<td>-0.26</td>
</tr>
<tr>
<td>area (standardized)</td>
<td>0.10</td>
<td>(0.065, 0.15)</td>
<td>depth (standardized)</td>
<td>-0.17</td>
</tr>
<tr>
<td>depth (standardized)</td>
<td>0.085</td>
<td>(0.060, 0.11)</td>
<td>double trawl</td>
<td>0.28</td>
</tr>
<tr>
<td>double trawl</td>
<td>0.59</td>
<td>(0.51, 0.67)</td>
<td>$Z$</td>
<td>0.24</td>
</tr>
<tr>
<td>triple trawl</td>
<td>1.16</td>
<td>(0.93, 1.39)</td>
<td>0-group</td>
<td>0.37</td>
</tr>
</tbody>
</table>
Table 4: DIC, CVD, ML and MSE values for the shrimp models and bycatch models. S, T and S-T represent spatial, yearly and spatio-temporal effects, respectively. For the spatio-temporal effect, model (5) with $q = 1$ is used, if not otherwise specified. The models with bold text correspond to the selected models.

<table>
<thead>
<tr>
<th>Random effects</th>
<th>Shrimp model</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>No random effects</td>
<td>21162 -38164 -38326 1.06</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>18627 -36915 -37206 0.752</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S-T</td>
<td>14026 -35231 -36007 0.506</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S and S-T</td>
<td>13504 -35145 -35881 0.493</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S, S-T and T</td>
<td>13509 -35145 -35881 0.493</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S and S-T with $q = 2$</td>
<td>14637 -35285 -35986 0.505</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S and S-T with eq. (6)</td>
<td>15328 -35460 -36088 0.522</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bycatch model</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>No random effects</td>
<td>55086 -27543 -27689 1.91</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>53651 -26822 -27054 1.57</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S-T</td>
<td>48036 -24239 -25142 0.779</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S and S-T</td>
<td>48018 -24194 -25076 0.767</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S, S-T and T</td>
<td>47955 -24187 -25076 0.765</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S, S-T with $q = 2$ and T</td>
<td>48320 -24277 -25165 0.785</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S, S-T with eq. (6) and T</td>
<td>49105 -24571 -25331 0.846</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5: Estimates and 95% credibility intervals of the hyperparameters.

<table>
<thead>
<tr>
<th>Shrimp catch</th>
<th>Bycatch of cod</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hyperparameter</td>
<td>Mean</td>
</tr>
<tr>
<td>$\alpha_0$</td>
<td>0.97</td>
</tr>
<tr>
<td>$\kappa_0$</td>
<td>0.011</td>
</tr>
<tr>
<td>$\sigma_\gamma$</td>
<td>0.23</td>
</tr>
<tr>
<td>$\sigma_\gamma$</td>
<td>0.62</td>
</tr>
<tr>
<td>$\theta_t$</td>
<td>71 (mode) unknown</td>
</tr>
<tr>
<td>$\theta_s$</td>
<td>63 (mode) unknown</td>
</tr>
<tr>
<td>$\sigma_v^2$</td>
<td>0.20</td>
</tr>
</tbody>
</table>

Table 6: Predicted bycatch ratios in the Hopen area 1st. of December 2005.

<table>
<thead>
<tr>
<th>New obs.</th>
<th>Simple model</th>
<th>Aldrin et al. (2012)</th>
<th>Our model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pred. 90%C.I.</td>
<td>Pred. 90%C.I.</td>
<td>Pred. 90%C.I.</td>
<td>Pred. 90%C.I.</td>
</tr>
<tr>
<td>0</td>
<td>1.3 (0.1, 4.7)</td>
<td>1.9 (0.2, 5.5)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1.8 (0.3, 7.1)</td>
<td>2.8 (0.5, 7.5)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>4.2 (0.9, 24.8)</td>
<td>4.9 (1.1, 12.8)</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>3.5 (0.6, 10.1)</td>
<td>4.5 (1.2, 10.3)</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>4.6 (1.3, 9.5)</td>
<td>4.5 (1.7, 9.0)</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>4.2 (1.8, 6.4)</td>
<td>4.7 (2.0, 9.0)</td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>4.4 (2.2, 7.5)</td>
<td>4.8 (2.1, 9.1)</td>
<td></td>
</tr>
</tbody>
</table>
Table 7: Coverage of 90% and 99% prediction intervals for the shrimp catch, bycatch and bycatch ratio. The coverage is defined as the percentage of times the prediction intervals overlap with the real observations when removing and predicting every tenth trawl haul observation.

<table>
<thead>
<tr>
<th>Target</th>
<th>Inside P.I.</th>
<th>Under P.I.</th>
<th>Over P.I.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shrimp 90%</td>
<td>90.6%</td>
<td>5.7%</td>
<td>3.7%</td>
</tr>
<tr>
<td>90% Bycatch</td>
<td>90.6%</td>
<td>4.6%</td>
<td>4.8%</td>
</tr>
<tr>
<td>Ratio</td>
<td>92.4%</td>
<td>3.7%</td>
<td>3.8%</td>
</tr>
<tr>
<td>Shrimp 99%</td>
<td>97.6%</td>
<td>1.9%</td>
<td>0.5%</td>
</tr>
<tr>
<td>99% Bycatch</td>
<td>97.7%</td>
<td>1.1%</td>
<td>1.2%</td>
</tr>
<tr>
<td>Ratio</td>
<td>98.4%</td>
<td>1.0%</td>
<td>0.6%</td>
</tr>
</tbody>
</table>

Table 8: Prior distributions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior</th>
<th>Parameter</th>
<th>Prior</th>
</tr>
</thead>
<tbody>
<tr>
<td>log(σ²)</td>
<td>N(0,10)</td>
<td>1/σ²</td>
<td>gamma(1,0.00005)</td>
</tr>
<tr>
<td>log(κ)</td>
<td>N(0,10)</td>
<td>1/σγ²</td>
<td>gamma(1,0.00005)</td>
</tr>
<tr>
<td>1/σ²</td>
<td>gamma(1,0.00005)</td>
<td>θl and θs</td>
<td>∝ 1</td>
</tr>
<tr>
<td>1/σγ²</td>
<td>gamma(1,0.00005)</td>
<td>log(1+a)</td>
<td>N(0,10⁻¹⁵)</td>
</tr>
</tbody>
</table>

Figure 1: Map of the Barents Sea with observations of shrimp trawls represented as red dots. Blue triangles indicate observations that have been removed from the original data. The polygon described by the black lines in the middle of the Barents Sea illustrates the Hopen area where we estimate the bycatch ratio in the decision making section.
Figure 2: Map of locations in the Barents Sea containing estimates of the 0-group of cod in four different years.

Figure 3: The triangulation grid used for approximating the Matern covariance function of the spatial effect of shrimp catch and bycatch of juvenile cod.
Figure 4: The seasonal effect with 95% credibility intervals for (a) the bycatch in Lyngen (south in the area investigated), (b) the bycatch in Hopen (north in the area investigated) and (c) the shrimp catch.

Figure 5: The spatial effect of (a) bycatch of juvenile cod and (b) the shrimp catch. Both maps are given in UTM coordinates.
Figure 6: Estimated bycatch ratio in the Hopen area after 1st of December 2005 based on information collected before 6th of December 2005. The solid black line shows the mean and the dotted red lines 90% credibility intervals. The blue solid horizontal line gives the upper limit of allowed bycatch ratio.

Figure 7: Predictions with 90% prediction intervals of bycatch ratio (7) of the trawl hauls taken each month in the Hopen area in 2005 and 2006. The blue solid horizontal line shows the upper limit of allowed bycatch ratio. The red crosses are the observed bycatch ratios.