A STOCHASTIC TIME AND SPACE MODEL FOR EARTHQUAKES

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Abstract

A stochastic model for earthquake occurrence focusing on the spatio-temporal interactions between earthquakes is discussed. The model is a marked point process model in which each earthquake is represented as a marked point in space and time. The marks are given by the magnitudes of the earthquakes but other observed properties of the earthquakes, such as information on the fault lines, can straightforwardly be included. The parameters of the model is estimated based on Bayesian updating of priors, using empirical data to derive posterior distributions. In the model we discuss the spatial and temporal dependencies between fore- and aftershocks. In addition the effect of strain build-up and subsequent release following an earthquake is discussed. An algorithm for simulating earthquakes from the model is presented along with simulation results for the region of Southern California. With an improved set of simulations the ambition of the model is to make more precise predictions on the occurrence of earthquakes. The prediction results may give clues as to whether such predictions of earthquakes is at all possible.

1 INTRODUCTION

Earthquake forecasting in the strict sense with the exact prediction of the time, the location, and the magnitude of an earthquake has been a difficult area of research for several decades. One outcome of this research, however, is that we today know much more about why earthquake prediction is difficult (Kagan, 1997). This difficulty is in part tied to concepts such as self-similarity, criticality and nucleation processes: All earthquakes start small, and while we know much about the limits to growth, we do not know in sufficient details when and why it stops before that.

In this paper we outline a stochastic model for earthquake occurrence which is focusing on the spatio-temporal interactions between earthquakes. We believe that by including the increased knowledge of earthquake processes in more advanced stochastic models the prediction capabilities for earthquakes can be improved. The model can be extended to use more extensive catalogs (including lower magnitudes), and other geophysical and geological data. This may improve predictions, particularly predictions over short periods of time. The model is a marked point process model (Cressie, 1993), in which each earthquake is represented by its magnitude and coordinates in space and time.

There are many possible parametrisations for the model. The principles behind the estimation and algorithms are independent of a particular parametrisation, however. Another freedom of the model is the choice of prior distributions. If the choice of priors turn out to be controversial it is always possible to choose flat priors that, however, give less information with the subsequent risk of smaller precision in the predictions. The model is based on Bayesian approaches with user specified prior distributions for all parameters, while empirical data are used for deriving posterior distributions.

2 MARKED POINT PROCESS MODEL

Marked point processes are commonly used stochastic models for representing a finite number of events located in time and space. Earthquakes can very well be fitted into a marked point process model. Each earthquake has, in addition to a location in time and space, parameters representing the magnitude and quite often also information about the earthquake fault lines. Point process models for earthquakes have previously been discussed by Vere-Jones (1995) and Ogata (1998). The model presented in this paper treats fore- and aftershocks in a similar fashion to Ogata. In addition, the model takes into account the effect of strain build-up. The ultimate goal is to include as much as possible of known physical processes into the model.

2.1 THE MODEL

In our notation an earthquake is represented by $E = (x, M)$ and t, where $x = (x_1, x_2)$ is the epicentre coordinates (x_1 = longitude,x_2 = latitude), M is the moment magnitude, and t is the time. An earthquake catalogue $H_T = \{(E_i, t_i)\}_{t_i<T} = \{(x_i, M_i, t_i)\}_{t_i<T}$ consists of all observed earthquakes above a certain magnitude in a specified region, and in a given time period (T_0, T).

The two major assumptions made in the proposed model are:

- The intensity $\lambda_1(E, t|H_t, \beta)$ of an earthquake $(E, t) = (x, M, t)$ is a function of previous earthquakes H_t in the region and some parameters β to be determined by a Bayesian updating. If additional data or physical knowledge is available, this should be included in this intensity.
- The time averaged intensity $\lambda(E) = \lambda(x)\lambda(M|x)$ as a function of magnitude and position is known. This can be estimated without using H_T. The model would benefit from including a Bayesian updating of the time averaged intensity, but this would increase the number of parameters and hence the CPU time considerably.

It is natural to let $\lambda(M|x)$ be determined by the well-known Gutenberg-Richter law (Vere-Jones, 1995) for the distribution of magnitudes such that $\lambda(M|x) \propto 10^{a-bM}$, with a and b constants. The value of the scaling parameter b is usually in the interval (0.7, 1.2). We here assume that the intensity λ_1 is given by the following form:

$$\lambda_1(E, t|H_t, \beta) = \lambda_2(E|\beta)(\lambda_3(E, t|H_t, \beta) + \lambda_4(E, t|H_t, \beta)), \tag{1}$$

where λ_2 is a scale factor independent of time, λ_3 represents the increase in the intensity after an earthquake used for modelling the fore- and aftershocks, and λ_4 represents the release of strain following an earthquake. If the release of strain is omitted, λ_4 should be replaced by 1, while if the fore- and aftershock treatment is omitted, λ_3 should be replaced by 0.

The intensity λ_3 is used to model the fore- and aftershocks. Let M_i be the magnitude of a shock in the catalogue at the time t_i, and M the magnitude of a subsequent shock. Foreshocks M_i are then modelled by $\lambda_3(E, t|H_t, \beta) > 0$ for earthquakes $M > M_i$ for $t > t_i$, while aftershocks M are modelled by $\lambda_3(E,t|H_t,\beta) > 0$ for earthquakes $M < M_i$ for $t > t_i$. We will assume that λ_3 has the form

$$\lambda_3(E,t|H_t,\beta) = \sum_{(E_i,t_i)\in H_t} g(E,t,E_i,t_i,\beta), \tag{2}$$

where

$$g(E,t,E_i,t_i,\beta) = \beta_1 g_1(M,M_i,\beta_2)\, g_2(t,t_i,\beta_3,\beta_4)\, g_3(x,x_i,\beta_5).$$

The functions g_1, g_2, and g_3 represent magnitudial, temporal, and spatial effects, respectively. Note that the summation implies that if there is a big earthquake followed by a series of smaller earthquakes all of these earthquakes contribute to the intensity. A typical form of g_1 is $g_1(M,M_i,\beta_2) = \exp(\beta_2 M_i)$, which gives both fore- and aftershocks. This gives the same magnitude distribution as the time averaged intensity. For the temporal effect we assume that $g_2(t,t_i,\beta_3,\beta_4) = 1/(t - t_i + \beta_4)^{\beta_3}$. The spatial effect can be represented by a function based on the distance between the epicentres, *i.e.*, $g_3(x,x_i,\beta_5) = \exp(-\beta_5||x - x_i||^2)$.

It seems to be generally accepted that there is more regularity in the occurrence of earthquakes than can be accounted for in a Poisson model (Working Group on California Earthquakes Probabilities, 1995). The assumption is that in any particular region, strain is slowly building up and then released during to earthquakes. This effect can be incorporated into a point process model. We first define a state variable S that can be connected to strain. The interpretation of S may be different than the standard definition of strain but this will be its general nature. For simplicity we here refer to S as strain. We define S by

$$S(x,t,H_t,\beta) = \phi(x,\beta)\, t - \sum_{(E_i,t_i)\in H_t} h(x,E_i,\beta),$$

with

$$\phi(x,\beta) = \int h(x,E',\beta)\lambda(E')dE',$$

$$h(x,E',\beta) = \exp(\beta_7 M')\exp(-\beta_8||x - x'||^2).$$

Here ϕ represents the average strain build-up per unit time and h the release in strain for each earthquake. The strain release h is factored into two terms related to the magnitude of the earthquake and a spatial effect, respectively. Thus, S represents the strain at any point (x, t) in space and time given all the previous earthquakes contained in the catalog H_t. It is assumed that S builds up linearly and then decreases instantaneously with each earthquake. The strain has a variance that is independent of time, and λ_4 is an increasing function of S given by

$$\lambda_4(E, t|H_t, \beta) = 1 + \beta_6 S + (\beta_6 S)^2/2. \tag{3}$$

The quadratic term $\propto (\beta_6 S)^2$ ensures that the intensity λ_4 always is positive, no matter the value of β_6 and the fact that S can be negative. The effect of λ_4 is to reduce the variability in the time periods between very big earthquakes compared to the simple Poisson model. The variability in the time periods between big earthquakes becomes smaller and smaller with increasingly large values of β_6 and β_7. The parameter β_8 specifies the surrounding region of an earthquake in which strain is released.

2.2 POSTERIOR DISTRIBUTIONS FOR THE PARAMETERS

From the real catalogue H_T of the period (T_0, T) it is possible to find the posterior distributions of the parameters β. These posterior distributions represent the best guesses for the parameters and should be used in all predictions. The posterior distributions for the parameters β, given the data in the catalog H_T, are defined by the equation $f(\beta|H_T) \propto f(\beta)f(H_T|\beta)$. The likelihood $f(H_T|\beta)$ can be calculated from

$$f(H_T|\beta) = \exp\left(-\int_{T_0}^{T}\int \lambda_1(E, t|H_t, \beta)dEdt\right)\prod_{i=1}^{n}\lambda_1(E_i, t_i|H_{t_i}, \beta) \approx \prod_{i=1}^{n}\lambda_1(E_i, t_i|H_{t_i}, \beta), \tag{4}$$

where the integral can be approximated by a constant. The first factor is due to periods (t_{i-1}, t_i) without earthquakes, while the second factor represents the intensities for the actual earthquakes.

2.3 RESULTS

The empirical data we have used is based on an earthquake catalogue over the time span 1932 – 1998 compiled by the Southern California Earthquake Center, SCEC (1999) which covers the region of Southern California as shown in Fig. 1.

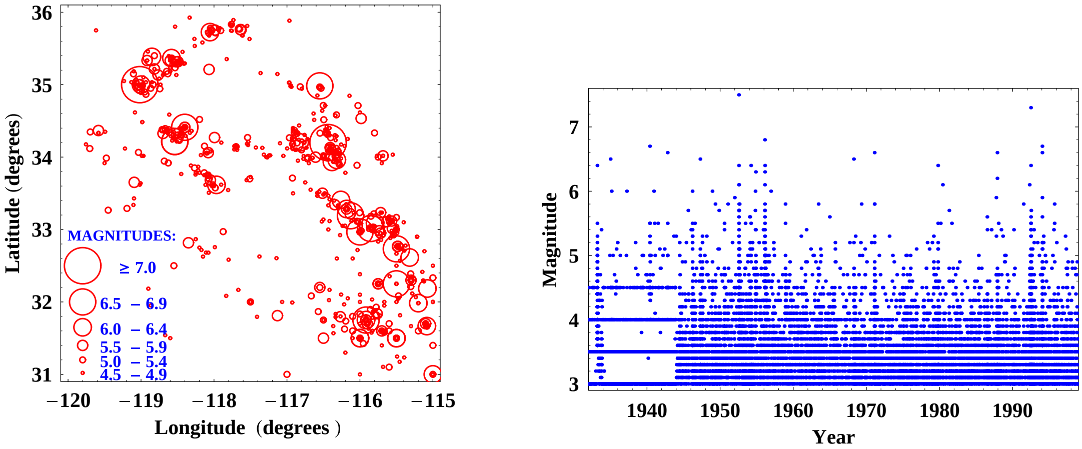


Figure 1: The location of earthquakes with $M \geq 4.5$ (left), and the magnitude vs. time for earthquakes with $M \geq 3.0$ (right) in Southern California in the period 1932 – 1998.

As a first step we have simulated earthquakes from the Poisson model, *i.e.*, setting $\lambda_1(E,t) = \lambda(E)$. The 5×5 degree area is divided into 1600 grid cells, with each cell corresponding to a size of about 14×12 km. The intensity $\lambda(E)$ for each grid cell is calculated from the empirical data. An average b-value of 0.93 for the Gutenberg-Richter relation is estimated from the same data. The simulation results for a 10-year period are shown to the right in Figs. 2 and 3. To the left in these figures are shown the observed data for the 10-year period 1989–1998. The simple Poisson model provides us with a constant background intensity. In the marked point process model, the spatio-temporal interactions between earthquakes will also be included.

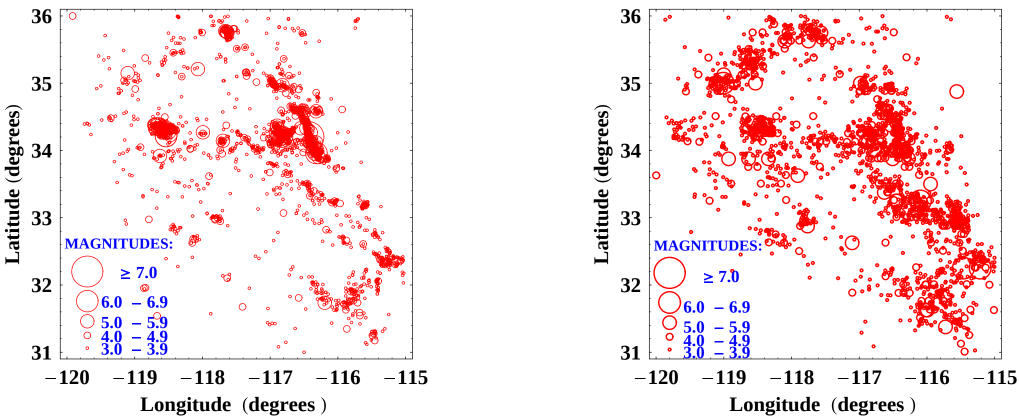
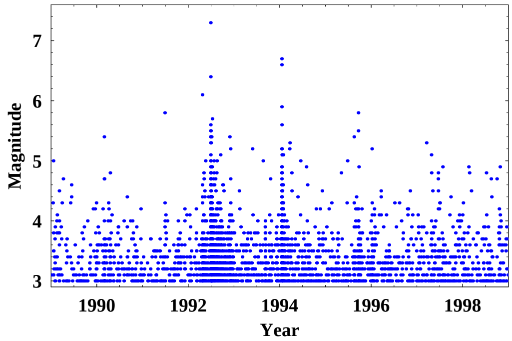


Figure 2: Earthquakes of $M \geq 3.0$ in the period 1989 – 1998 in Southern California (left), and simulated earthquakes over a 10-year period using the Poisson model (right).



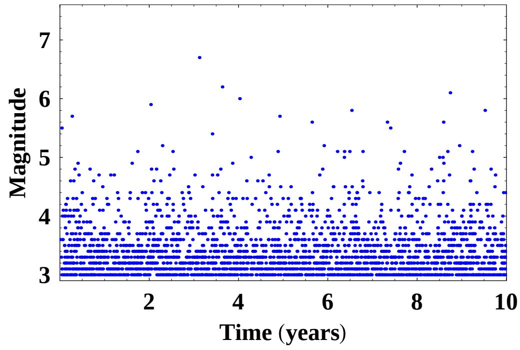


Figure 3: Magnitude vs. time of actual earthquakes in the period 1989 – 1998 (left), and simulated earthquakes over a 10-year period (right) in Southern California.

At the next step we have implemented an algorithm for estimating the maximum likelihoods of the β-parameters. In the first approximation we have assumed that $\lambda_2(E|\beta)$ is a constant. Using the 50-year period 1949–1998 of the catalogue, a calculation of the maximum likelihoods yields $\beta_1 = 160.8, \beta_2 = 0.8628, \beta_3 = 1.191, \beta_4 = 0.01662$, and $\beta_5 = 1242$ when the strain build-up is omitted and $\lambda_4 = 1$. An interpretation of this is that, compared to the background activity, the increase of intensity due to an earthquake of $M = 5.8$ is twice that of a $M = 5.0$ and that the increased intensity is halved at a distance of 2.6 km from the epicentre of the earthquake. It is also implied that increase of intensity is halved 19 min after an earthquake. Including the building-up of strain for earthquakes of magnitudes $M \geq 6.0$ the calculation yields $\beta_1 = 207.2, \beta_2 = 0.8633, \beta_3 = 1.196, \beta_4 = 0.01613, \beta_5 = 1307, \beta_6 = 1.722, \beta_7 = 0.5703$, and $\beta_8 = 3.596$. With these estimates it is now possible to include both the interaction term λ_3 and the strain release term λ_4 in the simulation model. The ultimate goal is to perform a large number of simulations in order to make predictions.

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