ADAPTIVE CHAINS

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ABSTRACT Adaptive chains are chains that are able to learn from all previous elements in the chain. It is an extension of Markov chains. It is proved convergence of adaptive chains that satisfies a strong Doeblin condition (i.e., the transition density r from x_i to y satisfies $r(y, x_1, x_2, \dots, x_i) \geq a_i \pi(y)$ for all $x_1, \dots, x_i, y$ in the state space.) By using the previous iterations of the adaptive chain, it is possible to increase a_i which will improve convergence compared with Markov chains. It is also proved a decreasion rate in the covariance between element x_i and x_{i+j} as j increases.

The results may also be applied on regeneration chains where only the history before the last regeneration is used. Particularly interesting is the adaptive Metropolis-Hastings algorithm. Adaptive simulated annealing is also described and convergence is proved when the temperature decreases proportional with $M/\log i$. The convergence is due to contraction properties of integral operators with a stationary distribution and that satisfies a strong Doeblin condition. The algorithm is particularly useful when it is necessary with many samples from the same distribution like in Bayesian estimation, and in applications where it is very expensive to calculate the limiting density like inverse problems and optimisation.

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