Recovery of surfaces of revolution from range and intensity images

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Abstract

The problem of estimating a surface of revolution from a noisy and sparse range image is considered. It is assumed that the axis of revolution is vertical, the surface is viewed from above, and the surface consists of a fixed number of horizontal segments. A method for estimating the surface based on an image model formulated in a Bayesian framework is described. Experiments on simulated and real data are presented. The real images were acquired by a sensor producing both range and intensity measurements. Inclusion of intensity data in the model improved estimation accuracy.

1 Introduction

Recovery of surfaces from range images has been an active research area for several years. In this paper, we consider very simple surfaces, namely surfaces of revolution consisting of a fixed number of horizontal segments. Such surfaces can often be described by a small number of parameters, and therefore high estimation accuracy can be expected even in cases when the image is noisy and sparse. The present author became interested in this estimation problem through a research project on classification of empty bottles.

We attempt to answer the following questions.

1. Given an estimator, how is estimation accuracy related to resolution, noise, and the true values of the parameters?
2. How should range and intensity images be combined in order to obtain satisfactory estimation accuracy?

Efrat and Gotsman (1994) presented analytic bounds on the accuracy that can be achieved on estimates of disk parameters from noise-free digital images. One of the first papers on the use of range and intensity data was Duda *et al.* (1979). A more recent paper on this issue is Zhang and Wallace (1993).

In Section 2, we describe our basic model assumptions and explain briefly how the surfaces can be recovered. In Section 3, we present experimental results on simulated and real data. Concluding remarks are in Section 4.

2 Methodology

In this section, we describe a method for recovering a surface of revolution. The method is based on a statistical image model formulated in a Bayesian framework. It is assumed that the axis of revolution is vertical, and we consider surfaces that can be represented by $\theta = (r, c, x)$ where $r = (r_1, r_2, \ldots, r_n)$, r_j's are nonnegative numbers such that $r_1 < r_2 < \ldots < r_n$, c is the location of the axis, $x = (x_1, x_2, \ldots, x_{n-1})$, x_j is the height at the points where the distance to the axis is between r_j and r_{j+1}, and n is a fixed number. We want to estimate θ from image data.

Let R_0 be the disk in the plane with radius r_1 and centre c. For $j = 1, 2, \ldots, n-1$, let R_j be the set points in the plane where the distance to c is between r_j and r_{j+1}, and R_n is the set of points where the distance is greater than r_n. Let $t_1, t_2, \ldots, t_m$ be points in the plane not necessarily uniformly spaced. We let y_i denote the observed record at t_i and y the corresponding vector, interpreted as a realization of a random vector, $Y = (Y_1, Y_2, \ldots, Y_m)$. It is assumed that the Y_i's are conditionally independent given θ, and the Y_i's are identically distributed within each R_j. Hence, the conditional density of the observed records given the surface is

$$f(y|\theta) = \prod_{j=0}^{n} \prod_{t_i \in R_j} f_j(y_i|\theta).$$

In addition, we assume that $f_j(y_i|\theta) = f_j(y_i|x_j)$ for $j = 1, 2, \ldots, n-1$, $f_0(y_i|\theta) = f_j(y_i)$, and $f_n(y_i|\theta) = f_n(y_i)$.

Prior knowledge is often present on the parameters. For instance we might know that x_j is greater than x_k. Thus a Bayesian approach is appropriate. We assume that the prior density π can be written as

$$\pi(\theta) = \pi_r(r)\, \pi_c(c)\, \pi_x(x).$$

Let $\hat{\theta}$ be the maximum a posteriori estimate of θ. $\hat{\theta}$ is our choice of inference about θ. We explain briefly how $\hat{\theta}$ can be computed. Let $\tilde{x}(r, c)$ be the vector x that maximizes $f(y|r, c, x)\, \pi_x(x)$ for a given choice of (r, c). It is often easy to find $\tilde{x}(r, c)$. For instance, if f_j is univariate Gaussian with expectation x_j, and π_x is uniform, $\tilde{x}_j$ is simply the average of the Y_i's from R_j. It follows that

$$\hat{\theta} = (r^\star, c^\star, \tilde{x}(r^\star, c^\star))$$

where $(r^\star, c^\star)$ maximizes the function L defined by

$$L(r, c) = f(y|(r, c, \tilde{x}(r, c)))\, \pi_r(r)\, \pi_c(c)\, \pi_x(\tilde{x}(r, c)).$$

In this paper, the domain of L is taken to be a continuum. Because the set of pixels is discrete it follows that L is piecewise constant. Hence $(r^\star, c^\star)$ is not unique. However, if the set of pixels is large, the diameter of the set of possible $(r^\star, c^\star)$ is small. One of the maximum points of L is selected. In order to minimize L we apply the technique of simulated annealing. We employed the routine called "amebsa" in Press *et al.* (1992).

An alternative estimator is the conditional expectation of θ given y. That approach may be pursued in the future.

	$\sigma = 1$	$\sigma = 10$
$r = 3$	0.08	0.53
$r = 7$	0.03	0.29

Table 1: *Standard deviation of various radius estimates.*

3 Experiments

3.1 Simulated data

Simulations were conducted in order to get an understanding of the estimation problem. We considered recovery of cylinders. A cylinder is a surface of revolution of the simplest kind. We tested the method on two cylinders. The height of each cylinder is 10, and the radii are 3 and 7, respectively. The height of the background is 0. Images of the cylinders were generated by sampling the scene at a 15×15-array of points and adding independent noise. The noise was Gaussian with zero expectation and standard deviation σ. We generated images with σ equal to 1 and 10.

We estimated the radius, the height of the cylinder, and the height of the background. As prior information we used that the height of the cylinder is greater than the height of the background, the radius is between 0.5 and 8.5, and the axis passes through a square with side length 2. The prior distribution was uniform. The standard deviation of the various radius estimators are shown in Table 1.

As expected, the estimation accuracy decreases as the noise level increases. Note also that we obtain higher accuracy for $r = 7$ than for $r = 3$. The same is true for the estimate of the height.

3.2 Real data

The performance of the method described in Section 2 was investigated on a set of empty plastic bottles without caps. The shape of the bottles is indicated in Fig. 1. When seen from above, a bottle consists of 4 segments: *bottom*, *peak*, *collar*, and *shoulder*. Two types of bottles were used in this study: *Coke* and *Pepsi*, and the data set consisted of 100 samples of each type.

Images of the bottles were acquired by a laser range finder. The resolution of the height measurements is 2mm. The analysis was restricted to circular regions in the image of radius 35mm containing a single bottle. The distance from a pixel to its closest neighbour is between 0.68mm and 0.71mm. However, because the pixels are not uniformly spaced, each region does not contain more than 1200 pixels.

Because the height is not measured properly on the *shoulder*, the slanting part of the surface, we were only interested in the upper part of the bottle consisting of *peak* and *collar*. Thus our goal was to estimate $r_1 =$ the distance from the axis to the peak, $x_1 =$ the height of the peak, $r_2 =$ the distance from the axis to the edge between the peak and

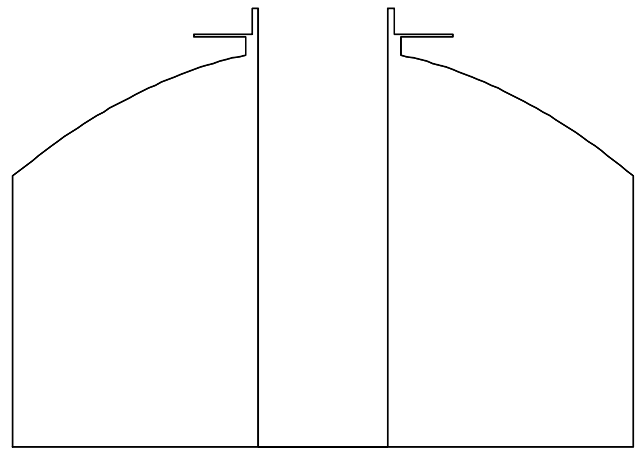

Figure 1: *Vertical cross section through a bottle.*

the collar, x_2 = the height of the collar, and r_3 = the distance from the axis to the outer boundary of the collar.

We used the prior distribution π given by $\pi(\theta) = \pi_r(r_1, r_2, r_3)\,\pi_x(x_1, x_2)\,\pi_c(c)$ where π_r is uniform on the set of (r_1, r_2, r_3) such that $7 \leq r_1 \leq 13$, $1 \leq r_2 - r_1 \leq 7.5$, and $3 \leq r_3 - r_2 \leq 11$, π_x is uniform on the set of (x_1, x_2) such that $x_1 - x_2 \geq 10$, and π_c is uniform on a square with side length 10. Throughout this section the length unit is mm.

In addition to height, the intensity of the reflected light was recorded. Most of the information concerning shape is contained in the height image. However, additional information is available since discontinuities in the surface cause discontinuities in the intensity image. Moreover, the quality of the height measurements deteriorates with decreasing intensity. The latter fact was incorporated in the image model. For simplicity, we assumed that the intensity distribution was the same for all segments, that is

$$f_j(y^{height}, y^{intensity}) = h_j(y^{height} | y^{intensity})\, g(y^{intensity})$$

for pixels in $R_j, j = 0, 1, 2, 3$. Note that it is not necessary to know g. The observed height was modelled as $Y^{height} = X + W$, where W is a Gaussian random variable with zero expectation and variance dependent on $Y^{intensity}$. For pixels in R_0 and R_3, X is a random variable uniformly distributed on $[0, 60]$ and $[0, 300]$, respectively, while $X = x_j$ for pixels in $R_j, j = 1, 2$.

The conditional variance of height given intensity was estimated by employing the *Nadaraya-Watson estimator*, see Nadaraya (1964) or Watson (1964). The estimate is based on measurements from surfaces where the height is known. The relationship is

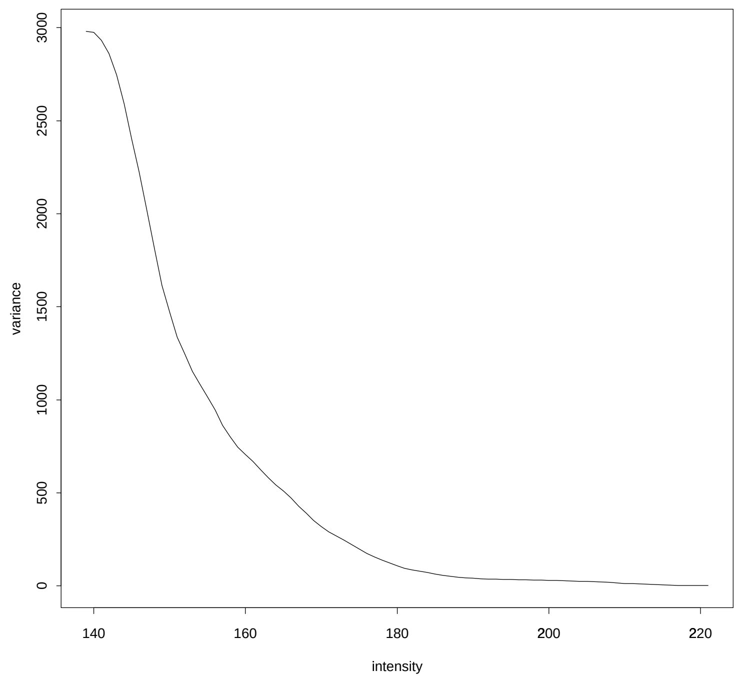

Figure 2: *Relationship between intensity and height variance.*

shown in Fig. 2. Note that the standard deviation is approximately 1mm for high intensity values and 50mm for low intensity values.

Figures 3- 4 show scatterplots of the data from a single bottle and indicate the values of the corresponding estimates. We see that the estimated surface fits the range data well.

We also attempted to estimate the surface parameters without utilizing the intensity data. Then we assumed that the height variance was constant. However, the results were poor.

The estimation results are summarized in Table 2. The standard deviation of the estimate of r_2 is relatively large. This is likely due to the fact that the intensity is low near the discontinuity between the *peak* and the *collar*, and hence the error of the height measurements near this discontinuity are large compared to the difference between x_1 and x_2. Note also that r_3 was more accurately estimated than r_1, and that the results for *Pepsi* were better than the results for *Coke*.

4 Concluding comments

In this paper, we have described a method for recovering a simple surface of revolution using a Bayesian approach. The method was tested on simulated images of cylinders and real range and intensity images of bottles. The performance depends on resolution and noise level.

Concerning the estimation of the cylinders, the absolute error seems to decrease as

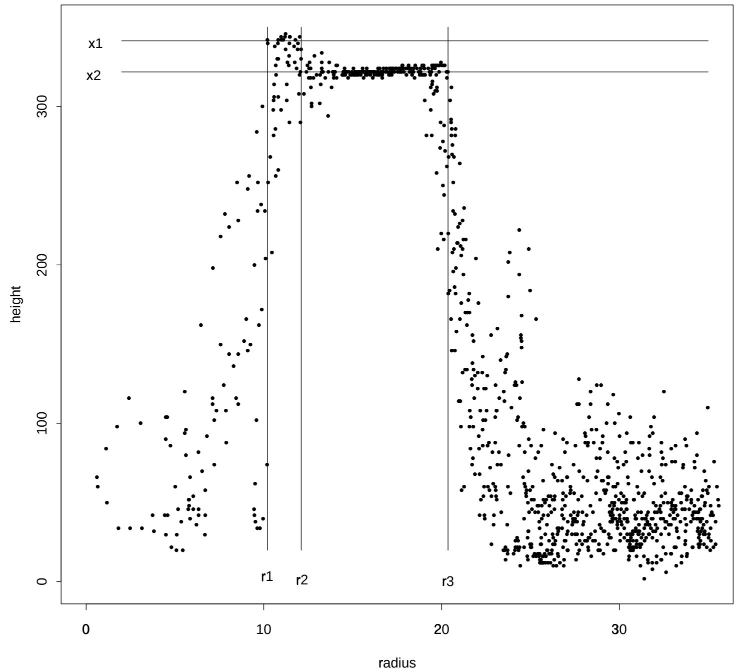

Figure 3: *Data from a single bottle. Distance to the estimated axis is plotted vs measured height. The lines indicate estimates of the parameters.*

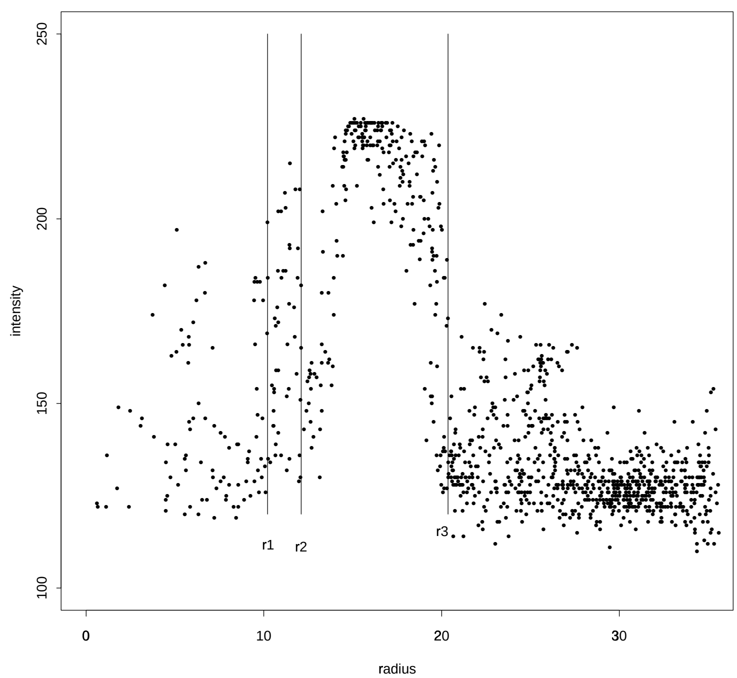

Figure 4: *Data from a single bottle. Distance to the estimated axis is plotted vs measured intensity. The vertical lines indicate estimates of the radii.*

	$r1$	$r2$	$r3$	$x1$	$x2$
Coke	9.9	12.6	20.5	336	321
	0.37	1.00	0.32	5.1	4.0
Pepsi	9.7	12.7	18.4	345	327
	0.34	0.78	0.29	3.9	3.6

Table 2: *Summary of the estimation results. Mean and standard deviation are shown for each parameter. The unit is mm.*

the radius increases. This might also be true for more general surfaces of revolution. The method does also work on real data provided that noise modelling is done properly. Modelling the height variance as a function of intensity is a fruitful approach. Further research is necessary in order to understand the impact of resolution, noise, and use of prior knowledge on the performance of the method.

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