Markov Mesh Simulations with Data Conditioning through Indicator Kriging

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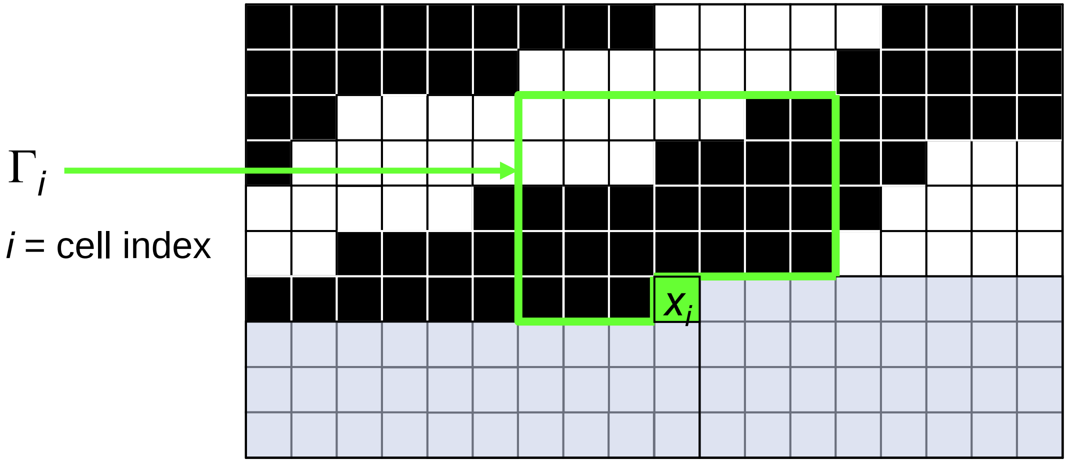
Markov mesh models

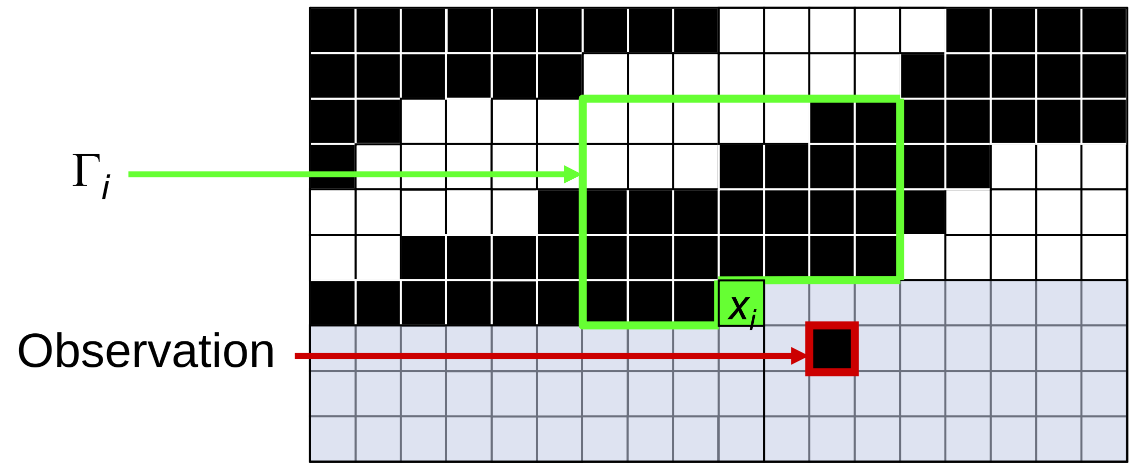


- Grid models
- Unilateral simulation
- Very fast
- Simulation probability:

$$P(x_i \mid \boldsymbol{x}_{j<i}) = P(x_i \mid \boldsymbol{x}_{j\in\Gamma_i})$$

(Colin Daly, Geostats 2004)

Conditioning in Markov mesh



If unconditioned:

$$P(x_i \mid \boldsymbol{x}_{j<i}) = P(x_i \mid \boldsymbol{x}_{j\in\Gamma_i})$$

For conditioning:

$$\text{wanted:} \quad P(x_i \mid \boldsymbol{x}_{j<i}, \boldsymbol{x}_w)$$

Our starting point:
- Unconditioned parametrized model exists
- Need method for conditioning

Main idea for data conditioning

$$P(x_i \mid \boldsymbol{x}_{j<i}, \boldsymbol{x}_w) = \frac{P(x_i \mid \boldsymbol{x}_{j<i}, \boldsymbol{x}_w)}{P(x_i \mid \boldsymbol{x}_{j<i})} P(x_i \mid \boldsymbol{x}_{j<i})$$

Approximate by indicator kriging

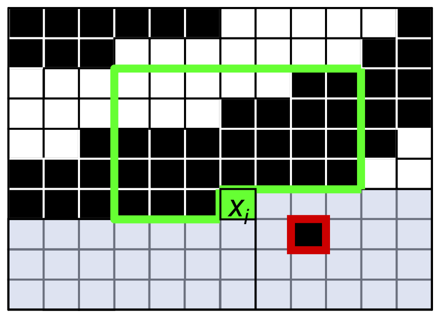
Use ordinary Markov mesh

$$P(x_i \mid \boldsymbol{x}_{j<i}, \boldsymbol{x}_w) \approx \frac{Z(x_i \mid \boldsymbol{x}_{j<i}, \boldsymbol{x}_w)}{Z(x_i \mid \boldsymbol{x}_{j<i})} P(x_i \mid \boldsymbol{x}_{j\in\Gamma_i}) \equiv \Psi(x_i \mid \boldsymbol{x}_{j<i}, \boldsymbol{x}_w) P(x_i \mid \boldsymbol{x}_{j\in\Gamma_i})$$

Two methods

Approximate

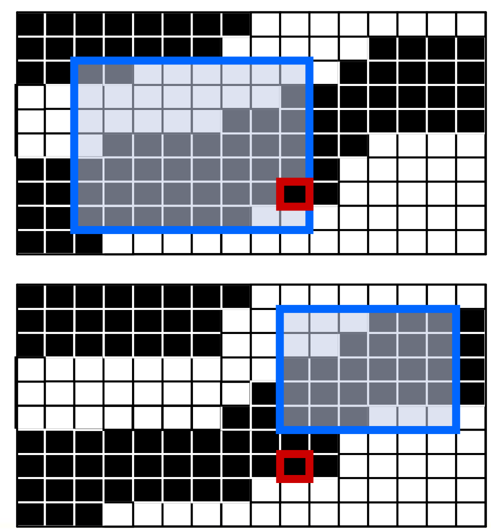
- unilateral



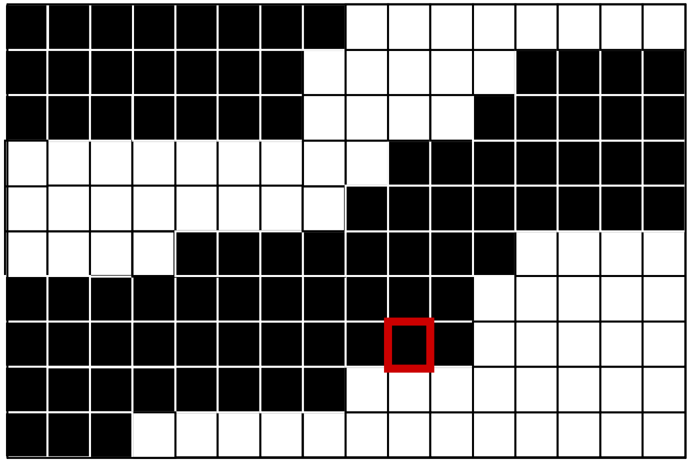
$$P(x_i \mid \boldsymbol{x}_{j<i}, \boldsymbol{x}_w) \approx \Psi(x_i \mid \boldsymbol{x}_{j<i}, \boldsymbol{x}_w) P(x_i \mid \boldsymbol{x}_{j\in\Gamma_i})$$

Accurate

- iterative McMC
- block update, Metropolis-Hastings



Iterations



For each iteration step

- let v denote existing grid configuration
- pick a set of grid cells Ω that are allowed to be changed in this step
- scan through these cells, assigning values according to the approximate algorithm; Γ_i, observations, edge cells
- gives proposal configuration μ

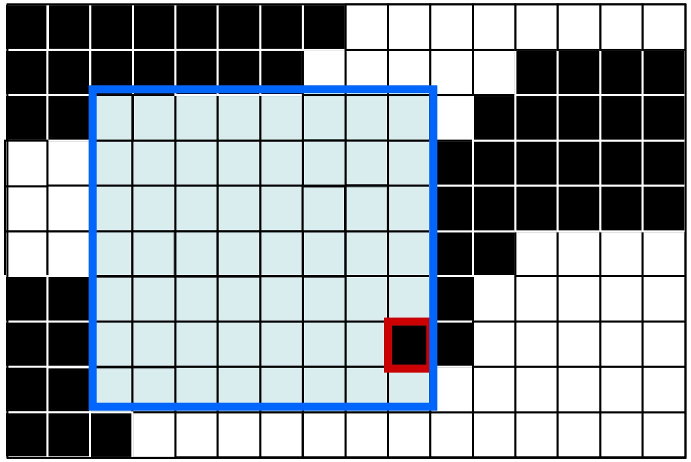
$$q_\mu = \prod_{i \in \Omega} \Psi(x_i \mid \boldsymbol{x}_{j<i}, \boldsymbol{x}_w, \boldsymbol{x}_{\text{edge}}) P(x_i \mid \boldsymbol{x}_{j \in \Gamma_i})$$

$$P_\mu = \prod_{\text{all } i} P(x_i \mid \boldsymbol{x}_{j \in \Gamma_i})$$

- accept new state with probability

$$\alpha = \min\left(\frac{q_v P_\mu}{q_\mu P_v}, 1 \right)$$

Iterations



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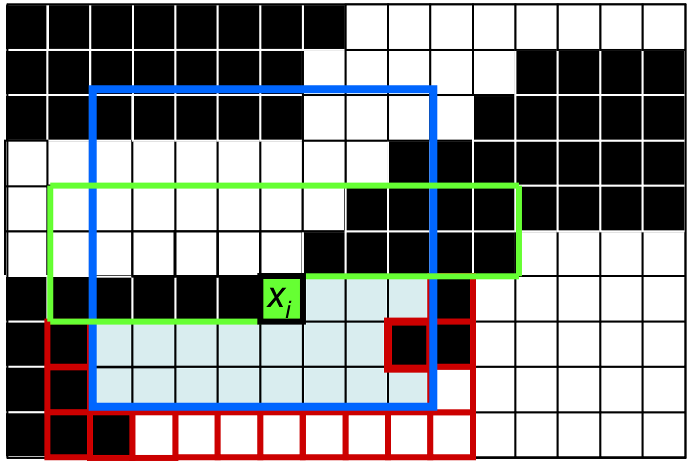
Iterations



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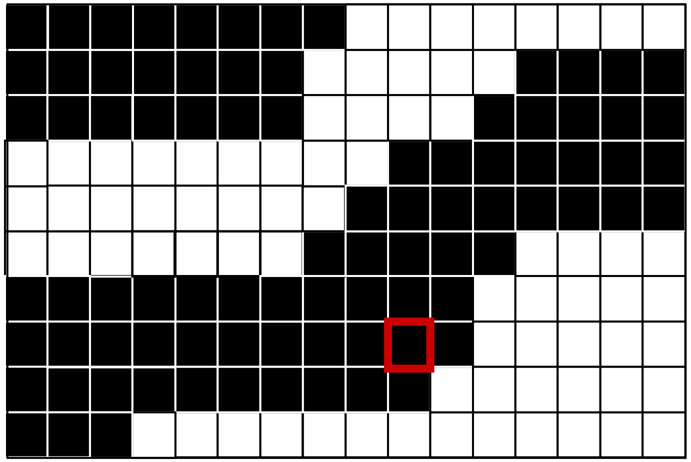

Each proposal grid configuration respects all observations

For each iteration step

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- pick a set of grid cells Ω that are allowed to be changed in this step
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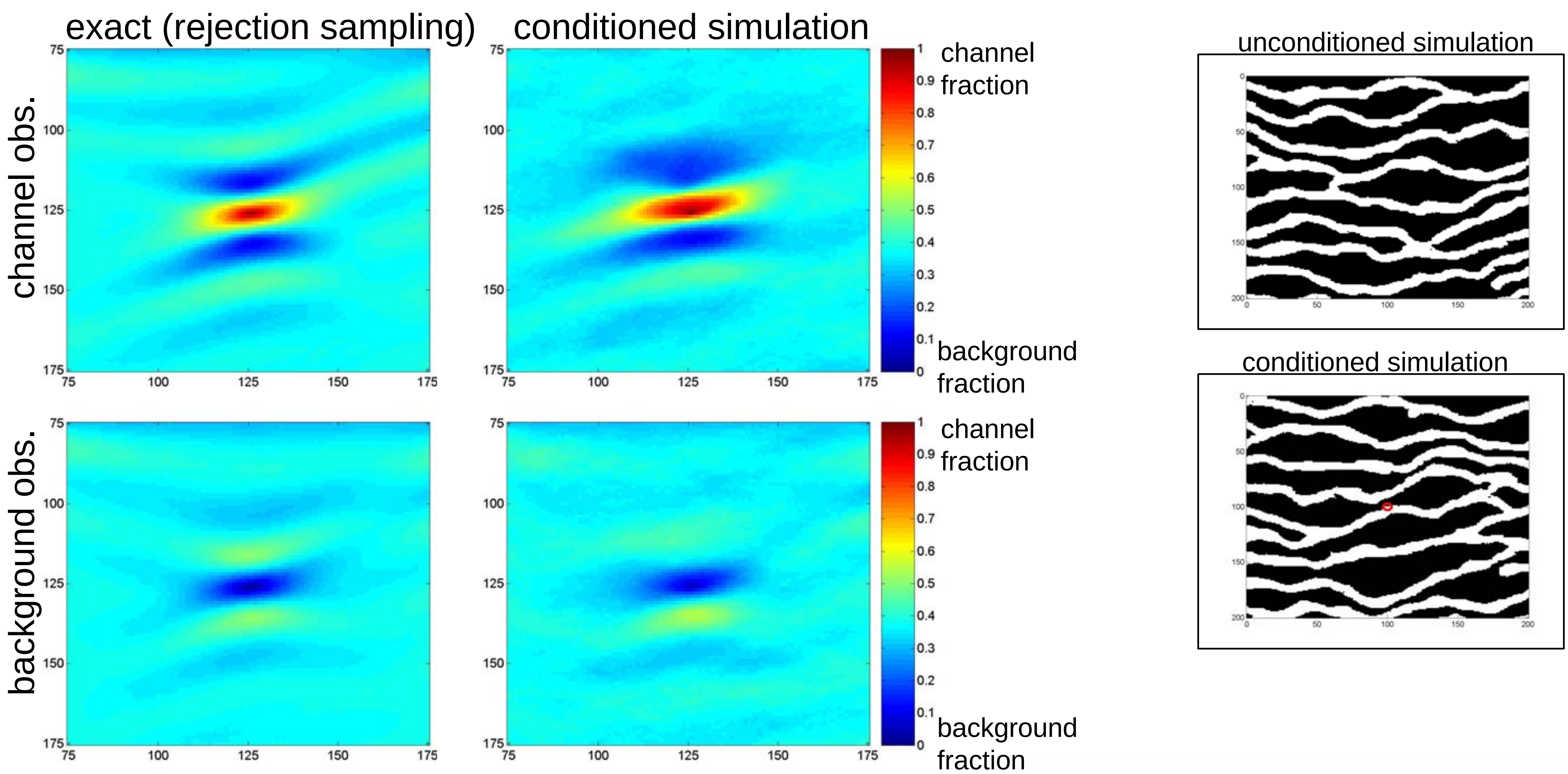
Approximate method, isolated observation

Cell wise average (2000 simulations)

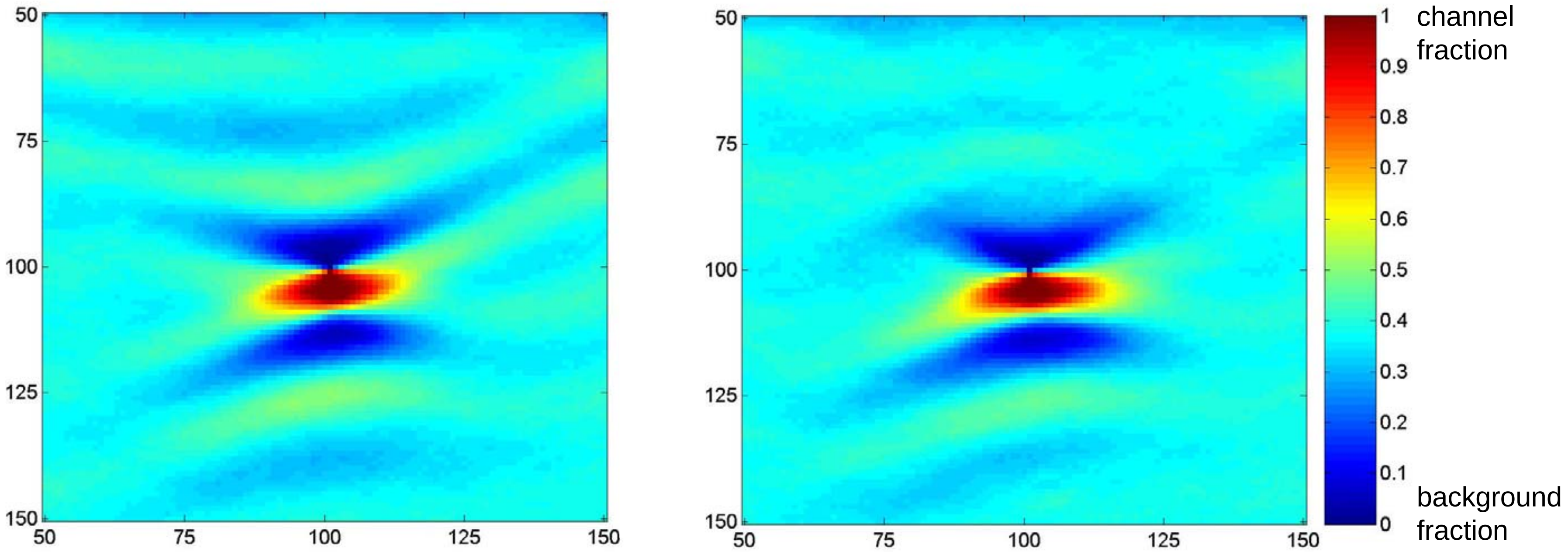
Single configurations



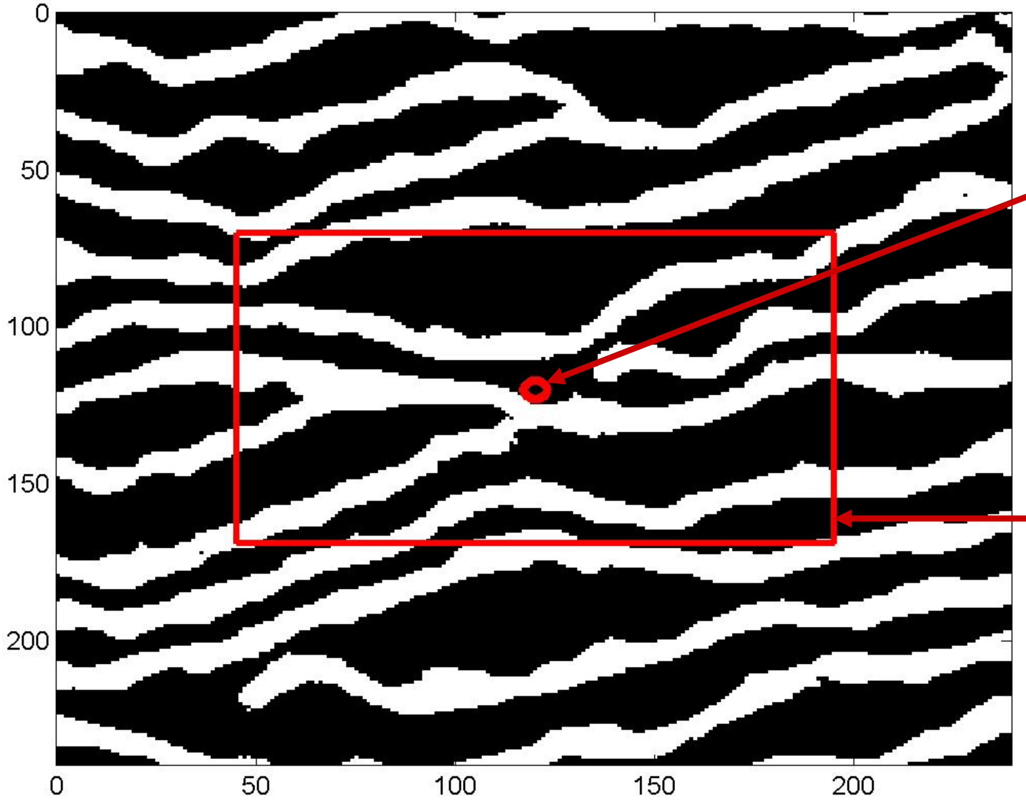
Approximate method, two neighbouring observations

Cell wise average (2000 simulations)

exact (rejection sampling) | conditioned simulation



Iterative method used for local update



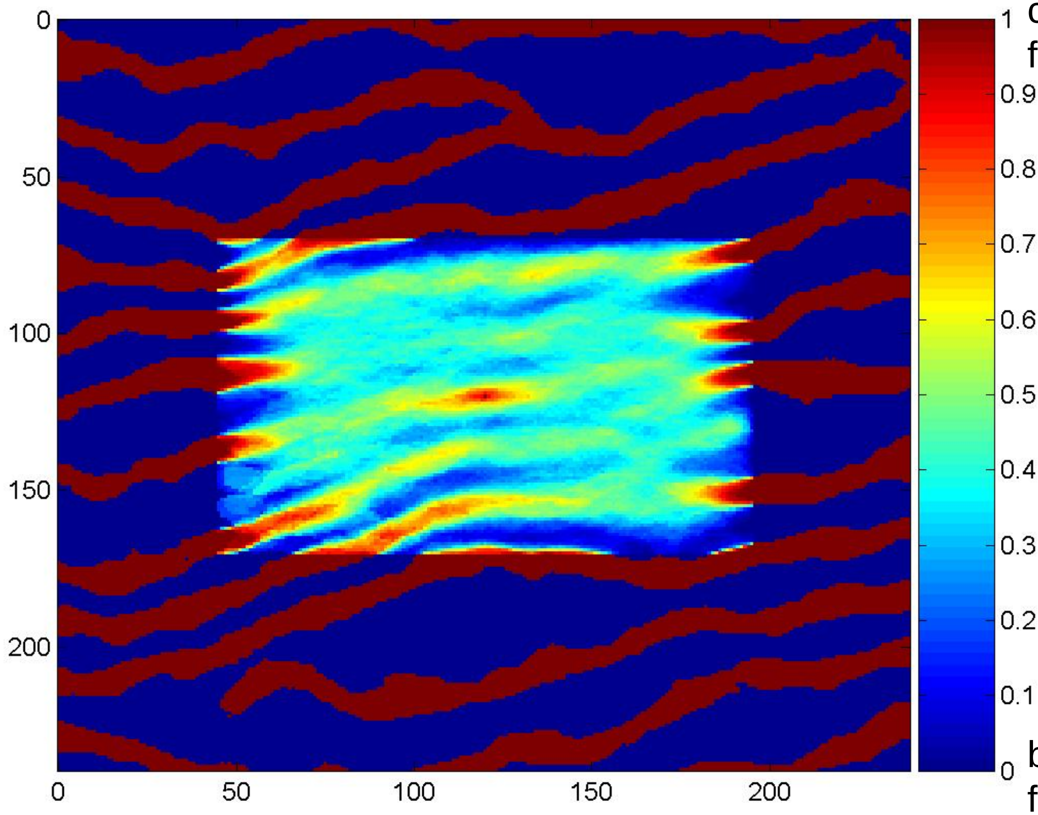
Assume we have:
• existing grid configuration
• new observation(s)

Want to:
• adjust existing configuration locally, around new observation(s)

Method:
• use iterative method on subset of initial grid

Local update, isolated observation

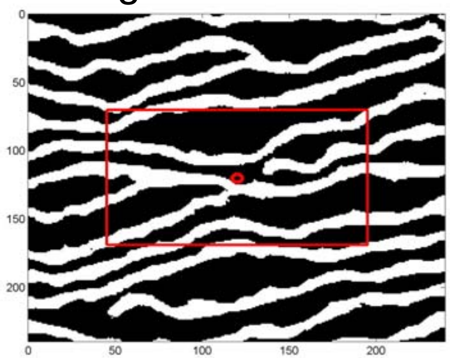
Cell wise average (3000 iterations)



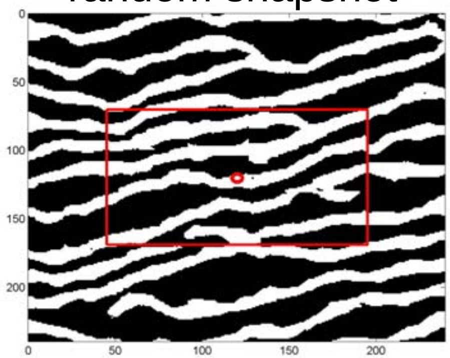
channel fraction

background fraction

initial grid + observation



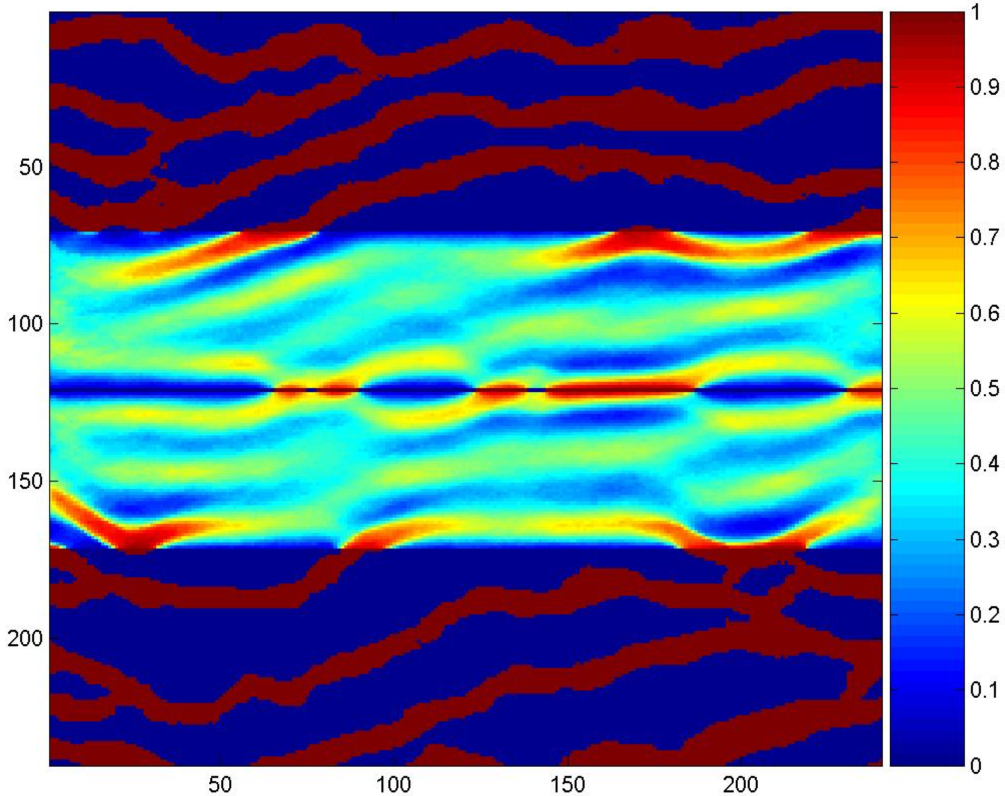
random snapshot



accept rate: 21%

Local update, line of observations

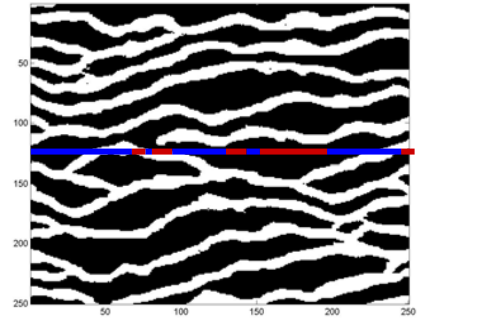
Cell wise average (5000 iterations)



channel fraction

background fraction

initial grid + observations



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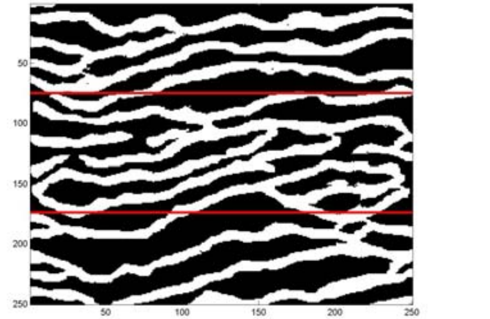

accept rate: 24%

Conclusions

- Established fast method for conditioning to observations in Markov mesh models
- Iterative method well suited for local update

Acknowledgements

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