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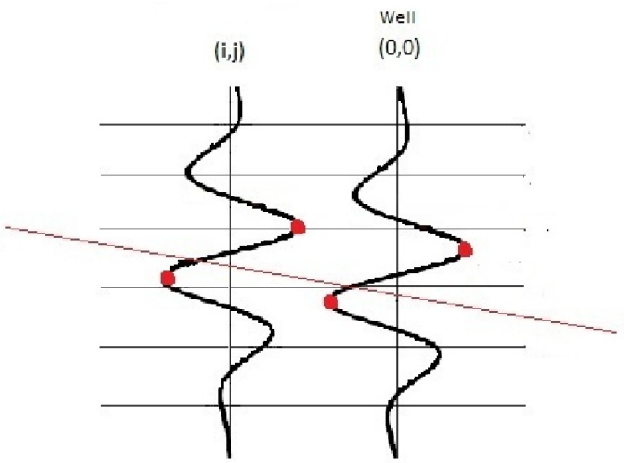
Note

Computation of horizon gradients from seismic



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Abstract

The reflection of seismic signals depends on the orientation of the transition surface. Working with 3D wavelets, we assume a locally planar surface from where we compute the gradients. The gradients are computed from the reflector surface given by seismic amplitude data. First, by locating the reflector surface in the well and in the seismic traces close to the well, then fitting a plane to the corresponding locations. For each log observation in the well, gradients are computed by interpolating between the gradients of the located reflector surfaces. Further, the estimated gradients are used as uncertain data in a Gaussian Markov random field, to obtain a smooth estimate where reflection planes do not intersect close to the well.

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Contents

1 Introduction

When working with seismic data and the convolution model, reflections are often considered to come from a horizontal transition between two layers. When working with 3D wavelets, this assumption is relaxed to having a locally planar surface as the transition surfaces. The reflection will then depend on the orientation of the plane, that is, the gradients.

Wavelets are estimated by computing reflections from a well log (using a nearly vertical well), and find the wavelet that gives a synthetic seismic closest to the observed seismic at the well location. In order to do this here, we need to know the gradients of the geology at the different depths in the well. In this note we present how we estimate the gradients of the reflector surface based on seismic amplitude data.

The extreme values of seismic amplitudes represent a change in the elastic parameters, and the continuity of these extreme values match the reflector surface. The local gradient of the reflector surface can therefore be estimated from a set of seismic traces in the neighborhood of the wells.

The gradients are estimated for each log observation in the well. Also, when there are several angle gathers of seismic amplitudes, the gradients are estimated independently for each seismic cube. We model the gradients such that the uncertainty in their estimation and the dependency between the gradients are properly accounted for. Gaussian Markov random fields (Rue and Held, 2005) are used for this purpose. This class of models fits our data and has nice computational properties.

2 Algorithm

The algorithm for finding the gradients have three main steps:

1. Find the extreme values of the seismic amplitude in the traces in and around the well, and match these.
2. Estimate the gradients based on the matched extreme amplitudes.
3. Use the estimated gradients as uncertain data in a Gaussian Markov random field, to obtain a smooth estimate where reflection planes do not intersect close to the well.

In the following, we look at each of these steps in more detail.

2.1 Detecting extreme values

An extreme value in the seismic traces is the time location, T, where the seismic amplitude is higher in absolute value than the amplitude at the two nearest sampled time locations. We find the extreme values in the neighborhood traces and match them with extreme values in the well trace. By smoothing the traces we eliminate most of the small

scale variation and obtain more similarity between the extreme values. Figure 1 displays a seismic trace before and after smoothing, where we have applied a gaussian kernel smoother.

An extreme value in a neighboring trace is assumed to match an extreme value in the well trace when; they are no more than 5 times the sampling density apart, the difference in seismic amplitude between the two does not exceed half their sum, and finally that they peak in the same direction.

2.2 Computing gradients

The gradient at time location $T(k)$ is represented by the x- and y-component $\{a(k), b(k)\}$. To estimate these, we use a neighborhood Ω_0 of traces around the well with size n_0. The well observations are given at n_{obs} locations $\{(x_0(k), y_0(k)\}$ for all $k = 1, ..., n_{\text{obs}}$. Figure 2 visualizes a cross-section of the x-component $a(k)$.

The gradient is computed at each of the n_{obs} locations, based on the displacement of the extreme values between the trace in the well location and the n_0 traces in the neighborhood. The displacement is computed by interpolating between the two nearest extreme values within the trace.

Let $T_0(k)^-$ and $T_0(k)^+$ be the time for the two nearest extreme values to $T_0(k)$ in the well location. The corresponding extreme values in trace $(x_{ij}(k), y_{ij}(k))$ are $T_{ij}(k)^-$ and $T_{ij}(k)^+$, where (i, j) represent the lateral indices of the trace location. The relative position of $T_0(k)$ between the nearest extreme values is

$$\omega = \frac{T_0(k) - T_0(k)^-}{T_0(k)^+ - T_0(k)^-}.$$

We apply this weight on each trace (i, j) to find the correspond time to $T_0(k)$,

$$T_{ij}(k) = T_{ij}(k)^- + \omega \cdot \left(T_{ij}(k)^+ - T_{ij}(k)^-\right).$$

The time displacement at k in trace (i, j) is then

$$\Delta T_{ij}(k) = T_0(k) - T_{ij}(k).$$

Figure 3 displays a section of the well trace and the (i, j)-trace. The drawing locates the time expressions above, where the red dots represent the extreme values in both traces for position k.

A plane $\Delta T_{ij}(k) = a(k)x_{ij}(k) + b(k)y_{ij}(k) + c(k)$ is fitted to the n_0 displacement values for every $k = 1, ..., n_{\text{obs}}$, where $a(k)$ and $b(k)$ is the x- and y-gradient, respectively. We use standard multivariate regression with least square estimation. This procedure returns the estimated coefficients $\left(\hat{a}(k), \hat{b}(k), \hat{c}(k)\right)$ and the error from the estimation Σ_{ϵ_k}.

When there are more than one seismic cube available we estimate the gradients similarly for each cube. Let P be the number of available cubes. This yields a set of $P \times n_{\text{obs}}$ estimated gradients and P error matrices $\Sigma^p_{epsilon}$ for $p = 1, ..., P$. We want to compute an average of the P estimates, accounting for the error in the estimates. At the same time

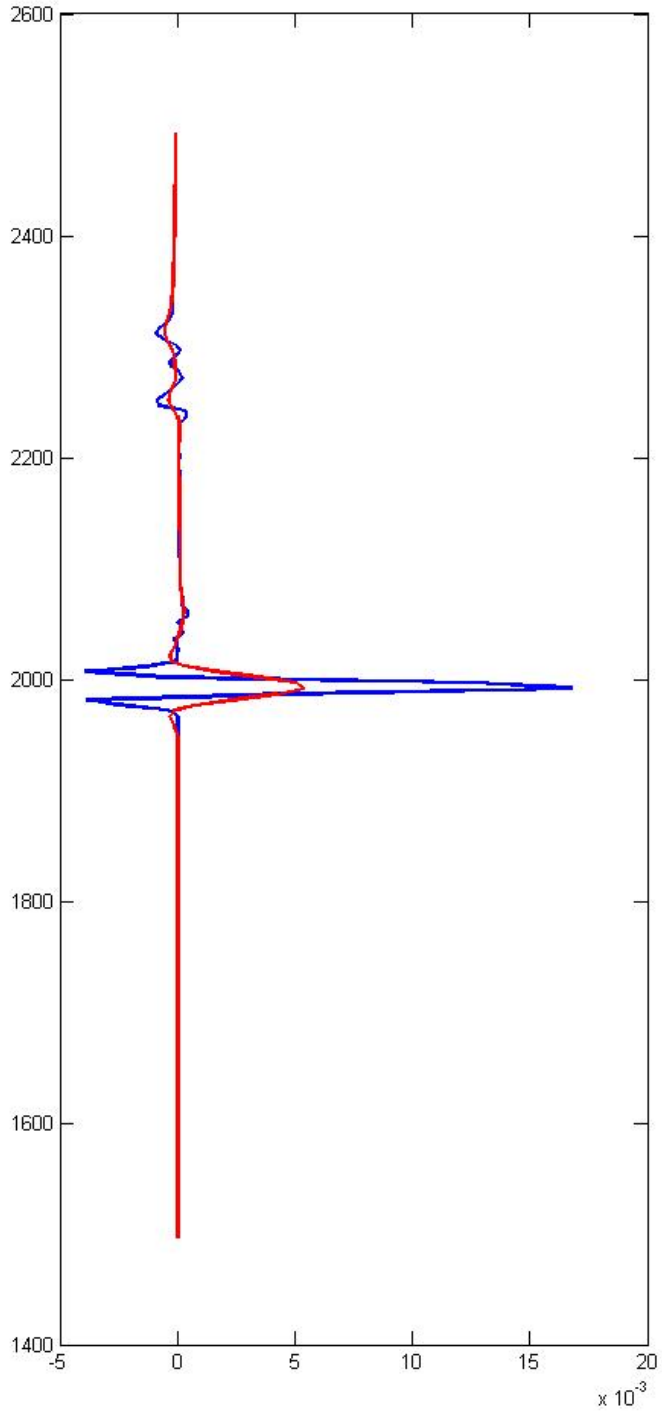


Figure 1. A seismic amplitude trace before (in blue) and after (in red) smoothing.

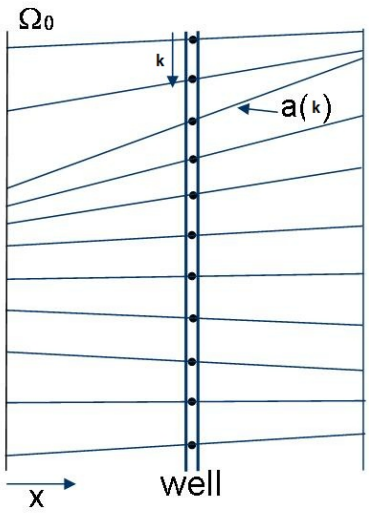


Figure 2. Well sampled at discrete times $T(k)$ with associated gradients in a (x, k)-cross section. Gradients are constant and non-crossing in Ω_0.

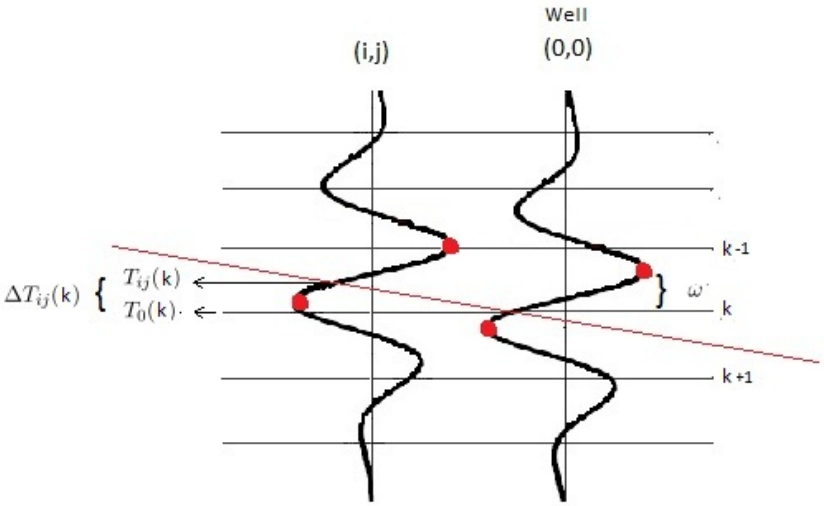


Figure 3. Illustration of the computation of displacement between trace (i, j) and the well trace.

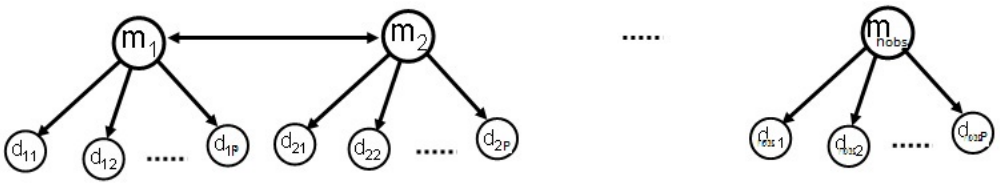


Figure 4. Graphical representation of the GMRF.

we need to consider the dependency between the gradients between the layers, such that the change in the gradient between consecutive layers is reasonable. In the following sections we describe how we have modeled the gradients, and how we put constraints on the model in order to avoid the gradients to cross within a limited area.

2.3 Into the framework of a Gaussian Markov random field

Gaussian Markov random fields(GMRF), (Rue and Held, 2005), are frequently used in hierarchical models in order to allow for dependence between a set of unknown parameters. A field $\mathbf{m}$ is connected to observations $\mathbf{d}$, which are assumed independent given $\mathbf{m}$. The distribution of interest is the posterior $p(\mathbf{m}|\mathbf{d})$, from where we extract the expectation.

The covariance matrix $\mathbf{\Sigma}$ yields direct information about the marginal dependency between the variables. The precision matrix $\mathbf{Q}$, on the other hand, is the inverse of the covariance matrix, $\mathbf{Q} = \mathbf{\Sigma}^{-1}$, and give information about the conditional independence. The advantage with GMRF is the sparseness of the precision matrix, and the numerically tractable methods that follows with sparse matrices.

We have estimated sets of gradients from the K given seismic amplitude cube. These estimated values are considered observations of the unknown gradient parameters $\mathbf{a} = \big(a(1), ..., a_(n_{\text{obs}})\big)$ and $\mathbf{b} = (b(1), ..., b(n_{\text{obs}}))$. We write

$$\begin{aligned} \hat{a}_p(k) &= a(k) + \epsilon_a^p(k) \\ \hat{b}_p(k) &= b(k) + \epsilon_b^p(k), \end{aligned}$$

where p refers to the p^{th} seismic cube. The error terms $\epsilon_a^p(k)$ and $\epsilon_b^p(k)$ are the error from the estimation of the gradients, and are taken from the diagonal in $\Sigma_{\epsilon_k}^p$.

Our priori assumptions on $\mathbf{a}$ and $\mathbf{b}$, is that they are gaussian distributed with mean zero and equal covariance structure $\mathbf{\Sigma}$. The gradients $\mathbf{a}$ and $\mathbf{b}$ are assumed independent, and the same computations applies for both. Without loss of generality we therefore use only one gradient $\mathbf{m}$ in our explanations, and let $(\mathbf{d}_1, ..., \mathbf{d}_P)$ be the corresponding estimate of $\mathbf{m}$ from the P angle gathers. We assume a first order Markov chain for $\mathbf{m}$, and the graphical representation of the GMRF is given in Figure 6.

We are interested in the expectation of $\mathbf{m}$ given the observations $(\mathbf{d}_1, ..., \mathbf{d}_P)$, $\mathrm{E}(\mathbf{m}|\mathbf{d}_1, ..., \mathbf{d}_P)$. The joint probability of $\mathbf{m}$ and $(\mathbf{d}_1, ..., \mathbf{d}_P)$ is

$$\pi(\mathbf{m}, \mathbf{d}_1, ..., \mathbf{d}_P) = \pi(\mathbf{m}) \times \pi(\mathbf{d}_1, ..., \mathbf{d}_P|\mathbf{m}).$$

Recall that the observations $(\mathbf{d}_1, ..., \mathbf{d}_P)$ are assumed independent given $\mathbf{m}$, so we can write

$$\pi(\mathbf{m}, \mathbf{d}_1, ..., \mathbf{d}_P) = \pi(\mathbf{m}) \times \prod_{p=1}^{P} \pi(\mathbf{d}_p|\mathbf{m}) \tag{1}$$

In general, if $\mathbf{x} \sim N(\boldsymbol{\mu}, \mathbf{Q})$ the exponential is on the form

$$-\frac{1}{2}\mathbf{x}^T\mathbf{Q}\mathbf{x} + \mathbf{x}^T\mathbf{Q}\boldsymbol{\mu} + \text{constant terms.} \tag{2}$$

When $\mathbf{m} \sim N(\mathbf{0}, {\mathbf{Q}_\mathbf{m}}^{-1})$ and $\mathbf{d_p} \sim N(\mathbf{m}, \mathbf{Q}_\epsilon^{p^{-1}})$ for $p = 1, ..., P$, the exponential of (1) is on the form

$$-\frac{1}{2}\mathbf{m}^T(\mathbf{Q}_\mathbf{m} + \sum_p \mathbf{Q}_\epsilon^p)\mathbf{m} + \mathbf{m}^T \sum_j (\mathbf{Q}_\epsilon^p \mathbf{d}_p) + \text{constant terms.} \tag{3}$$

Since $\pi(\mathbf{m}|\mathbf{d}) \propto \pi(\mathbf{m}, \mathbf{d})$, we compare (3) with (2) get that

$$\begin{aligned} \mathbf{Q}_{\mathbf{m}|\mathbf{d}} &= \mathbf{Q}_\mathbf{m} + \sum_p \mathbf{Q}_\epsilon^p \\ \boldsymbol{\mu}_{\mathbf{m}|\mathbf{d}} &= {\mathbf{Q}_{\mathbf{m}|\mathbf{d}}}^{-1}(\sum_p \mathbf{Q}_\epsilon^p \mathbf{d}_p). \end{aligned}$$

In order to compute $\boldsymbol{\mu}_{\mathbf{m}|\mathbf{d}}$ we need to specify the precision matrix $\mathbf{Q}_\mathbf{m}$. This matrix is subject to constraints, and in the next section we explain how we select the entries in $\mathbf{Q}_\mathbf{m}$.

2.3.1 Computation of $\mathbf{Q_m}$

In general, for a first order Markov chain the precision matrix $\mathbf{Q}_\mathbf{m}$ is

$$\begin{pmatrix} a & c & 0 & 0 & \cdots & 0 \\ c & b & c & 0 & \cdots & 0 \\ 0 & c & b & c & \cdots & 0 \\ \vdots & & \ddots & & & \\ 0 & & \cdots & c & b & c \\ 0 & & \cdots & 0 & c & b \end{pmatrix}, \tag{4}$$

If $\mathbf{Q}_{\mathbf{m}ij} = 0$ for $i \neq j$ then $\mathbf{m}_i$ and $\mathbf{m}_j$ are conditionally independent given the other variables.

The gradient lines must not cross in the area around the well. With a certain level of significance we can assure that does not occur, by laying constraints on the prior model, $\pi(\mathbf{m})$. Since the prior model is a first order Markov chain it can be expressed in terms of the autoregressive

$$m_k = \alpha m_{k-1} + \gamma e_k \text{ for } k = 2, ..., n_{\text{obs}}, \tag{5}$$

where $e_k \sim N(0, 1)$ for $k = 1, ..., n_{\text{obs}}$ and $m_1 = \sigma_m e_1$.

In matrix notation we can write $\mathbf{Lm} = \mathbf{e}$ such that

$$\begin{pmatrix} 1/\sigma_m & 0 & 0 & 0 & \cdots & 0 \\ -\alpha/\gamma & 1/\gamma & 0 & 0 & \cdots & 0 \\ 0 & -\alpha/\gamma & 1/\gamma & 0 & \cdots & 0 \\ \vdots & & \ddots & & & \\ 0 & & \cdots & -\alpha/\gamma & 1/\gamma & 0 \\ 0 & & \cdots & 0 & -\alpha/\gamma & 1/\gamma \end{pmatrix} \begin{pmatrix} m_1 \\ m_2 \\ m_3 \\ \vdots \\ m_{n_{\text{obs}-1}} \\ m_{n_{\text{obs}}} \end{pmatrix} = \begin{pmatrix} e_1 \\ e_2 \\ e_3 \\ \vdots \\ e_{n_{\text{obs}}-1} \\ e_{n_{\text{obs}}} \end{pmatrix},$$

where $\mathbf{L}$ a the lower triangular matrix. The precision matrix $\mathbf{Q_m}$ is the matrix product $\mathbf{L}^T\mathbf{L}$, which gives the relation between the autoregressive parameters and the entries in the precision matrix in (4)

$$a = \frac{1}{\sigma_m} + \frac{\alpha^2}{\gamma^2} \tag{6}$$

$$b = -\frac{\alpha}{\gamma^2} \tag{7}$$

$$c = \frac{\alpha^2 + 1}{\gamma^2}. \tag{8}$$

The time location of the gradient in a distance D from the well is $T_D(k) = T(k) + m_k \cdot D$ for observation k, and similarly for $k-1$ we get $T_D(k-1) = T(k-1) + m_{k-1} \cdot D$. Within a given minimum distance D_{min} we need $T_{D_{min}}(k) > T_{D_{min}}(k-1)$ to ensure that the gradients do not cross. We can write

$$\begin{aligned} T(k) + m_k \cdot D_{min} &> T(k-1) + m_{k-1} \cdot D_{min} \Rightarrow \\ m_{k-1} - m_k &< \frac{T(k) - T(k-1)}{D_{min}}, \end{aligned}$$

where from the relation in (5) we get $m_{k-1} - m_k \sim N(0, 2\sigma_m^2(1-\alpha))$. Note that the sampling density is constant throughout the traces such that $T(k) - T(k-1)$ can be set to ΔT. With a certain level of confidence we require that $m_{k-1} - m_k < \frac{\Delta T}{D_{min}}$, such that

$$z_q < \frac{\Delta T}{D_{min}} \cdot \frac{1}{\sqrt{2\sigma_m^2(1-\alpha)}}$$

where z_q is the quantile value for confidence level q. In addition, from (5) we have the relation $\text{Var}(m_k) = \alpha^2\text{Var}(m_{k-1}) + \gamma^2$ where $\text{Var}(m_k) = \text{Var}(m_{k-1}) = \sigma_m^2$.

With two equation and two unknowns we can compute α and γ and further a, b and c such that the gradients with $q\%$ confidence do not cross within D_{min}. The resulting expression for α and γ is

$$\begin{aligned} \alpha^2 &> \left(1 - \frac{1}{2\sigma_m^2}\left(\frac{T(k) - T(k-1)}{z_q D_{min}}\right)^2\right)^2 \\ \gamma^2 &> \sigma_m^2(1-\alpha^2). \end{aligned}$$

3 Examples and parameter choice

We are given two seismic angle gathers at offset-angle 0 and 10. The sample density of the seismic cubes are 50 meters in both x- and y-direction and 5ms vertically. We have one well with 200 log observations.

There are two values that must be set. The first one is the prior variance σ_m, and the second one is the minimal distance constraint D_{min}. We set default values to $\sigma_m = 1$ms/meter and $D_{min} = 100$meters, and Figure 5 shows the gradient estimated for the two angle gathers in blue, and the resulting gradient estimated with the default values in red.

The impact of adjusting D_{min} is visualized in Figure 6. The gradient estimated from the two angle gathers are still displayed in blue, and the various resulting gradients are displayed in different colors explained by the legend in the figure. It can be seen from the plot that the higher the minimum distance D_{min}, the less variation is allowed between two consecutive gradients, m_k and m_{k-1}.

Figures 7 and 8 displays images of the X-cross-section and Y cross-section of traces, intersecting in the well location. And the black lines are the estimated gradients, using the default parameter values $\sigma_m = 1$ and $D_{min} = 100$. The images to the left show all n_{obs} blocks while the images to the right are sub-images of those.

4 Conclusions

The gradient of the reflector surface are used in wavelet estimation. These gradients are computed from seismic gathers around the wells, by matching the extreme values between the seismic traces. Gradients are computed from each seismic angle gather independently, and connected through a GMRF. It is important that gradient lines do not cross, and the GMRF framework allows us to specify a prior model to minimize the risk of that to occur.

References

Rue, H. and Held, L. (2005). *Gaussian Markov Random Fields*. Chapman and Hall, 1 edition.

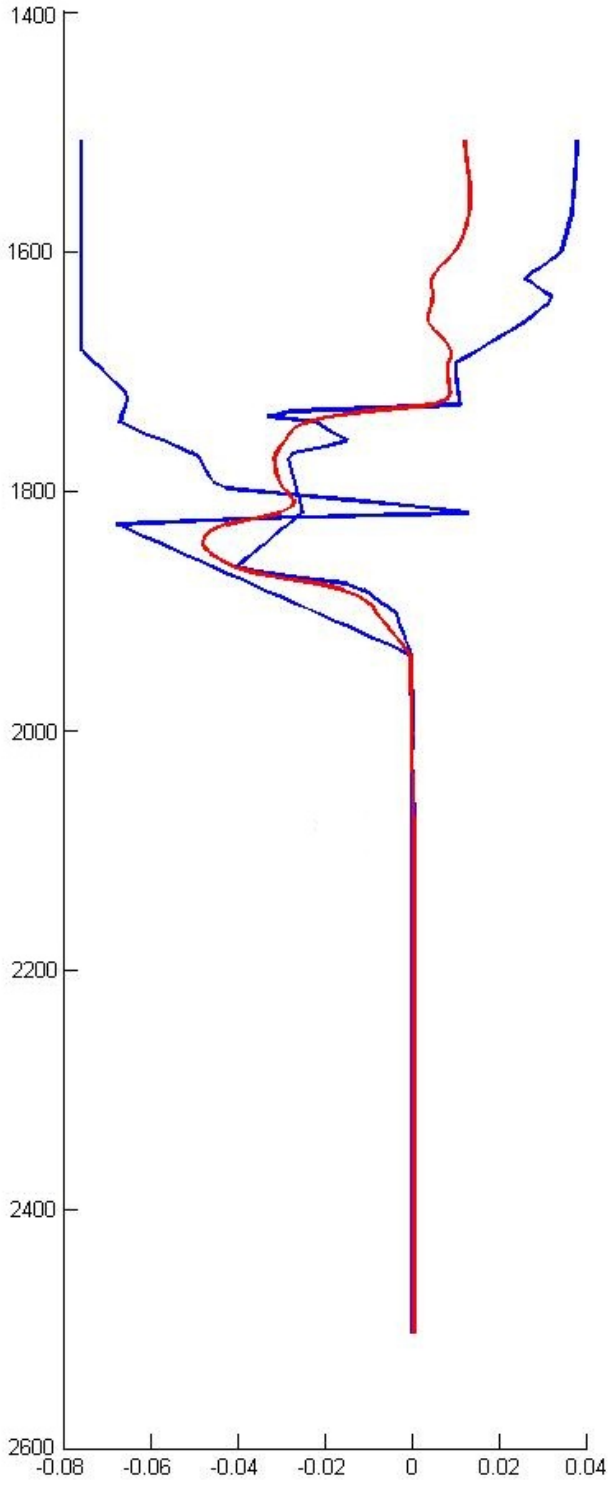


Figure 5. Gradients computed from both angle gathers in blue, and resulting gradient in red

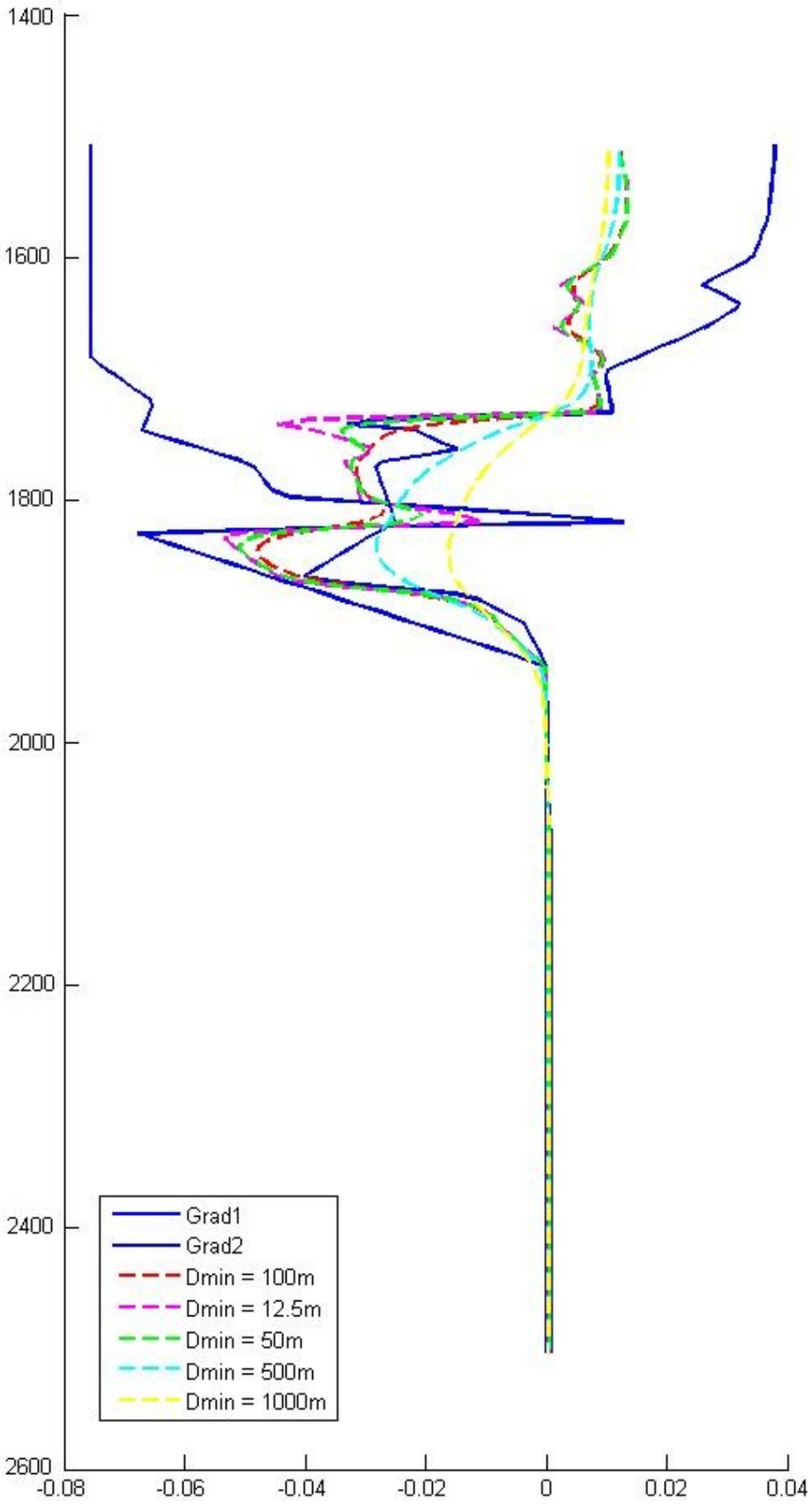
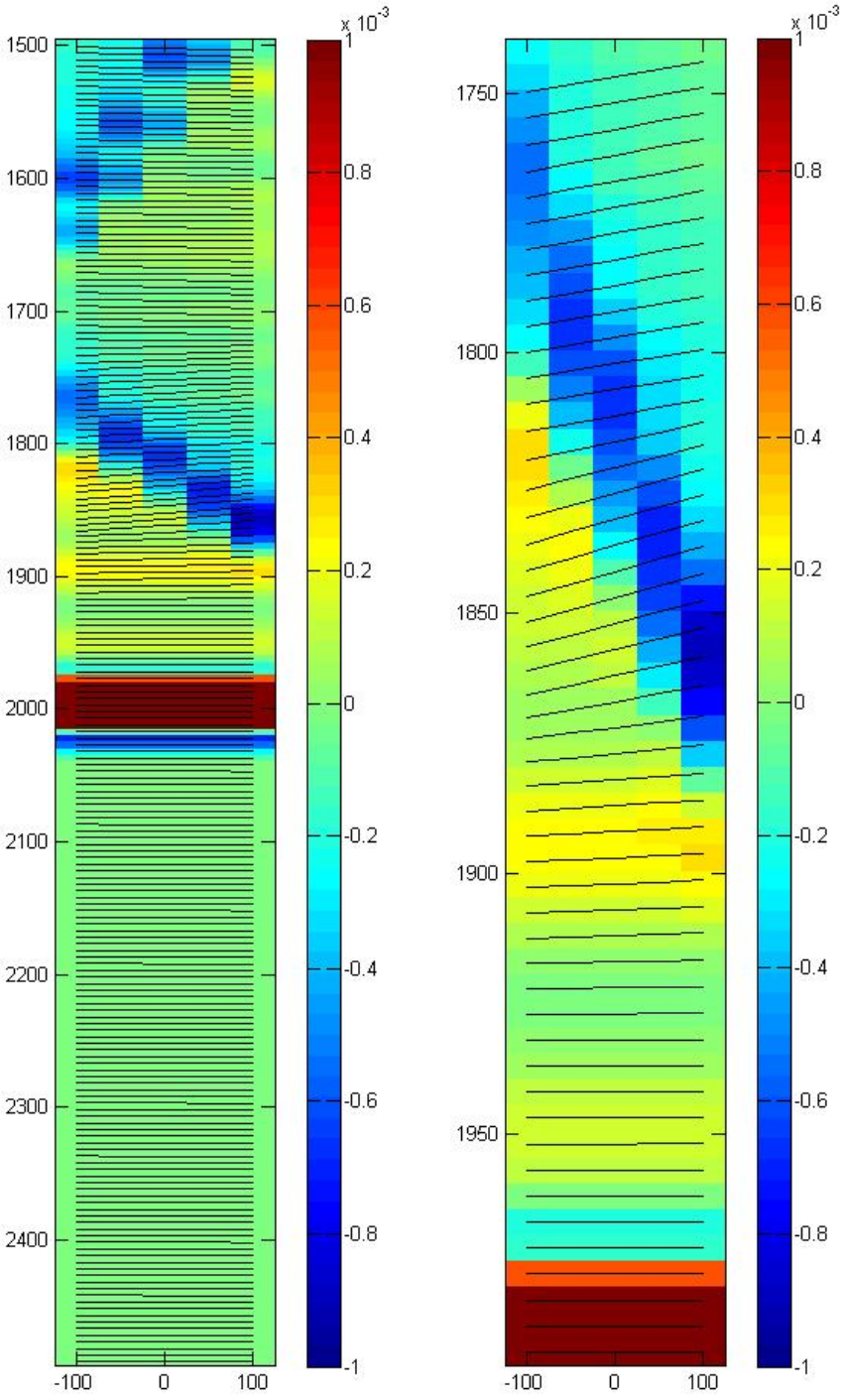
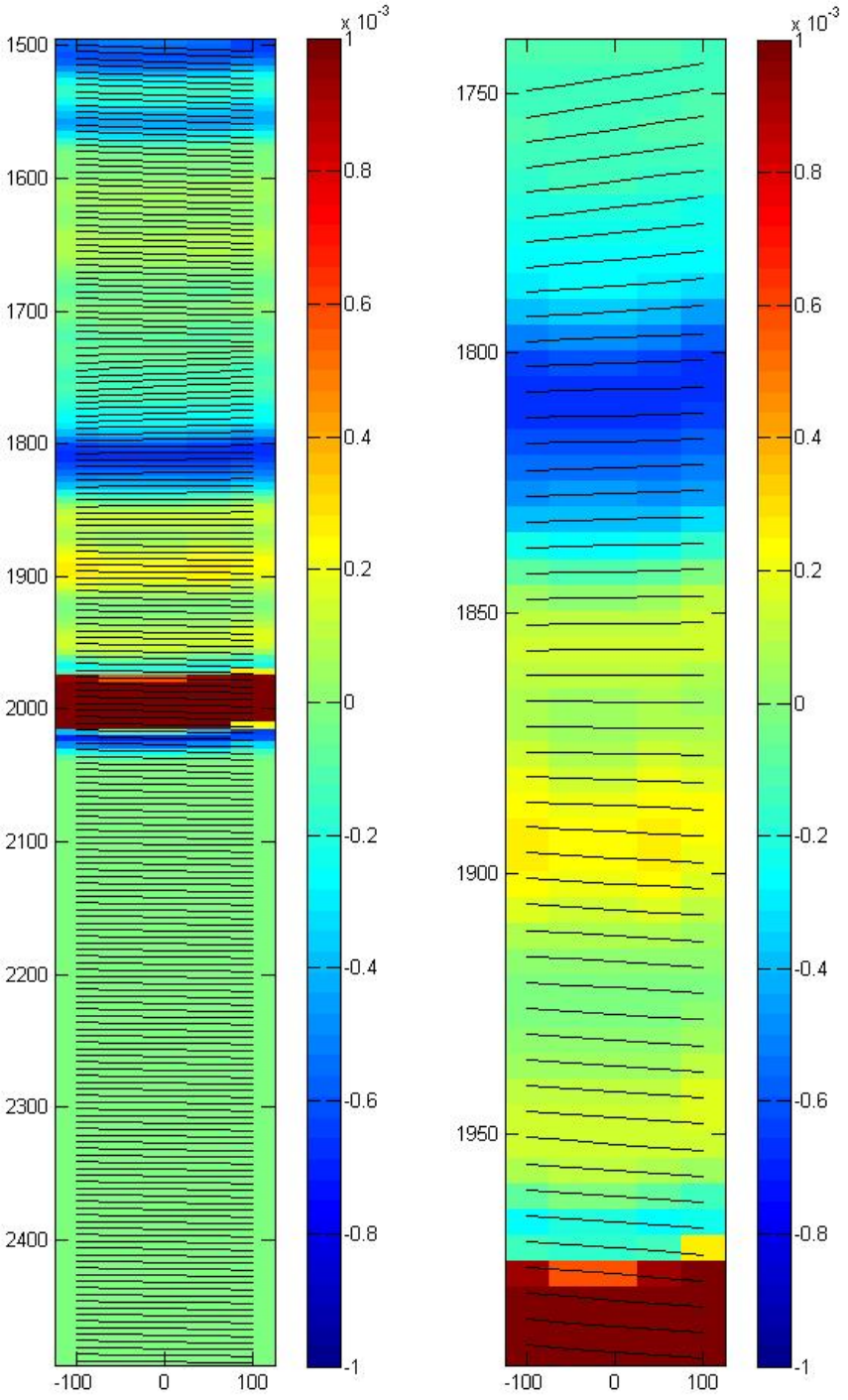


Figure 6. The effect of adjusting D_{min} in the model



(a) (b)

Figure 7. X cross-section of traces intersecting in the well location. The black lines are the estimated x-gradients. The right plot is a sub-plot of the left one with indices $k = 50, ..., 100$.



(a) (b)

Figure 8. Y cross-section of traces intersecting in the well location. The black lines are the estimated y-gradients. The right plot is a sub-plot of the left one with indices $k = 50, ..., 100$.