A Study of the Length and Age Distribution of Norwegian Spring Spawning Herring in the Vestfjord System Using a December 2002 Survey Data

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Sonia Mazzi and Gudmund Høst

mazzi@nr.no
Norwegian Computing Centre/Norsk Regnesentral
P.O.Box 114, Blindern
0314 Oslo, Norway

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Abstract

Surveys are carried out to assess the status of the stock of Norwegian spring spawning herring. During these surveys fish are caught and certain measurements are taken on each captured unit, such as length and age assessment. Due to the cost, both in time and money, mostly lengths are obtained.

The distribution of lengths does not directly and clearly convey the necessary information to assess, for example the differences in the age distributions in different subregions, or different depths at different times, of the Vestfjord system, or the abundance by age in the region.

Length is a causal effect of age and this relationship is modelled so that ages can then be calibrated from lengths.

It is shown how age distributions can be obtained from length distributions via calibration. It becomes evident how the age distributions can more clearly deliver information about the status of the stock. Also, the age distributions are a better statistical object to use when comparing presumed strata of the population under study.

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Chapter 1

Introduction

The work described in this reports relates to the strategic program “Estimation of Absolute Abundance of Fish”, funded by the Research Council of Norway through contract No. 143249/140. The aim of the program is to provide a survey-based estimate of abundance by age, including uncertainty bounds. In general, the relevant survey data consist of measurements of local fish densities and biological samples of individual fish ages and lengths.

In this work, we focus on the uncertainty in the biological sampling of Norwegian spring-spawning herring. This is the largest fish stock in the Northeast Atlantic, with current spawning stock of about six million tons. The stock spawns along the Norwegian coast, feeds in the Norwegian Sea and overwinters in the Vestfjord system in Northern Norway. See Røttingen (1990).

The current procedure for estimating age distribution of the stock is to pool age samples from 10-15 trawl stations, sometimes including age samples from commercial catches as well. The location and number of trawl stations to include is chosen subjectively based on previous experience. Furthermore, the age samples are usually collected from the upper part of the school of fish, since the quality of the fish scales used for age readings quickly deteriorates with the residence time in the trawl. The time of day for trawling is usually not taken into account in the routine assessment of the age distribution.

While overwintering in the Vestfjord system, diurnal variability in the vertical distribution of the herring density may be observed from acoustic data. See, for example, Vabø (1999), and Huse and Korneliussen (2000). The density of fish may also vary spatially within the fjord system. However, little is known about the variability of age or length distributions in space, time and depth. Such variability may give rise to systematic and non-systematic errors in the estimated age-distributions.

During 10th–16th December, 2002, an experiment was conducted in the Vestfjord system to study the variability of length and age with location, time of the day, and depth. The rationale for studying also length distributions is that individual age and length is related through growth. Furthermore, length measurements are both less costly and less time-consuming to collect than age readings. Thus, more trawl stations may be covered with length-only measurements than with age-length measurements. Finally, length measurements may be collected from greater depths, since residence time in the trawl is not a concern for length-only samples. The current experiment involved sampling at different locations at different times during the day and at different depths.

Some important questions that the present data may provide answers to are:

- How many trawl stations are needed to obtain a reliable age distribution?
- At what hours of the day should biological samples be collected?
- What depths should be sampled for length-only?
- What is the value of additional length-only samples as compared to additional age samples?

The issues raised in these questions will be addressed along the report. The report is organised as follows. In Chapter 2 a description of the collected data is given. Here we start seeing that the population under study might be stratified in homogeneous and significantly different groups. Chapter 3 provides a methodology to obtain ages from length data. In Chapter 4 we apply the methodology described in Chapter 3 to the length data described in Chapter 2. Chapter 5 contains some remarks on the future planning of sampling experiments in light of the results of the previous analysis.

Chapter 2

Descriptive Analysis of the Data on Catch of Herring from the December 2002 Sampling Experiment

2.1 Spatial frame of the sampling experiment

The sampling was carried out in the Vestfjord system, between 15 and 18 degrees of longitude East and 68 and 69 degrees of latitude North. There were 72 sampling positions in the area, clustered in 5 locations: (see Figure 2.1)

- Vestfjord (23 sampling positions)
- Tysfjord (16 sampling positions)
- Barøya (27 sampling positions)
- Ofotfjord (5 sampling positions)
- Other location in the inner part of the Ofotfjord (1 sampling position)

The samples were obtained using a multi-sampler system, allowing the sampling of three batches of fish at separate depths during one towing operation. In what follows, each towing operation will be referred to as a trawl station. From each trawl station, 100 fish are sampled at random from each of the three depth-specific batches, and their lengths are recorded. Some batches contained less than 100 fish. All fish were measured for length. The depth of the three batches was determined relative to the position of the targeted school. The aim was to get catches from the top, middle, and bottom areas of the schools. This was determined using the aid of echograms. The coverage of the data samples is given in Table 2.1.

2.2 Temporal frame of the sampling experiment

This experiment took place from the 10th until the 16th of December, 2002. The original plan for the sampling involved covering all periods of the day at all locations and depths, but several modifications had to be done for practical reasons. The resulting sampling took place at all hours of the day except 12-13, 15-16, and 21-22 hours. On December 12th sampling was more scarce, only carried out at 7-8 and 18-19 hours, see Table 2.2.

2.3 Measured variables

For each collected fish the following information was recorded:

- The time it was caught at: day, hour, minute.

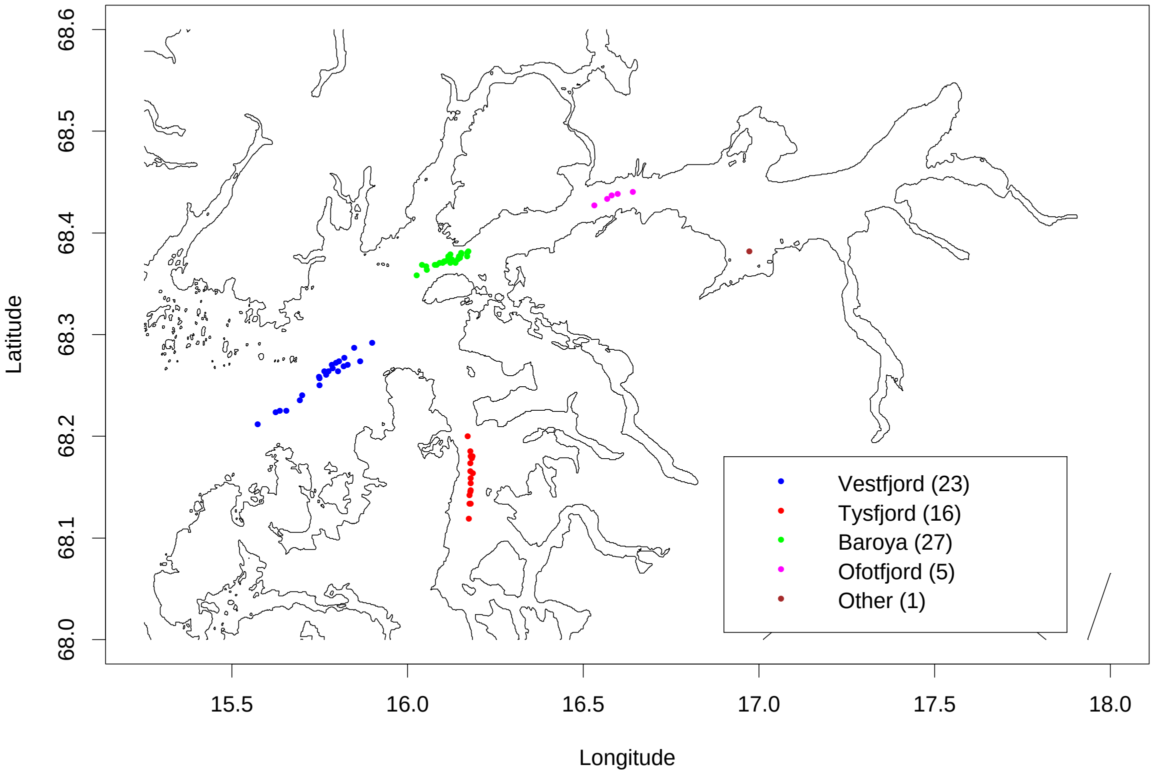

Figure 2.1: Map of sampling locations.

- The location it was caught at: latitude, longitude.
- The absolute depth it was at (in m).
- Its length (in mm).
- Only for some fish their age was determined by reading the number of rings of one of their scales (in years). The method employed is quite accurate.

2.4 The distribution of Length

In this section a descriptive analysis of the observed lengths will be given. The length distribution is studied for some combinations of the factors described in Sections 2.1 and 2.2. We aim to compare the populations in the different clusters where sampling occurred.

2.4.1 Distribution of length given Location category

Figure 2.2 shows histograms of observed lenghts at the different locations. It is seen that there is a tendency towards a larger proportion of smaller fish at Barøya and Ofotfjord locations than at Vestfjord and Tysfjord. This is confirmed by the density estimates in Figure 2.3, where a left shoulder is present in the Barøya and Ofotfjord density curves.

If there are differences between the populations in the different locations, then these differences must be taken into account when planning a sampling experiment as will be explained in Chapter 5.

Sample sizes for Vestfjord

Depth	Daylight(9-15)	Dark	Total
1	106	366	472
2	213	452	665
3	77	400	477
Total	396	1218	1614

Sample sizes for Tysfjord

Depth	Daylight(9-15)	Dark	Total
1	0	357	357
2	141	150	291
3	0	474	474
Total	141	981	1122

Sample sizes for Ofotfjord

Depth	Daylight(9-15)	Dark	Total
1	6	100	106
2	0	100	100
3	100	100	200
Total	106	300	406

Sample sizes for Barøya

Depth	Daylight(9-15)	Dark	Total
1	400	300	700
2	620	300	920
3	209	1090	1299
Total	1229	1690	2919

Table 2.1: Sample sizes for the different sampling locations

Hour	0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	10-11	11-12
Nr fish	371	92	270	297	390	100	143	300	100	106	368	239
Nr batches	5	1	3	4	3	1	3	3	1	2	5	3

Hour	13-14	14-15	16-17	17-18	18-19	19-20	20-21	22-23	23-0
Nr fish	494	665	500	431	500	300	305	100	100
Nr batches	8	8	5	3	5	3	4	1	1

Day	10	11	12	13	14	15	16
Nr fish	657	1346	20	1257	1388	972	341
Nr batches	8	20	2	13	15	10	4

Table 2.2: Temporal coverage of samples

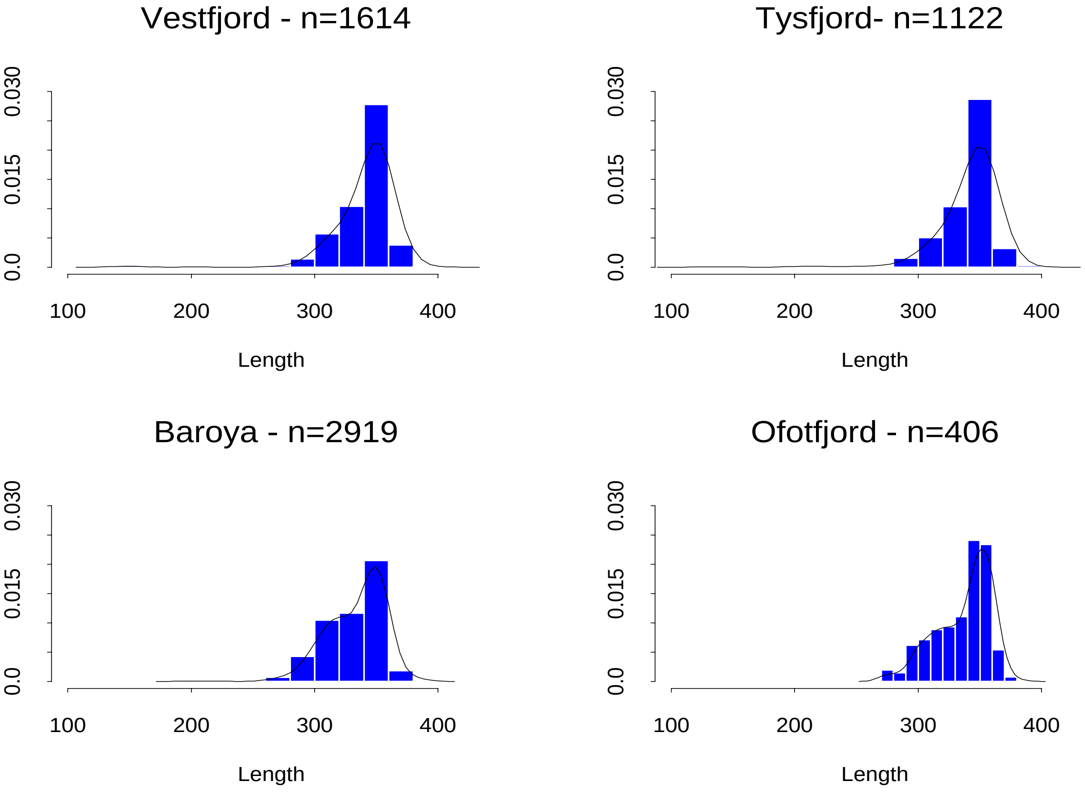

Figure 2.2: Histograms of Lengths of herring in the Vestfjord system by Location they were caught.

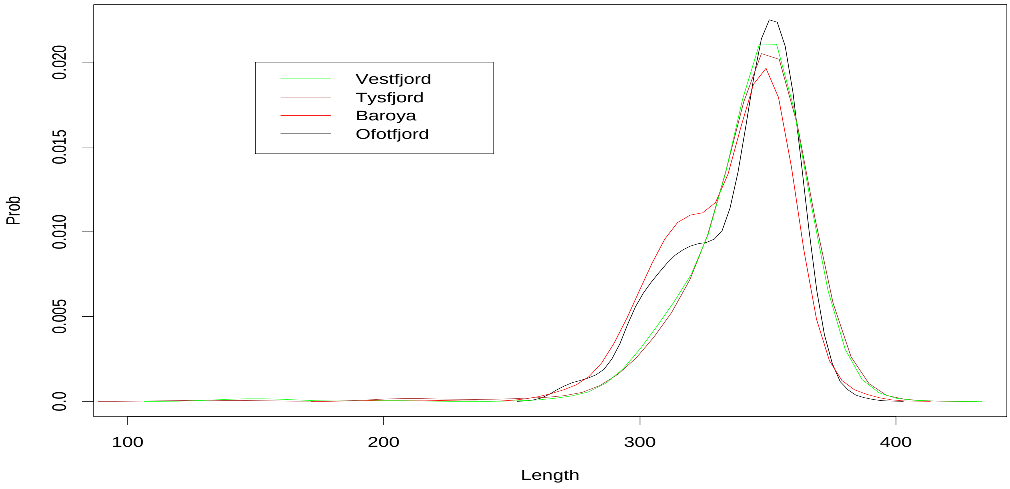

Figure 2.3: Smoothed histograms of Lengths of herring in the Vestfjord system by Location they were caught.

2.4.2 Distribution of Length given Depth category

We recall that the depth categories were determined according to the relative position of the sampled fish in the school where they belonged. Depth 1 correspond to the upper layer of the school, relative to the boat, or the layer closer to the surface of the water. Depth 3 corresponds to the layer of fish closest to the bottom of the sea, or deepest relative to the boat. Depth 2 corresponds to approximately the center of the targeted school of herring.

Figure 2.4 shows length at the three considered depths aggregated over all locations. It is important to observe that there is a slight tendency towards larger fish at the deepest category, Depth 3.

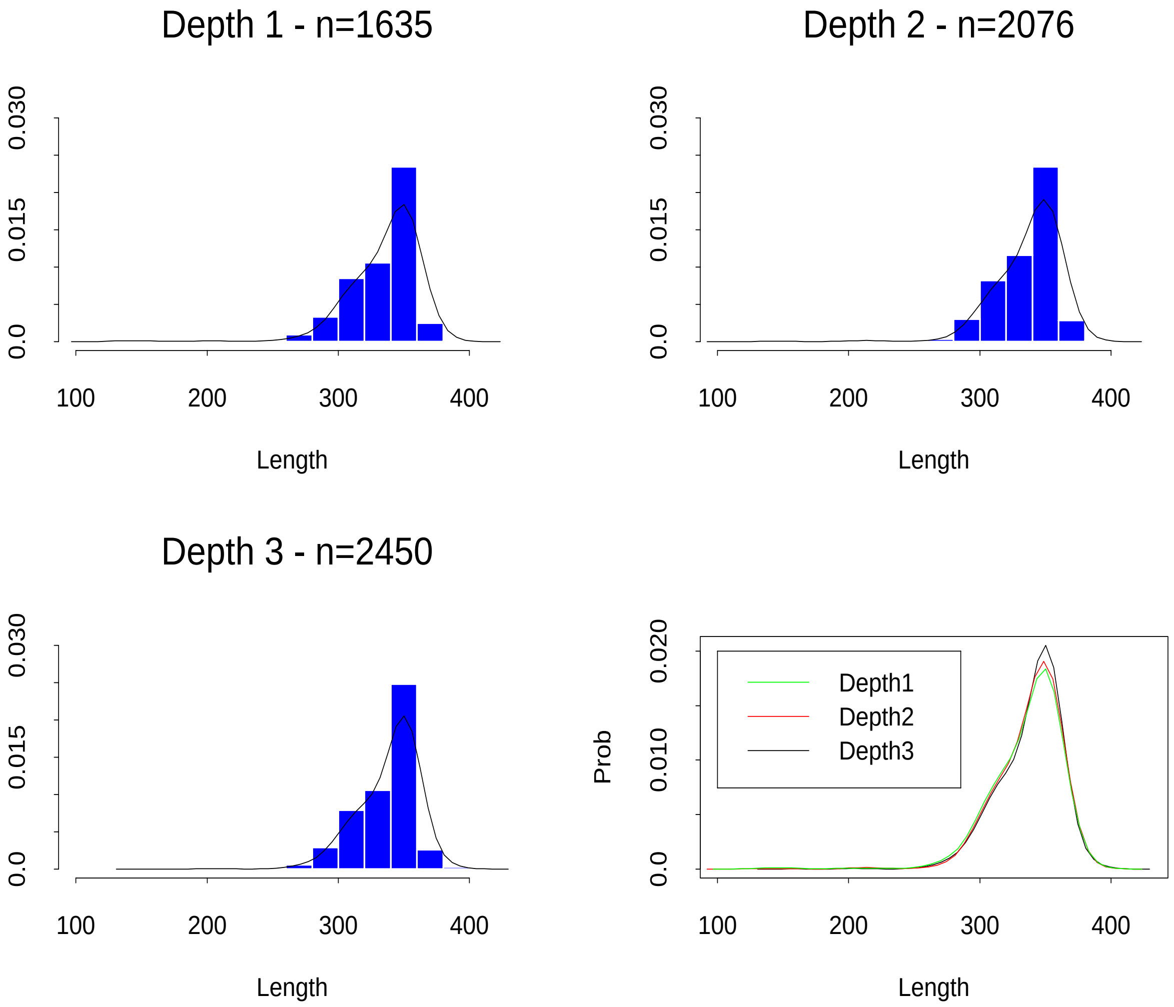

Figure 2.4: Histograms of Lengths of herring in the Vestfjord system by Depth category.

2.4.3 Distribution of Length given Location and Depth category

Similarities between the V-T areas are apparent here again. Note that the lack of smoothness in the histogram for Ofotfjord is due to the small sample sizes.

Specially in the Tysfjord area, we see how larger fish seem to place themselves at deeper depths.

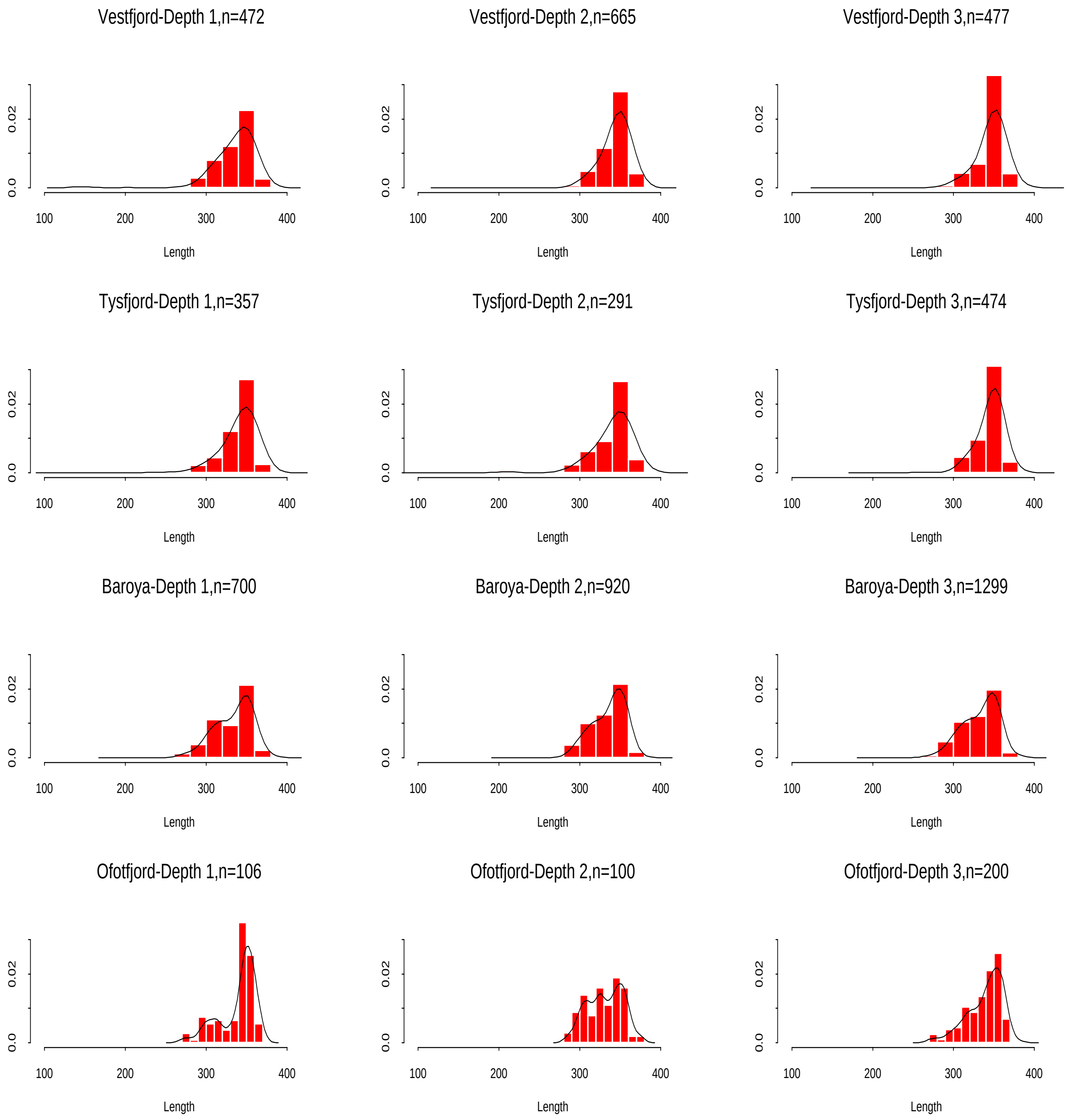

Figure 2.5: Histograms of Lengths of herring in the Vestfjord system by Location and Depth category.

2.4.4 Distribution of Length given Location, Depth, and Time category

The plots presented here give us an opportunity to explore the interactions between the considered factors. Note that we are only including data from Vestfjord and Barøya since for the other two regions the data is insufficient.

The time categories utilised are two: Day, which includes the times from 9.00 until 15.00 hours, and Night including the rest of the time of the day.

Vestfjord

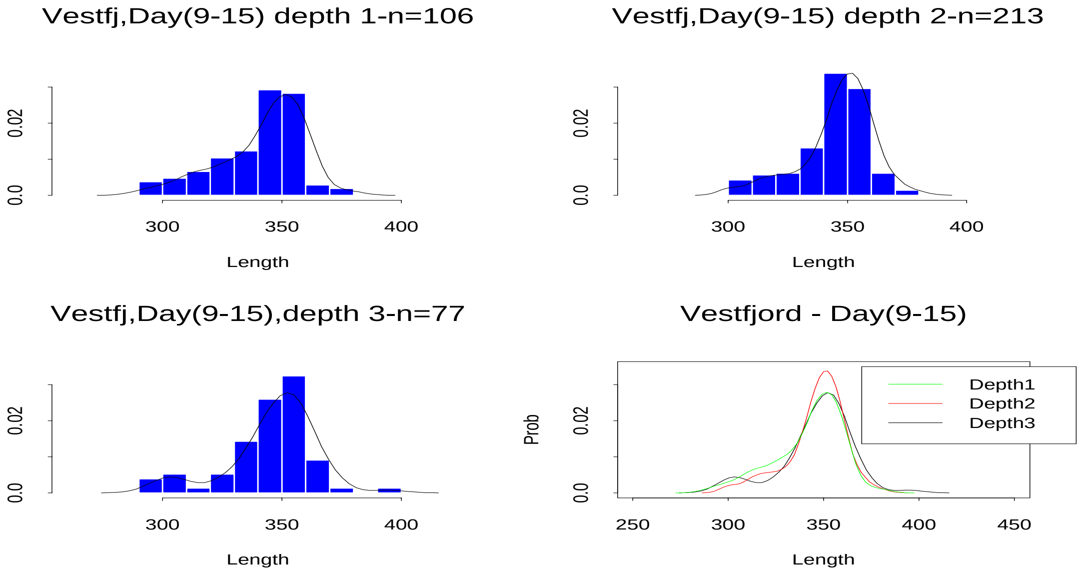

Figure 2.6: Histograms of Lengths of herring in the Vestfjord location by Depth and Time category Day.

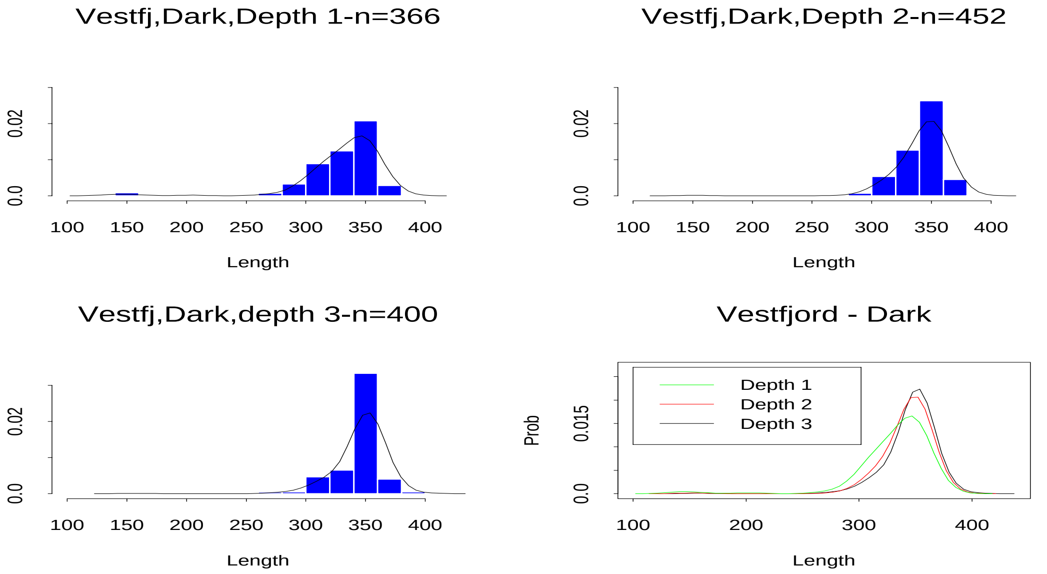

Figure 2.7: Histograms of lLengths of herring in the Vestfjord location by Depth and Time category Dark.

Barøya

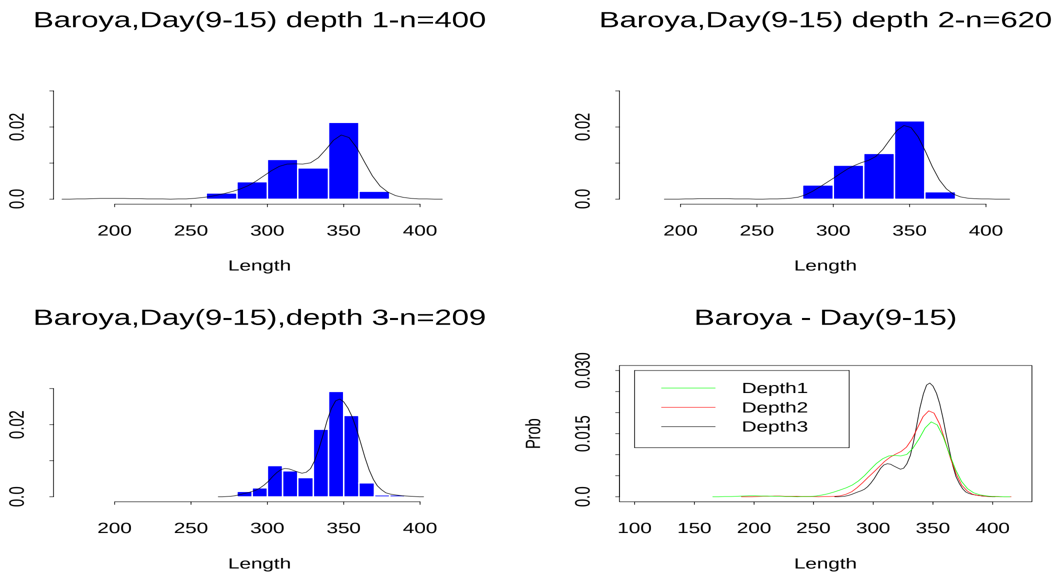

Figure 2.8: Histograms of lLengths of herring in the Barøya location by Depth and Time category Day.

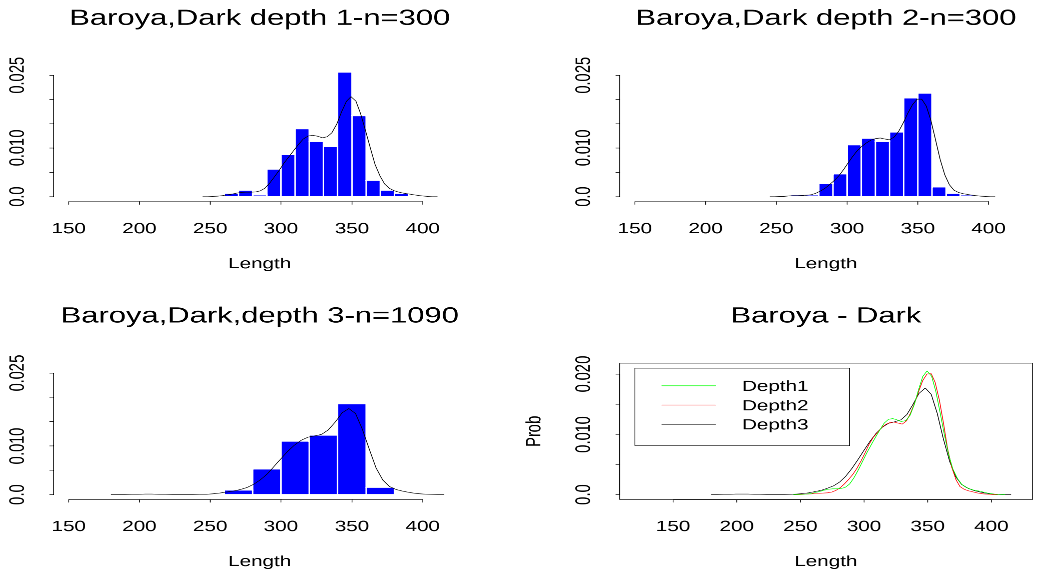

Figure 2.9: Histograms of Lengths of herring in the Barøya location by Depth and Time category Dark.

2.5 Description of the Age-Length sampled data

In total, 528 measurements of the age of herring were obtained. These observations were obtained in two locations namely, Barøya (436 measurements) and the location in the inner bay of the Ofotfjord (92 measurements). The observed frequencies and relative frequencies can be found in Table 2.3. The data is plotted in Figures 2.10 and 2.11. The lines in Figure 2.12 represent the smoothed age-length data using a non-parametric smoother.

Age Group (in years)	Barøya		Inner Ofotfjord		Total	
	Count	Proportion	Count	Proportion	Count	Proportion
1	1	0.0023	2	0.0217	3	0.0057
2	0	0.0000	1	0.0109	1	0.0019
3	26	0.0596	15	0.1630	41	0.0777
4	135	0.3096	39	0.4239	174	0.3295
5	41	0.0940	8	0.0870	49	0.0928
6	64	0.1468	14	0.1522	78	0.1477
7	1	0.0023	0	0.0000	1	0.0019
8	15	0.0344	2	0.0217	17	0.0322
9	19	0.0436	1	0.0109	20	0.0379
10	73	0.1674	6	0.0652	79	0.1496
11	48	0.1101	2	0.0217	50	0.0947
12	9	0.0206	1	0.0109	10	0.0189
13	2	0.0046	1	0.0109	3	0.0057
14	0	0.0000	0	0.0000	0	0.0000
15	0	0.0000	0	0.0000	0	0.0000
16	0	0.0000	0	0.0000	0	0.0000
17	1	0.0023	0	0.0000	1	0.0019
18	0	0.0000	0	0.0000	0	0.0000
19	1	0.0023	0	0.0000	1	0.0019
Total	436		92		528	

Table 2.3: Observed frequencies and relative frequencies of Ages - Dec 2002.

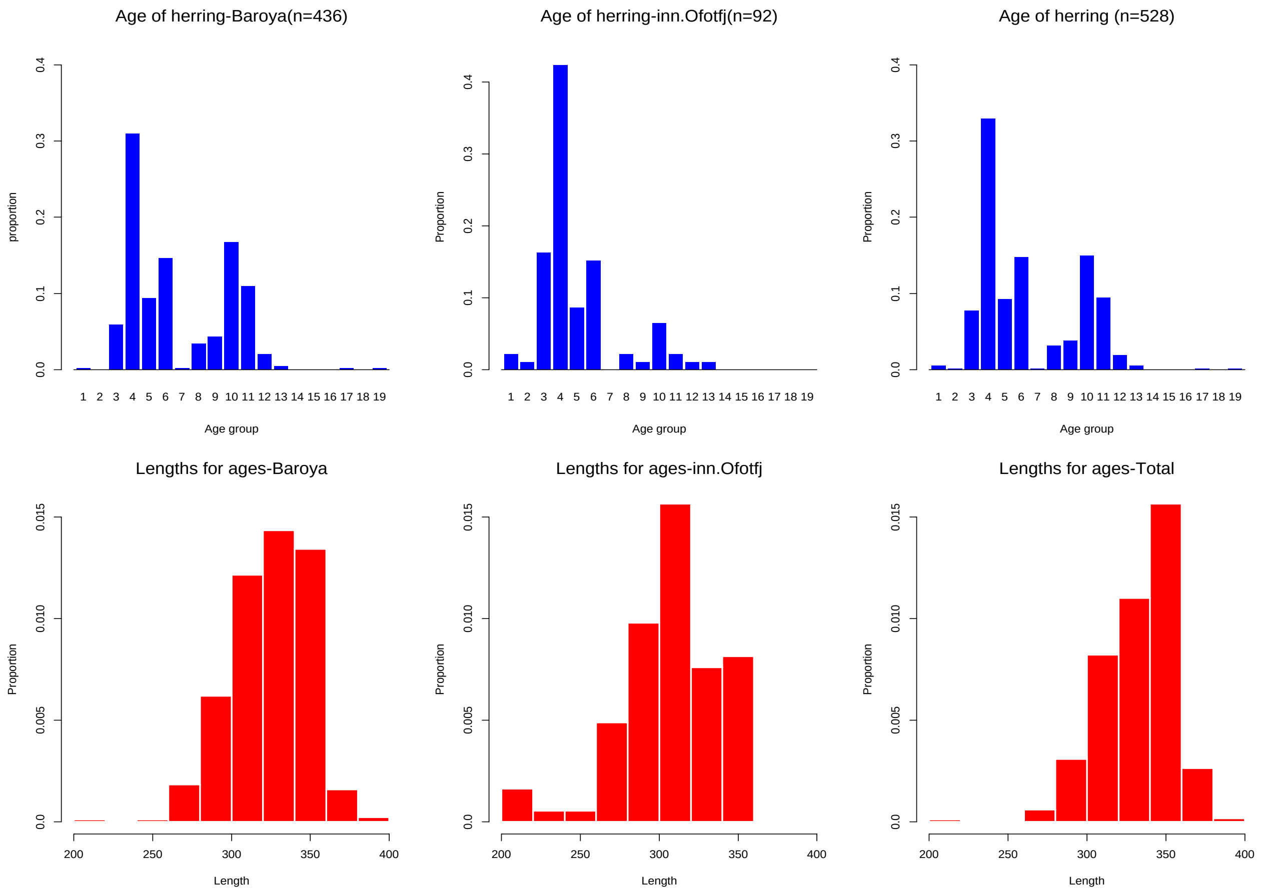

Figure 2.10: Histograms of observed relative frequencies of Age and corresponding Lengths of herring - Dec 2002.

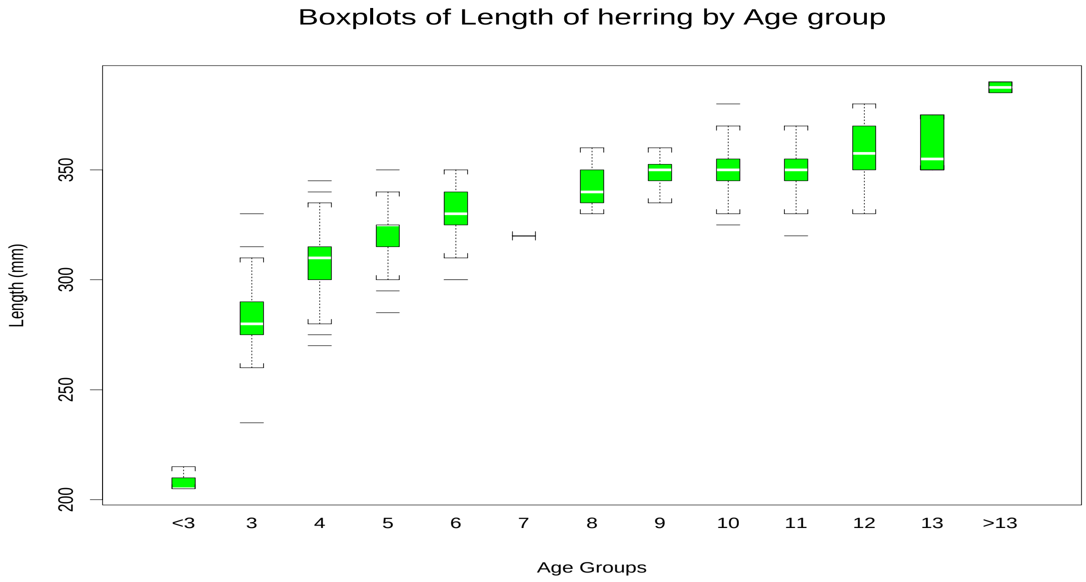

Figure 2.11: Boxplots of Length of herring by Age group - Dec 2002.

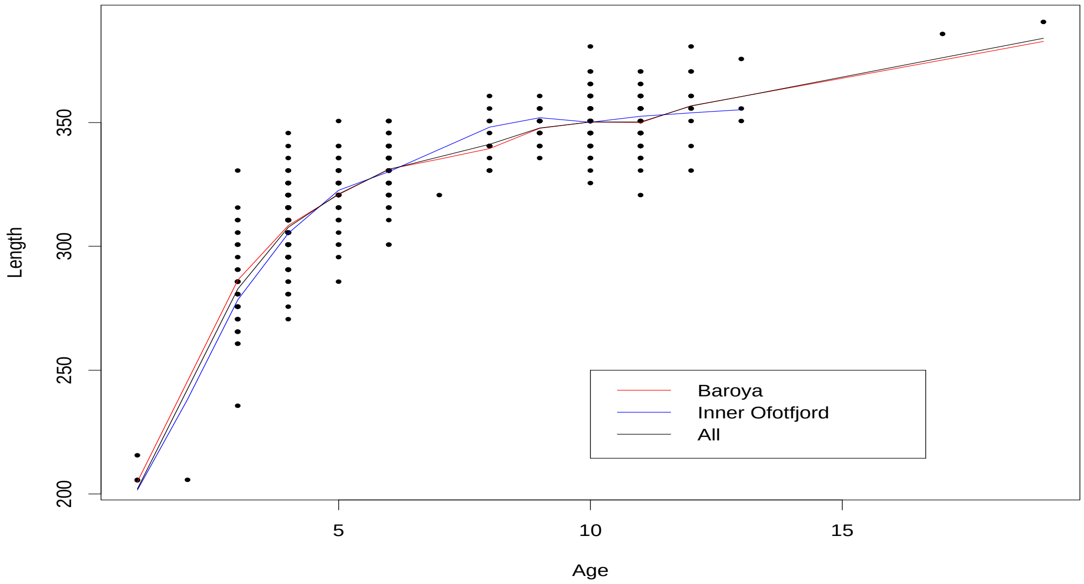

Figure 2.12: Scatter plot of Age vs. Length of herring - Dec 2002.

2.5.1 An earlier experiment - Nov 2002

Earlier trials were conducted on Nov 1, 23, 26, 29, 30 and December 1, 2, and 3, 2002. these trials were carried out in similar conditions as the ones described in the previous section. No sampling was done in the Vestfjord. Briefly, the experiment is summarised below. Figure 2.13 illustrates the geographical positions where sampling occurred.

- Tysfjord: 8 sampling positions, 677 samples obtained.
- Barøya: 3 sampling positions, 300 samples obtained.
- Ofotfjord: 4 sampling positions, 351 samples obtained.
- Inner Ofotfjord: 1 sampling position, 100 samples obtained.

Description of Age-Length data

In total, 948 measurements of age and length of herring were obtained. These observations were obtained in four locations namely, Barøya (279 measurements), the location in the inner bay of the Ofotfjord (98 measurements), Ofotfjord (313), and Tysfjord (258). The frequencies and relative frequencies can be found in Table 2.4. The data is also displayed in Figures 2.14, 2.15, 2.16, and 2.17.

Observing Figures 2.14 and 2.15, it is interesting to note how the observed age distributions are different in the Tysfjord and Barøya-Ofotfjord areas. The dominating age classes are 4 and 10 years. However the 10 year old class seems to be the most abundant in the Tysfjord area wheras the 4 year old class seems to be the most abundant in the Barøya-Ofotfjord area. In spite of the differences in the age distributions, we expect the age-length relationship to be the same in both areas since the habitat does not differ widely between regions. This is further confirmed in Figure 2.17.

Age Group (in years)	Barøya		Inner Ofotfjord		Ofotfjord		Tysfjord		Total	
	Count	Prop	Count	Prop	Count	Prop	Count	Prop	Count	Prop
1	0	0.000	1	0.010	1	0.003	0	0.000	2	0.002
2	1	0.004	3	0.031	3	0.010	2	0.007	9	0.009
3	18	0.065	11	0.112	16	0.051	2	0.007	47	0.050
4	87	0.312	30	0.306	100	0.319	39	0.136	256	0.270
5	22	0.079	10	0.102	36	0.115	25	0.087	93	0.098
6	31	0.111	19	0.194	35	0.112	35	0.122	120	0.127
7	1	0.004	0	0.000	2	0.006	4	0.014	7	0.007
8	11	0.039	1	0.010	8	0.026	5	0.017	25	0.026
9	16	0.057	7	0.071	13	0.042	17	0.059	53	0.056
10	53	0.190	12	0.122	54	0.173	80	0.279	199	0.210
11	30	0.108	4	0.041	35	0.112	41	0.143	110	0.116
12	8	0.029	0	0.000	8	0.026	5	0.017	21	0.022
13	0	0.000	0	0.000	0	0.000	0	0.000	0	0.000
14	0	0.000	0	0.000	0	0.000	0	0.000	0	0.000
15	0	0.000	0	0.000	1	0.003	0	0.000	1	0.001
16	0	0.000	0	0.000	0	0.000	0	0.000	0	0.000
17	1	0.004	0	0.000	0	0.000	3	0.010	4	0.004
18	0	0.000	0	0.000	0	0.000	0	0.000	0	0.000
19	0	0.000	0	0.000	1	0.003	0	0.000	1	0.001
Total	279		98		313		258		948	

Table 2.4: Observed frequencies and relative frequencies of Ages - Nov 2002.

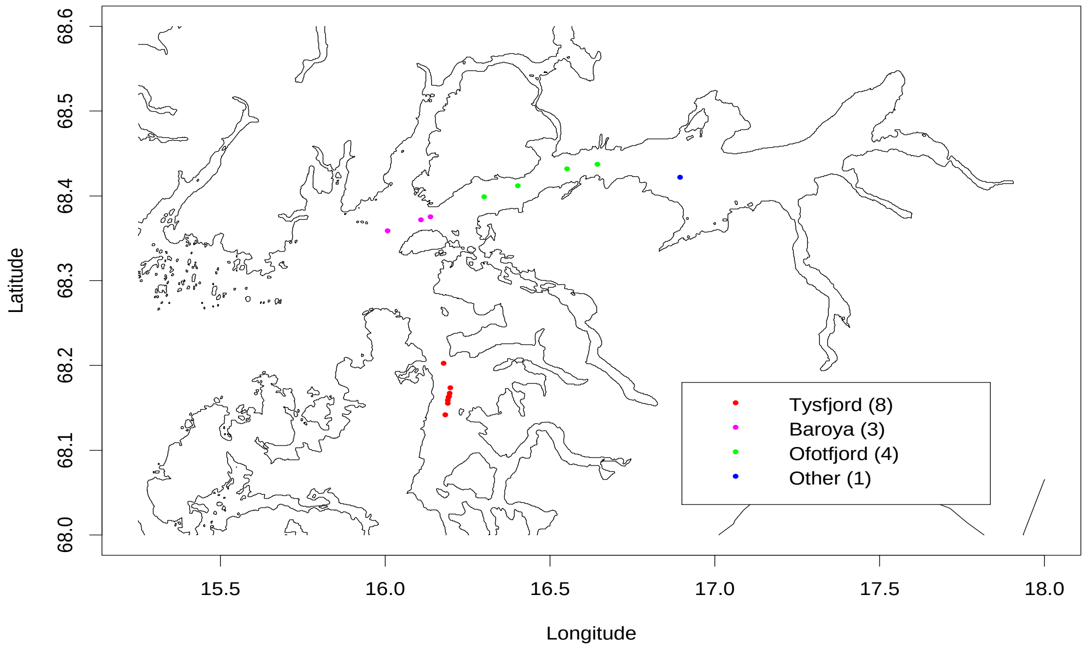

Figure 2.13: Map of sampling locations of the Nov 2002 experiment.

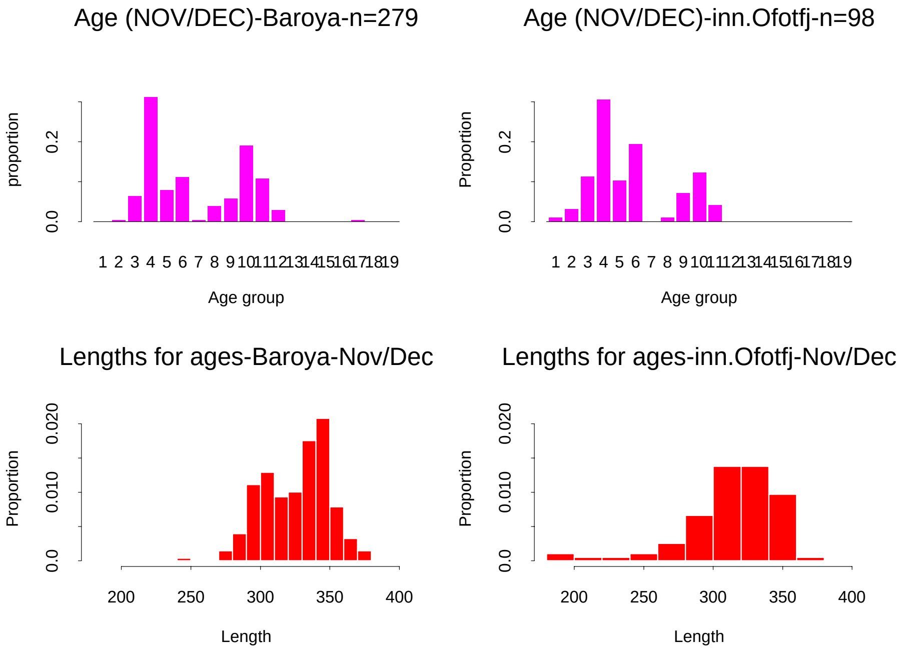

Figure 2.14: Histograms of relative frequencies of Age and corresponding Lengths of herring - Barøya and Inner Ofotfjord - Nov 2002.

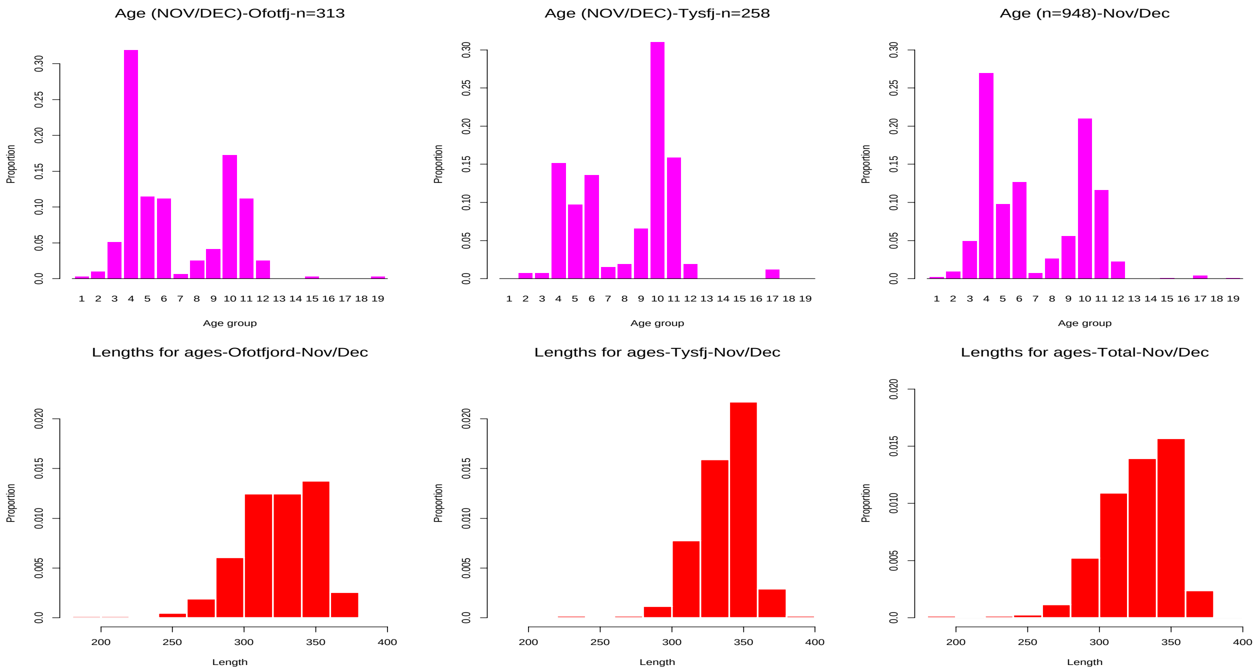

Figure 2.15: Histograms of relative frequencies of Age and corresponding Lengths of herring - Tysfjord, Ofotfjord, and all data - Nov 2002.

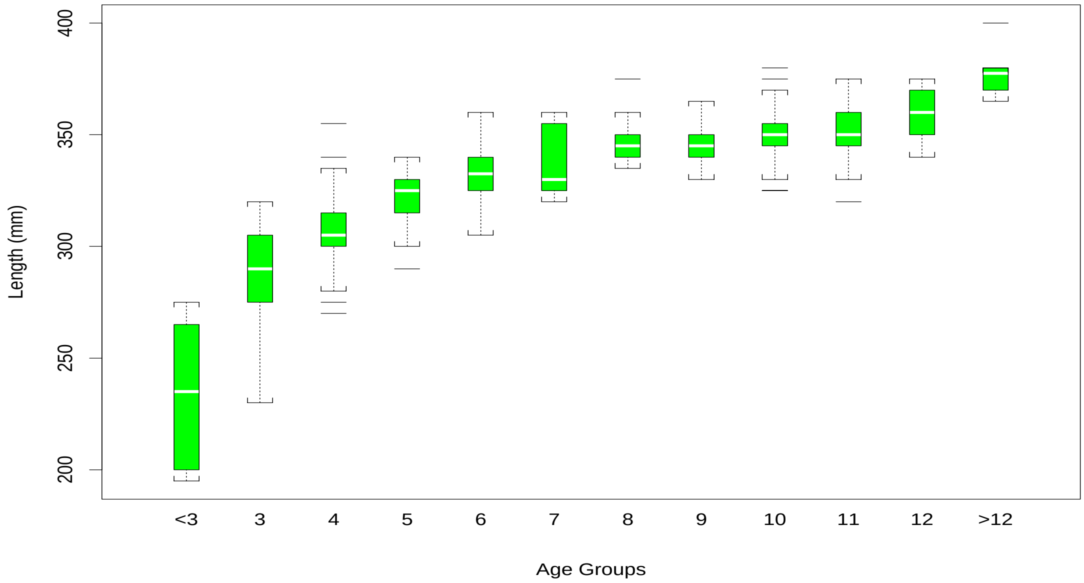

Figure 2.16: Boxplots of Length of herring by Age group - Nov 2002.

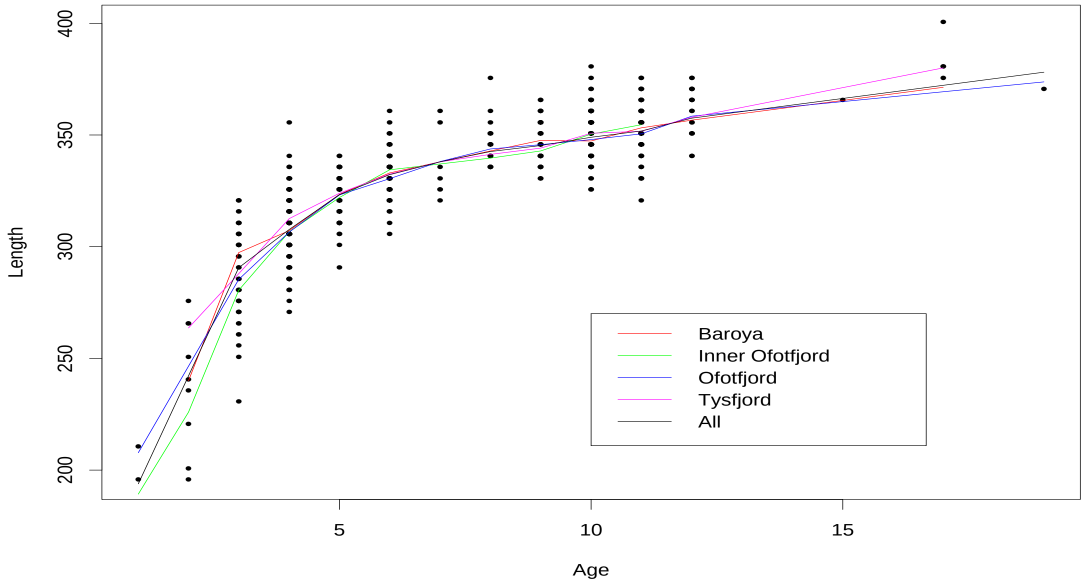

Figure 2.17: Scatter plot of Age vs. Length of herring - Nov 2002.

2.5.2 Pooling all the data, from the pilot and the main experiment

Our objective is to study the age-length relationship in order to deduce the age distributions from the length distributions. In this sense, the more age-length pairs we have at our disposition, the better. So, the age-length data from the two experiments, conducted in November and December 2002, will be merged.

In total, 1476 measurements of the age of herring were obtained. These observations were obtained in four locations namely, Barøya (715 measurements), the location in the inner bay of the Ofotfjord (190 measurements), Ofotfjord (313), and Tysfjord (258). The observed frequencies and relative frequencies can be found in Table 2.5. Figures 2.18, 2.19, 2.20, and 2.21 display the data in graphical way.

Age Group (in years)	Barøya		Inner Ofotfjord		Ofotfjord		Tysfjord		Total	
	Count	Prop	Count	Prop	Count	Prop	Count	Prop	Count	Prop
1	1	0.001	3	0.004	1	0.001	0	0.000	5	0.003
2	1	0.001	4	0.006	3	0.004	2	0.003	10	0.007
3	44	0.062	26	0.036	16	0.022	2	0.003	88	0.060
4	222	0.310	69	0.097	100	0.140	39	0.055	430	0.291
5	63	0.088	18	0.025	36	0.050	25	0.035	142	0.096
6	95	0.133	33	0.046	35	0.049	35	0.049	198	0.134
7	2	0.003	0	0.000	2	0.003	4	0.006	8	0.005
8	26	0.036	3	0.004	8	0.011	5	0.007	42	0.028
9	35	0.049	8	0.011	13	0.018	17	0.024	73	0.049
10	126	0.176	18	0.025	54	0.076	80	0.112	278	0.188
11	78	0.109	6	0.008	35	0.049	41	0.057	160	0.108
12	17	0.024	1	0.001	8	0.011	5	0.007	31	0.021
13	2	0.003	1	0.001	0	0.000	0	0.000	3	0.002
14	0	0.000	0	0.000	0	0.000	0	0.000	0	0.000
15	0	0.000	0	0.000	1	0.001	0	0.000	1	0.001
16	0	0.000	0	0.000	0	0.000	0	0.000	0	0.000
17	2	0.003	0	0.000	0	0.000	3	0.004	5	0.003
18	0	0.000	0	0.000	0	0.000	0	0.000	0	0.000
19	1	0.001	0	0.000	1	0.001	0	0.000	2	0.001
Total	715		190		313		258		1476	

Table 2.5: Observed frequencies and relative frequencies of Ages - Nov, Dec. 2002.

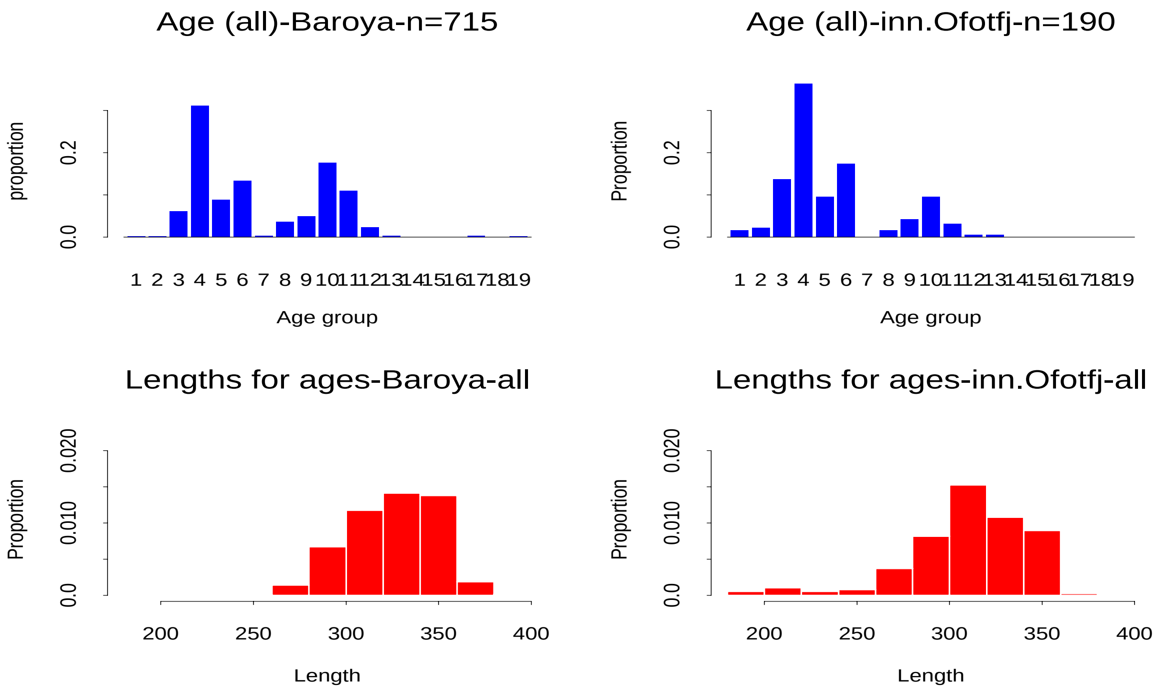

Figure 2.18: Histograms of Age and corresponding Lengths of herring - Barøya and Inner Ofotfjord - Nov, Dec 2002.

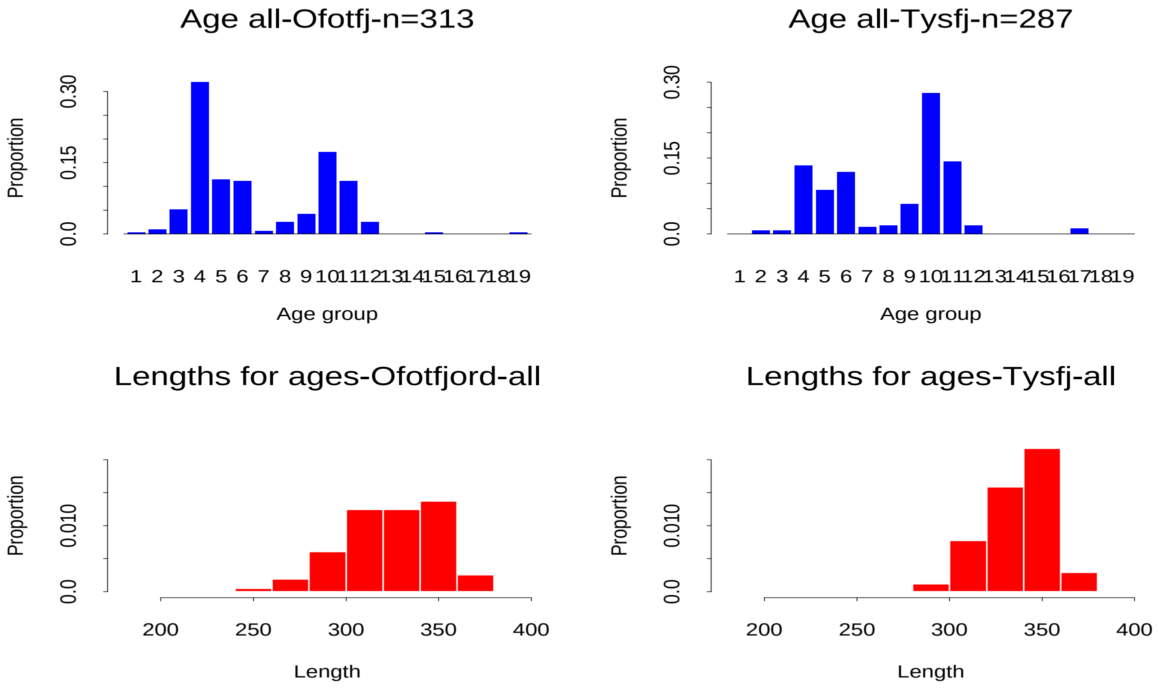

Figure 2.19: Histograms of Age and corresponding Lengths of herring - Tysfjord, Ofotfjord - Nov, Dec 2002.

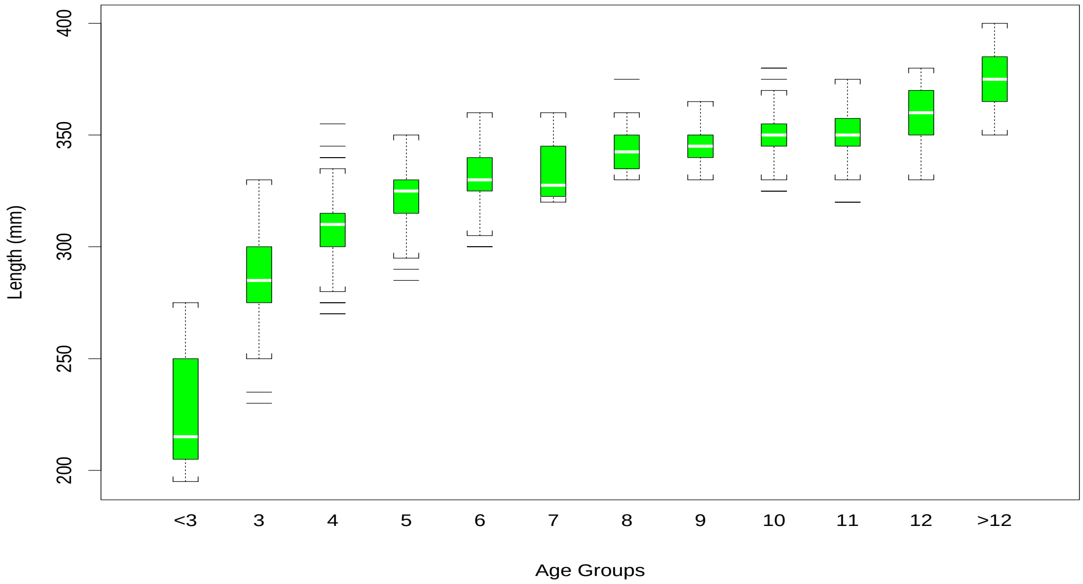

Figure 2.20: Boxplots of Length of herring by Age group - all data.

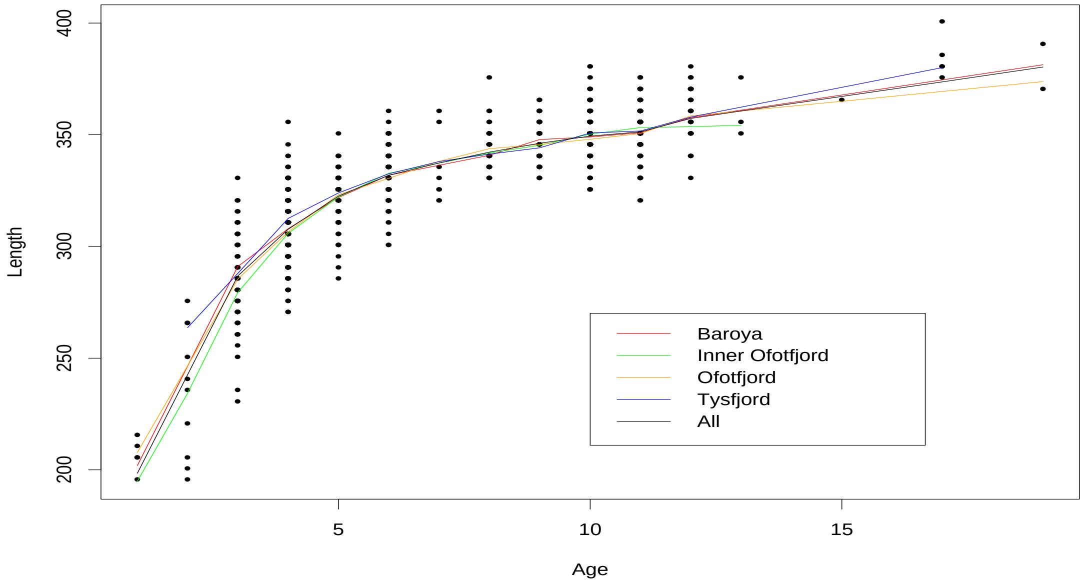

Figure 2.21: Scatter plot of Age vs. Length of herring. All data

Chapter 3

Obtaining the Age Distribution from Length Data

As will be seen, some attributes and differences within the population of herring in the Vestfjord system will become more clear if the age distribution, rather than the length distribution, is studied.

Utilizing the sampled data which was described before, we will deduce the age distribution of the population using a method known as calibration, which has the inverse concept of prediction.

The age-length pairs will be used to construct a calibration table, which will be the key to convert lengths into ages. The length data will be used to determine the age distribution that will characterise the population.

3.1 A model for the growth of fish in length

Using the entire collection of pairs of length and age, sampled as described in the previous section, we will first study the relationship between these two variables. For this purpose, a modified version of the von Bertalanffy (1938) growth model will be used. See also Sandland et. al. (1979). In what follows L will denote observed length.

The von Bertalanffy growth curve, widely used in fisheries, is the solution of the differential equation

$$\frac{dL(t)}{dt} = J - KL(t), \tag{3.1}$$

where L is the function that represents the length of the fish, and t represents time. The constants J and K are usually unknown. The ratio J/K is the limit of L as t tends to infinity and is interpreted as the maximum potential length that a fish can reach. The parameter K governs the rate of growth.

The von Bertalanffy model (3.1) is a deterministic model and may result somewhat inflexible to study real data. To allow for the unavoidable noise present in collected data, a more flexible approach is proposed. One possibility is to turn the deterministic equation (3.1) into a stochastic differential equation by the addition of white noise

$$dL(t) = [J - KL(t)]dt + d\,b(t) \tag{3.2}$$

where b is Brownian motion with variance $\sigma^2 t$.

Another possibility is to assume that length at age a, $L(a)$, is observed with error and that the true length, $\lambda(a)$, follows (3.2). Also, instead of time t, we consider its equivalent, age a. The resulting model is

$$L(a) = \lambda(a) + \nu(a)\eta(a), \qquad d\lambda(a) = [J - K\lambda(a)]\,da + d\,b(a), \tag{3.3}$$

where η is a white noise process with variance σ^2 and ν is non–random. The process λ is a latent process and hence is not directly observable. It represents the true length. The constants $\nu(a)$ represent the age-specific noise-to-signal ratios: we expect ν to be larger than one; if so the signal is sharper than the measurement noise. Note that different ages will be measured with different precisions and also the different age categories will be represented in rather different proportions in the sample. The constants $\nu(a)$ will pick up these effects.

Model (3.3) is a continuous state-space model and, moreover, a version of the Ornstein-Uhlenbeck process defined and exemplified in de Jong and Mazzi (2001). Inference on this model can be done using Kalman filtering techniques as described in de Jong (1991) and de Jong and Mazzi (2001).

3.1.1 Inferential results

The signal, λ, is predicted from the the observed pairs $(a, L(a))$ using the best linear predictor method. The parameters are estimated using maximum likelihood, assuming that the errors in the measurement and state equations are independent and Gaussian. Figure 3.1 depicts the estimated signal together with confidence bands which are given by ±2 times the square root of the mean squared error of prediction. The model parameter estimates are listed in Table 3.1 below.

J	K	σ^2	$\nu_1 = \nu_2 = \nu_3$	$\nu_4 = \nu_5 = \nu_6 = \nu_{11} = \cdots = \nu_{19}$	ν_7	$\nu_8 = \nu_9 = \nu_{10}$
111.64	0.30	23.81	4.40	2.25	3.13	1.81

Table 3.1: Parameter estimates of Model 3.3

The estimated asymptotic length, J/K, is 372 mm. The different values of ν represent different measurement errors which likely account for the rather small sample sizes at ages 1,2,3, and 7. Regarding the units, ν and K are dimensionless and J, J/K, and σ are in mm.

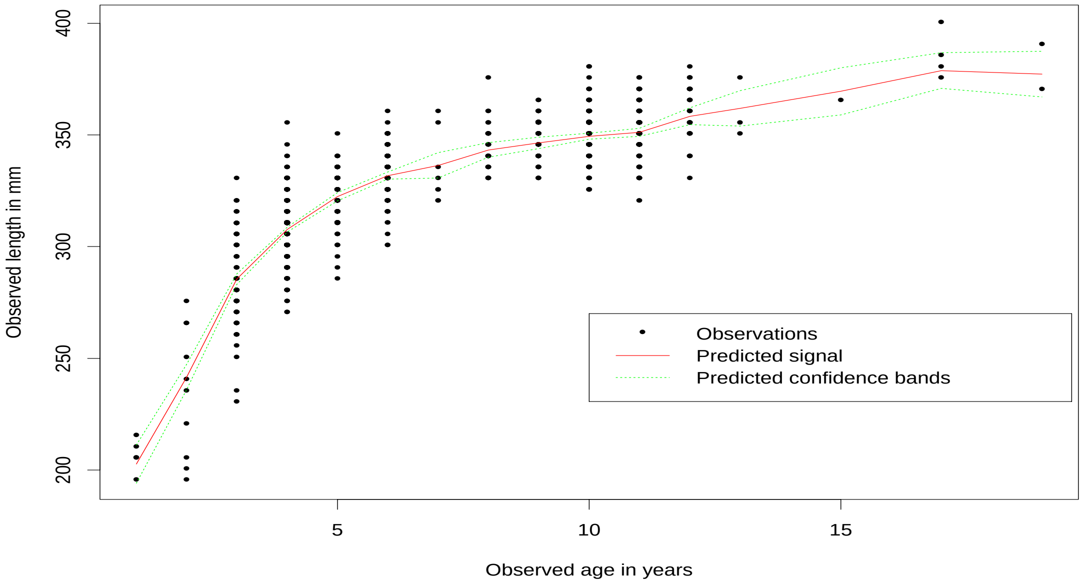

Figure 3.1: Estimated signal for model.

3.1.2 Model adequacy checking

In order to verify that Model (3.3) provides a good fit to the observed data, we can use the prediction residuals or innovations which, provided the model is adequate, after normalisation, are i.i.d standard normal. The ith innovation is defined as $L_i - E[\lambda_i | L_1, \ldots, L_{i-1}]$. The (standardised) innovations are obtained directly after one pass of the Kalman Filter.

It is observed that 5.71% of the residuals are outside $[-2, 2]$. Figure 3.2 (a) is a histogram of the standardised innovations superimposed with a kernel estimate of the density and the standard Gaussian density. Figure 3.2 (b) is a boxplot of the standardised residuals, Figure 3.2 (c) is a Q-Q plot and, finally, Figure 3.2 (d) has boxplots of the standardised residuals separated by corresponding observed age. No anomalies are detected in these plots pointing towards lack of fit.

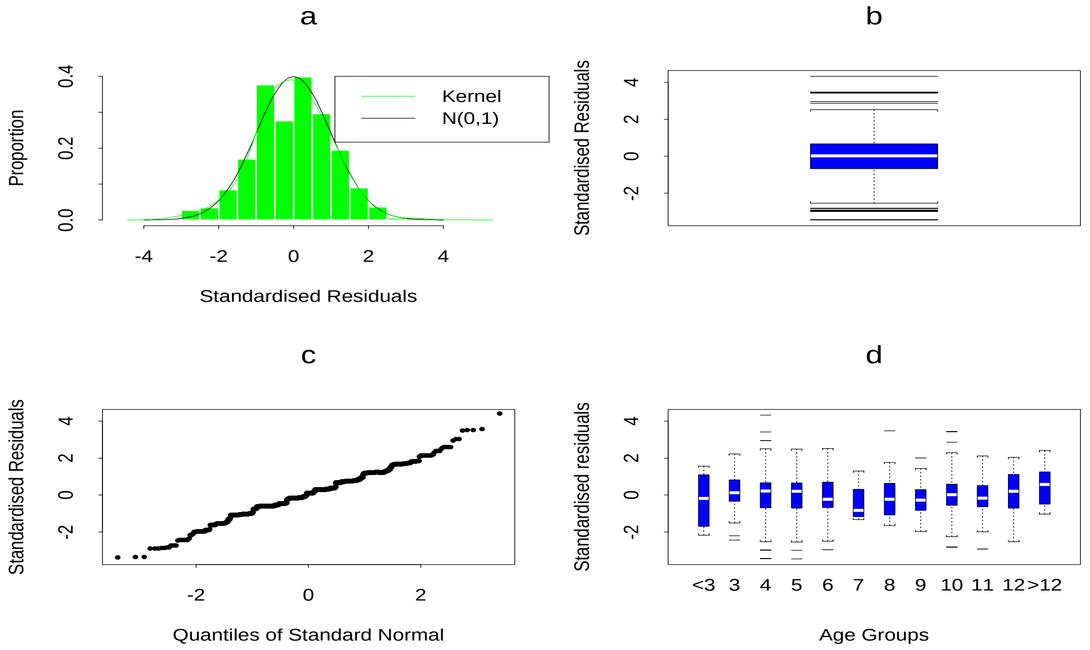

Figure 3.2: Some residuals plots for Model 3.3.

3.2 Calibration of age from length

The problem we are presented with is one of (non-linear) calibration: given the length of a fish, estimate its age. This is the inverse problem of prediction, which in our case would be: given the age of a fish predict its length.

Given a length L^*, and assuming that Model (3.3) holds, with the parameters J, K, and ν equal to those given in Table 3.1, we will predict the corresponding age, a^*, as

$$a^* = \arg\max_a l\left((\mathcal{A}, a), (\mathcal{L}, L^*)\right)$$

where $l((\mathcal{A}, a), (\mathcal{L}, L^*))$ is the likelihood of $(\mathcal{L}, L^*)$, observed at $(\mathcal{A}, a)$, where $\mathcal{A}$ and $\mathcal{L}$ are the observed vector of ages and corresponding lengths with which Model 3.3 was fitted.

This method gives the calibration Table 3.2 below.

Note that in this calibration, we have assumed that since no observations were made at ages 14, 16 and 18, then the chance of observing these ages somewhere else, this year, is minimal. Hence, there is no possibility of calibrating an age to be 14, 16, and 18. This was done mainly for comparative purposes (see Section 3.2.2). However, the method is general enough to allow for ages that were not observed in the sample $(\mathcal{A}, \mathcal{L})$ to be calibrated. Also, lengths outside the observed range of lengths in the sample $(\mathcal{A}, \mathcal{L})$ can be used to obtain their corresponding age. The minimum and maximum lengths in Table 3.2 are taken from our particular sample of lengths from which ages are to be calibrated.

3.2.1 Accuracy of the calibration method

In order to assess the accuracy of the calibration method described in Section 3.2 we take the observed lengths in the sample $(\mathcal{A}, \mathcal{L})$, used to fit Model 3.3, and calibrate the ages. Then we compare the calibrated age with the observed age. A deviation is defined as the calibrated age minus the observed age. Hence, if the deviation is positive we are overestimating the age.

From Table 3.4 we have that the method calibrates 43.5% of the ages exactly, and 33% within one year of the true age. More than 90% of the ages are calibrated within 2 years of the true age. Also, there is a slight tendency for underestimation, mainly caused by the fact that ages are calibrated only up to 17 years and the observed ages range from 1 to 19 years.

Length (mm)	Calibrated age (years)
125–225	1
230–265	2
270–290	3
295–315	4
320–325	5
330–335	6
340	8
345	9
350–355	10
360	12
365	13
370	15
375–400	17

Table 3.2: Calibration Table

It is expected, due to the shape of the growth curve, that the calibration will be more imprecise at higher ages. This can be seen from Table 3.5 and Figure 3.3 (c).

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
-9.00	-1.00	0.00	0.02	1.00	6.00

Table 3.3: Summary statistics of the vector of deviations.

Absolute Deviation =	Proportion
0	0.435
1	0.330
2	0.140
3	0.042
4	0.029
5	0.018
6	0.005
7	0.001
8,9	0.001

Deviation	Proportion
< 0	0.310
> 0	0.255
$\in [-1, 1]$	0.765
$\in [-2, 2]$	0.905
$\in [-3, 3]$	0.947
$\in [-4, 4]$	0.976

Table 3.4: Distribution of deviations

Deviation	Calibrated Age									
	< 3	3	4	5	6	8	9	10	12	≥ 13
$= 0$	0.357	0.482	0.796	0.300	0.443	0.089	0.107	0.489	0.063	0.081
$\in [-1,1]$	1	0.977	0.971	0.932	0.667	0.241	0.641	0.855	0.381	0.145
$\in [-2,2]$		1	1	0.968	0.797	0.821	0.901	0.936	0.857	0.355
$\in [-3,3]$				0.968	0.854	0.955	0.992	0.948	0.936	0.613
$\in [-4,4]$				0.968	0.932	1	0.992	0.991	0.968	0.791
$\in [-5,5]$				0.999	0.995		1	0.996	0.984	0.903
$\in [-6,6]$				1	1			1	1	0.935
$\in [-7,7]$										0.984
$\in [-8,8]$										0.984
$\in [-9,9]$										1

Table 3.5: Distribution of deviations, discriminated by Age.

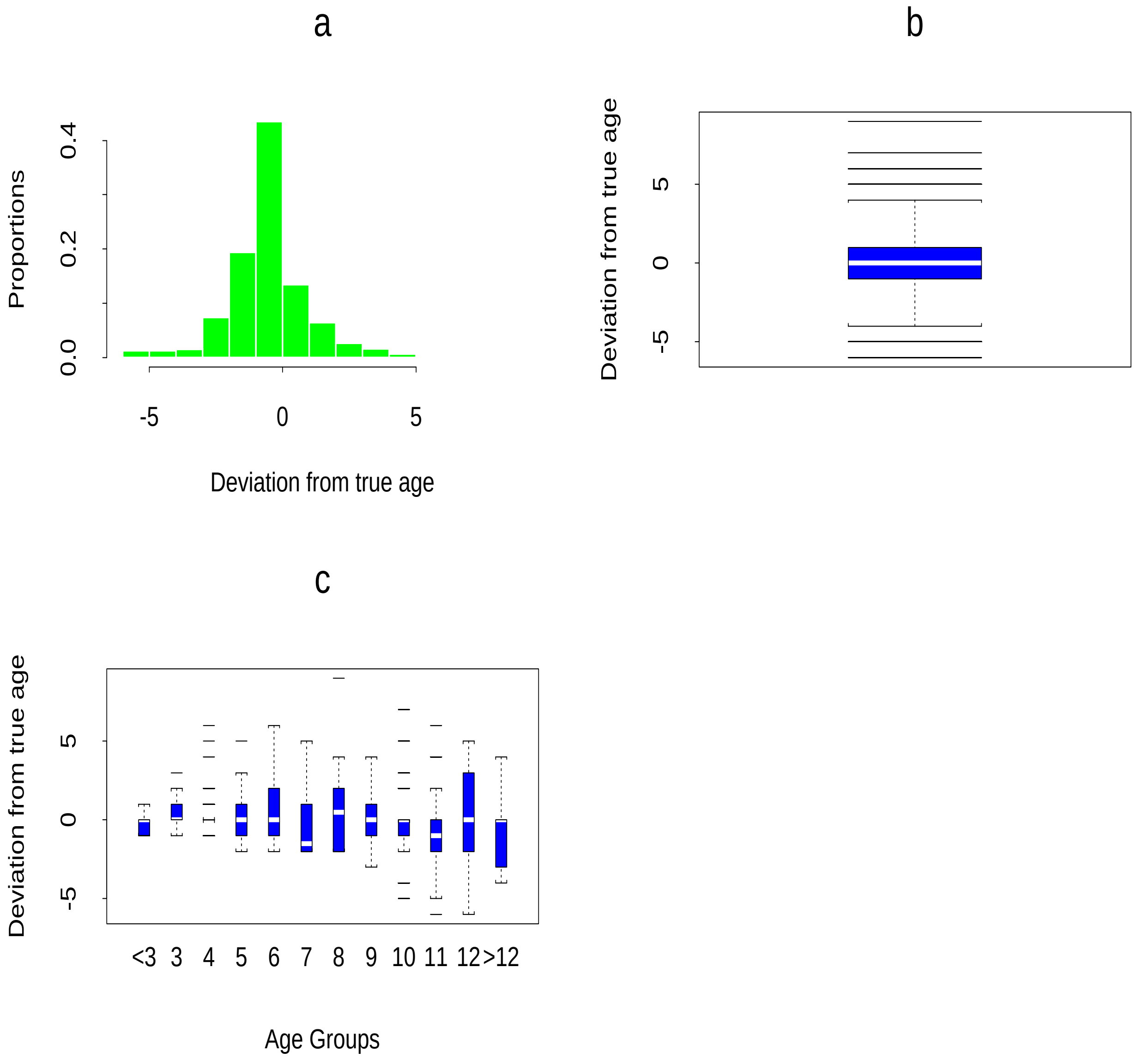

Figure 3.3: Plots to assess the accuracy of the calibration of Age.

3.2.2 Comparison with the IALK method

The IALK method, or Iterated Age Length Key was presented by Kimura and Chikuni (1987). In this method both age and length are treated as discrete variates. The method is described by the following algorithm:

(i) Assume that the age distribution is such that there is equal probability for each age category: if there are n age categories, $\hat{p}_0(a) = 1/n, \ a = 1, \dots, n$.

(ii) Using the observed length-at age distribution, $\hat{p}(l|a)$, and $\hat{p}_0(a)$, compute $\hat{p}(a|l) = \hat{p}(l|a)\hat{p}_0(a)/\sum \hat{p}(l|a)\hat{p}_0(a)$.

(iii) Apply the empirical length distribution, for which the corresponding age distribution is desired, to the age-length key calculated in (ii). Compute a new age distribution $\hat{p}_1(a) = \sum q_l \hat{p}(a|l)$, where q_l is the observed proportion of fish of length l.

(iv) Given $\epsilon > 0$, if $\max_a\{|p_0(a) - p_1(a)|\} < \epsilon$, take $p_1(a)$ as the estimated age distribution. Else, define $p_0(a) = p_1(a)$ and start from (ii) again.

The main difference between the IALK and the calibration method is that the latter assumes continuity of age and length, autocorrelation in the growth curves, and uses the age-length relationship in a much more informative way. Table 3.6 and Figure 3.4 show this clearly: the IALK method essentially only picks up the two peaks of the observed distribution.

In the IALK method, lengths outside the range of observed lengths with ages cannot be calibrated. Also, only observed ages can be predicted from lengths. The calibration method does not have these two restrictions.

Age class	Observed prop.	IALK prop.	Calibration prop.
1	0.003	0.003	0.006
2	0.007	0.005	0.013
3	0.060	0.015	0.058
4	0.291	0.474	0.256
5	0.096	0.000	0.129
6	0.134	0.060	0.130
7	0.005	0.000	0.000
8	0.028	0.000	0.076
9	0.049	0.000	0.089
10	0.188	0.441	0.159
11	0.108	0.000	0.000
12	0.021	0.000	0.043
13	0.002	0.000	0.019
14	0.000	0.000	0.000
15	0.001	0.000	0.012
16	0.000	0.000	0.000
17	0.003	0.001	0.011
18	0.000	0.000	0.000
19	0.001	0.001	0.000

Table 3.6: Observed and estimated Age proportions for IALK and Calibration methods.

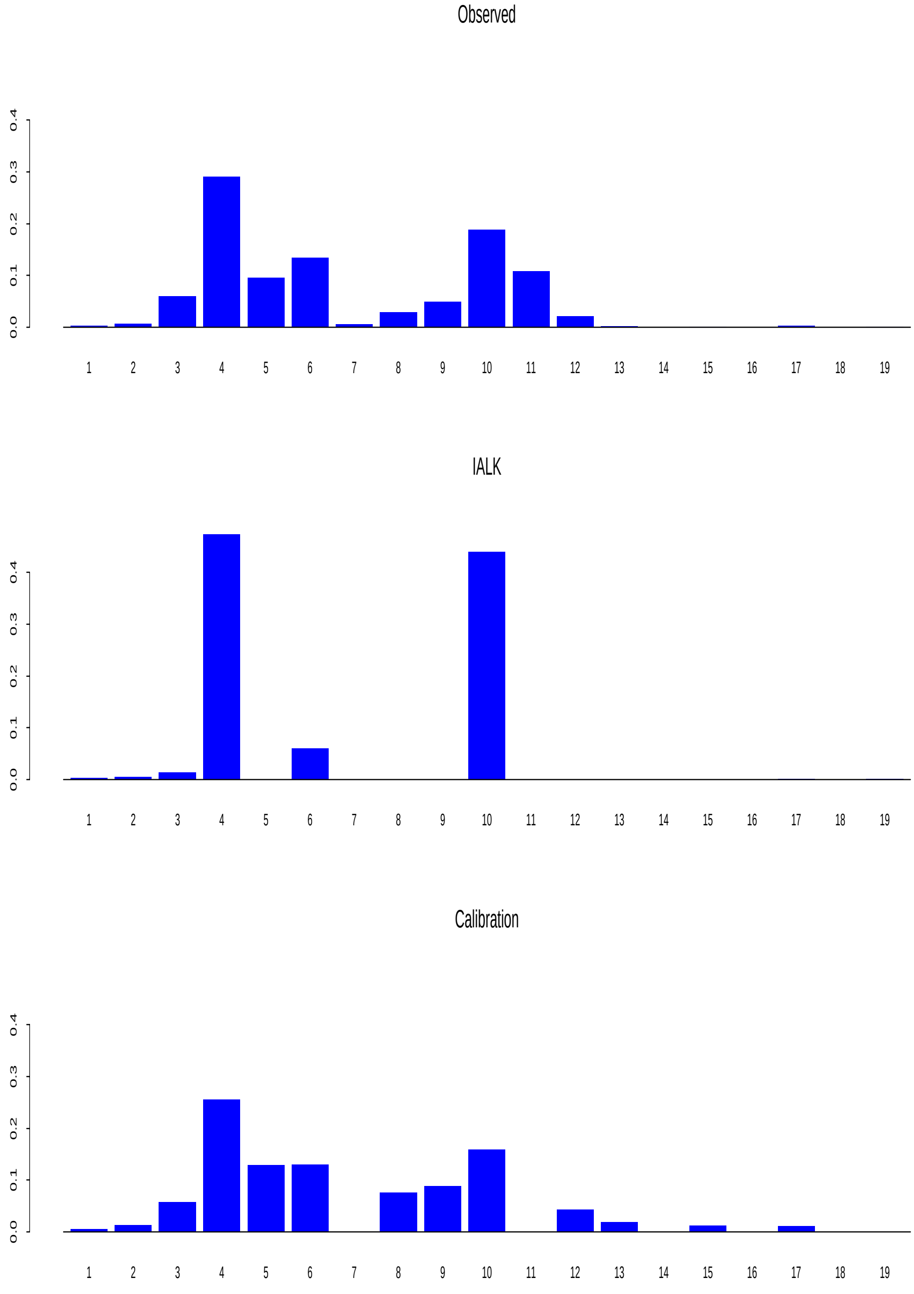

Figure 3.4: Plots of observed and estimated Age proportions for IALK and Calibration methods.

Chapter 4

Calibrated Age Distributions

In this section we calibrate the age distributions from the length data described in the previous sections.

The calibration method has the property that it can be applied to any length data, as long as the fish come from the same species, from a region where the fish had about similar chances to grow in length, and that the environment where the fish grew has not been greatly disturbed for an amount of years equal to at least the maximum age observed. In this sense when sampling for age-length data, it does not matter that the data comes from a stratum of the population with peculiarities that distinguish it from other strata.

Assuming that the population of herring that was sampled has lived in areas which provide equal opportunities for the herring to grow in length, and that the ecosystems along the Norwegian Sea, where they lived, have been reasonable stable in the last twenty years, we will extract the age distributions from the length data, concentrating specially in the different strata identified in Chapter 2.

In principle, this calibration could be applied to data previously collected or data resulting from future sampling experiments. However, because we have some discontinuities in the calibration table (no ages were calibrated at 7 and 11 years, and no data was observed at ages 14, 16, and 18) this is not recommended. We would need at least 3 consecutive years of data in order to obtain a smoother calibration table. This would allow to better estimate the right tail (ages greater than 10) of the age distribution.

Figure 4.1 shows the calibrated ages for the entire sample obtained in the December 2002 experiment. As a whole, we see that the class of 10 year old fish is dominating. The next dominating class is that of four year old fish.

However, as will be shown in what follows, this distribution does not prevail in the different regions of the fjord that were sampled.

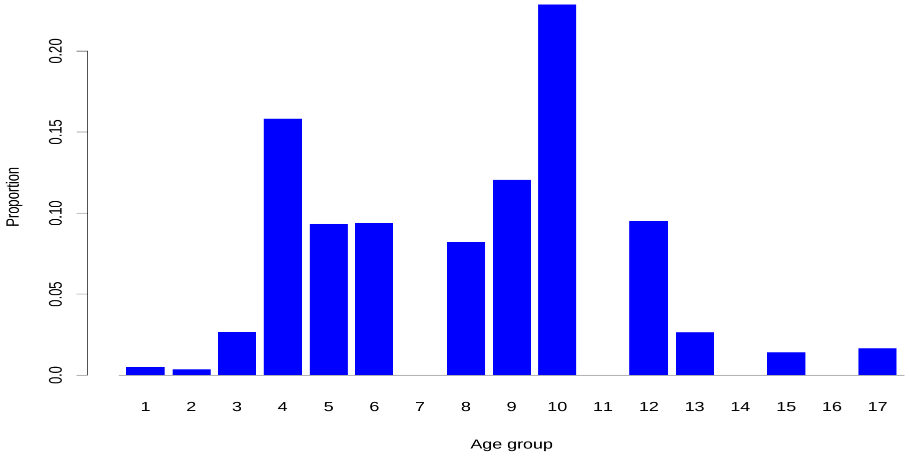

Figure 4.1: Calibrated Age of all data.

4.1 Calibrated age distribution by subregion

In this section it is clearly seen what had been noted observing the length distribution (see Chapter 2). We clearly see here how there seems to be a divide: the distribution of the ages of fish seems to be quite similar in the Vestfjord and Tysfjord, and also the distributions of Barøya and Ofotfjord. See Figures 4.2 and 4.3. Some χ^2 tests were run to further verify this. When the four populations are considered together, the χ^2 test result is that the four populations have different age distributions. From Table 4.1 we see that there are large discrepancies between the populations at ages 4 and 10. If we compare the age distributions of Tysfjord and Vestfjord subpopulations using a χ^2 test no differences are detected. When comparing the Barøya and Ofotfjord subpopulations, the result of the χ^2 test is that the distributions are different but this is mainly due to one single discrepancy in the > 10 age class for Ofotfjord. If we carry out the test excluding the last age category then no differences in the age distributions are detected. See Table 4.4

Note that the expected counts in the contingency tables are computed assuming that the age distributions of the considered groups are the same.

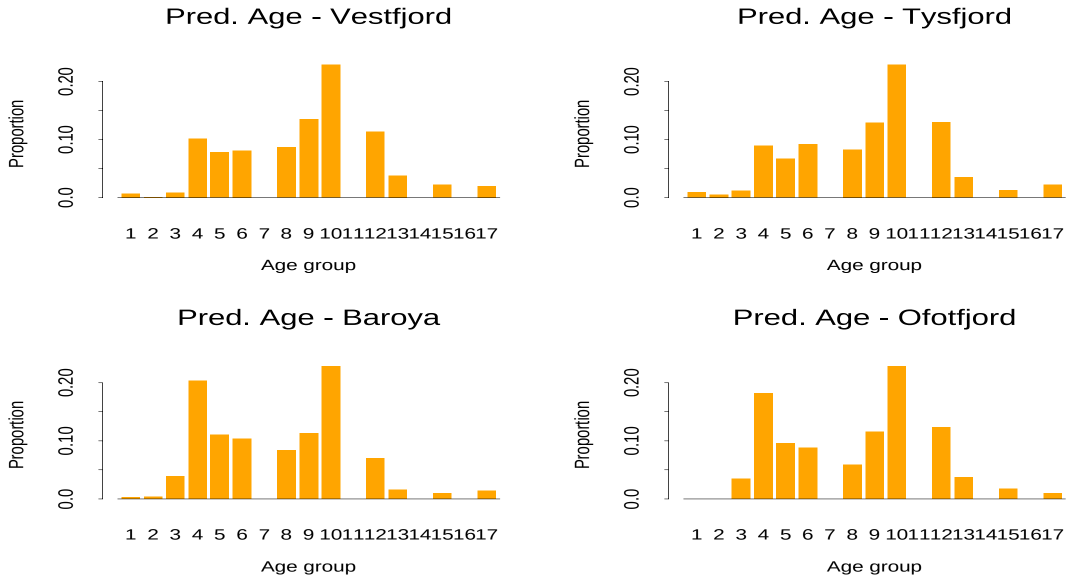

Figure 4.2: Calibrated Age by location.

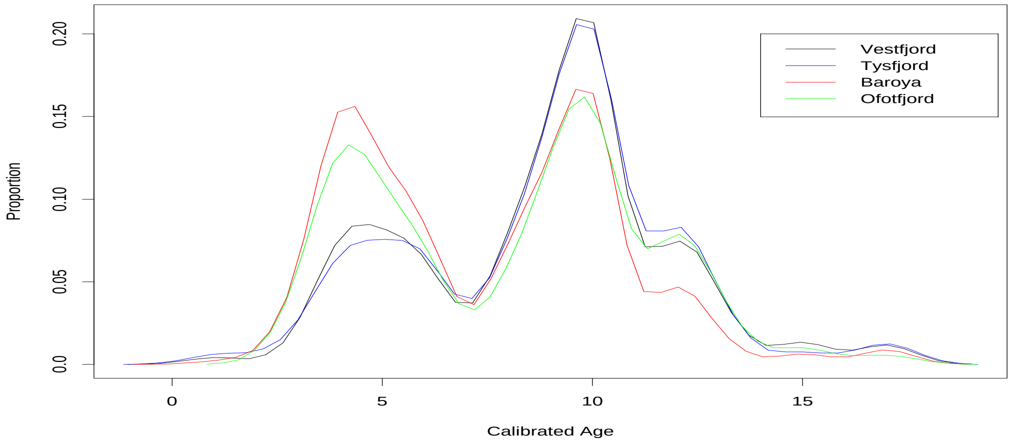

Figure 4.3: Kernel smooths of the calibrated age distributions by location

Area		Age								Row Total
		< 4	4	5	6	8	9	10	> 10	
Bar	Obs.count	130	595	323	303	246	331	671	320	2919
	Exp.count	(95.8)	(449.3)	(271.1)	(276.0)	(242.2)	(356.9)	(778.7)	(448.8)	
	Cal. prop.	0.045	0.2	0.11	0.1	0.084	0.11	0.23	0.11	0.48
Ofot	Obs.count	14	74	39	36	24	47	96	76	406
	Exp.count	(13.3)	(62.5)	(37.7)	(38.4)	(33.7)	(49.6)	(108.3)	(62.4)	
	Cal. prop.	0.034	0.18	0.096	0.089	0.059	0.12	0.24	0.19	0.067
Tys	Obs.count	29	100	75	103	93	145	353	224	1122
	Exp.count	(36.8)	(172.7)	(104.2)	(106.1)	(93.1)	(137.2)	(299.3)	(172.53)	
	Cal. prop.	0.026	0.089	0.067	0.092	0.083	0.13	0.31	0.2	0.19
Vest	Obs.count	26	164	126	131	140	218	497	312	1614
	Exp.count	(53.0)	(248.4)	(149.9)	(152.6)	(133.9)	(197.3)	(430.6)	(248.2)	
	Cal. prop.	0.016	0.1	0.078	0.081	0.087	0.14	0.31	0.19	0.27
Tot	Col Tot	199	933	563	573	503	741	1617	932	6061

Table 4.1: Cross-tabulation of calibrated number of fish at different Age classes, Tysfjord and Vestfjord

Area		Age								Row Total
		< 4	4	5	6	8	9	10	> 10	
Tysf	Obs.count	29	100	75	103	93	145	353	224	1122
	Exp.count	(22.5)	(108.3)	(82.4)	(95.7)	(95.5)	(148.9)	(348.6)	(219.8)	
	Cal. prop.	0.026	0.089	0.067	0.092	0.083	0.13	0.31	0.2	0.41
Vestf	Obs.count	26	164	126	131	140	218	497	312	1614
	Exp.count	(32.4)	(155.7)	(118.6)	(138.0)	(137.4)	(214.1)	(501.4)	(316.2)	
	Cal. prop.	0.016	0.1	0.078	0.081	0.087	0.14	0.31	0.19	0.59
Total	Col Total	55	264	201	234	233	363	850	536	2736

Table 4.2: Cross-tabulation of calibrated number of fish at different Age classes, Tysfjord and Vestfjord

Area		Age								Row Total
		< 4	4	5	6	8	9	10	> 10	
Bar	Obs.count	130	595	323	303	246	331	671	320	2919
	Exp.count	(126.4)	(587.3)	(317.8)	(297.6)	(237.0)	(331.8)	(673.3)	(347.6)	
	Cal. prop.	0.045	0.2	0.11	0.1	0.084	0.11	0.23	0.11	0.88
Ofot	Obs.count	14	74	39	36	24	47	96	76	406
	Exp.count	(17.6)	(81.7)	(44.2)	(41.4)	(33.0)	(46.1)	(93.6)	(48.3)	
	Cal. prop.	0.034	0.18	0.096	0.089	0.059	0.12	0.24	0.19	0.12
Tot	Col Total	144	669	362	339	270	378	767	396	3325

Table 4.3: Cross-tabulation of calibrated number of fish at different Age classes, Barøya and Ofotfjord

Comparison	χ^2 statistic	d. of f.	p-value
B-O-T-V	279.8	21	0.000
B-O (all ages)	24.0	7	0.001
B-O (no $>$ 10 age)	3.8	6	0.698
T-V	6.7	7	0.460

Table 4.4: χ^2 test results for Tables 4.1, 4.2, and 4.3.

4.2 Depth, Time, and Location

What makes the regions Barøya-Ofotfjord and Vestjord-Tysfjord different? Perhaps the main factor is the depth of the sea in these regions. The V-T area, and specially Tysfjord, is deeper than the B-O area. And the fish samples were collected also at deeper depths in V-T than in B-O See Figure 4.4, and Tables 4.5 and 4.6.

From what we have seen, and as will be illustrated in what follows, older herring seem to prefer not only to be in areas that are generally deeper but also to locate themselves at deeper depths relative to the group they belong to.

It seems to be apparent that older fish seem to prefer the deeper depths. See Figure 4.6. However, we cannot state that the three depths divide the population in three non-overlapping strata since there seem to be cyclical movements during the day between strata.

Given a fixed location and time, perhaps it is more sensible to take the depths as more or less fixed strata. However, we have no information as to how accurate this assumption is. Therefore, just the plots will be presented and no testing carried out.

The material presented in the following section should perhaps serve as a motivation for future studies.

Note that for Tysfjord and Ofotfjord there is not enough data, or no data at all, for the comparison.

Area	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
Barøya	150.0	200.0	260.0	256.7	303.0	370.0
Ofotfjord	85	215	305	253	320	340
Tysfjord	95.0	344.0	381.5	362.0	413.5	480.0
Vestfjord	250.0	315.0	355.0	353.1	398.5	432.0

Table 4.5: Summary statistics of the maximum depth of the groups of fish where samples were taken from.

Area	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
Barøya	300.0	482.5	533.0	496.0	544.0	547.0
Ofotfjord	425.0	531.0	533.0	511.6	534.0	535.0
Tysfjord	681.0	713.5	715.0	712.3	717.2	722.0
Vestfjord	408.0	541.5	588.0	568.9	603.0	625.0

Table 4.6: Summary statistics of the depths of the sea at which samples were taken.

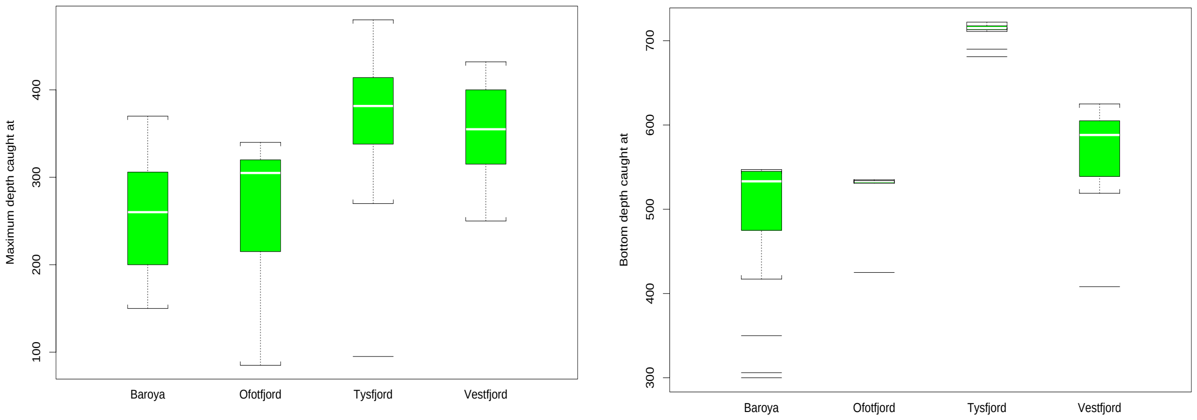

Figure 4.4: Boxplots of the maximum depths of the fish schools where samples were taken from and of the depths of the bottom of the sea where the samples of fish were taken.

4.2.1 Calibrated age distribution by depth

Note how it seems to be that we have a larger concentration of older fish at depth 3 than at the other two depths, which seem to have similar age distributions.

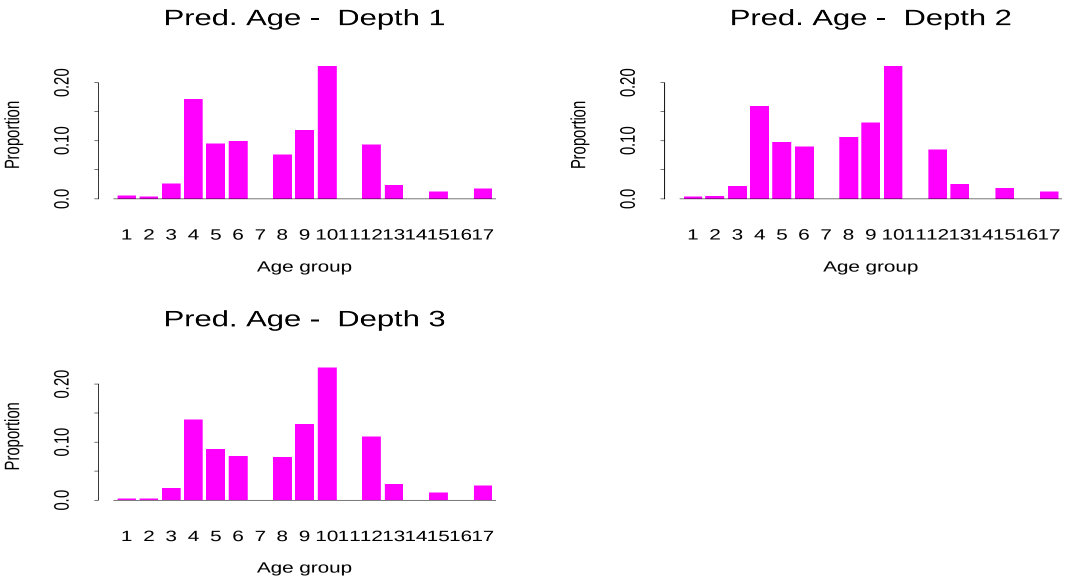

Figure 4.5: Calibrated Age by Depth.

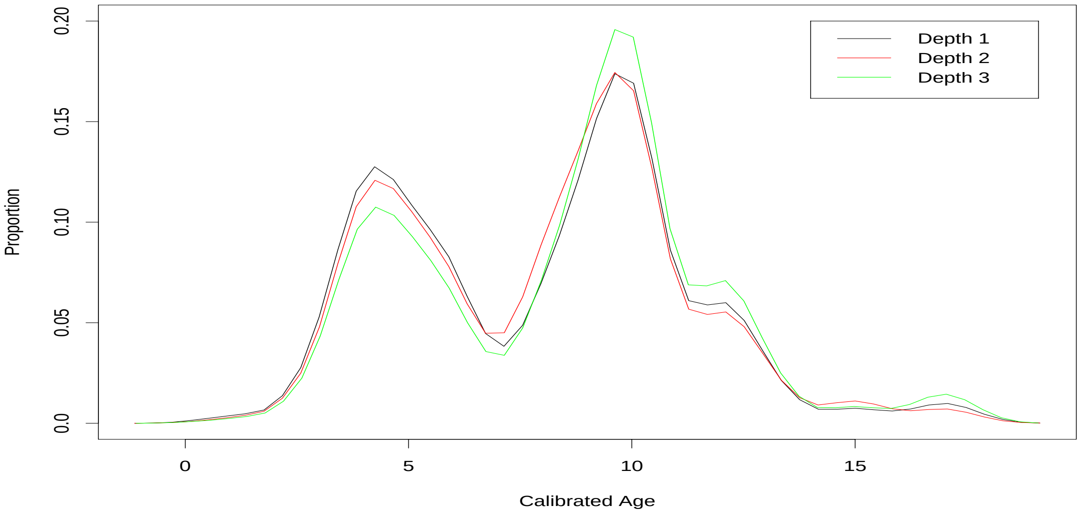

Figure 4.6: Kernel smooths of the calibrated Age by Depth.

4.2.2 Calibrated age distribution in the Vestfjord by depth and time

Specially during the dark hours, we see how we have a larger concentration of older fish at the deepest depth. The age of the fish seems to increase with the depth of the school at dark hours. This is not so obvious at daylight hours.

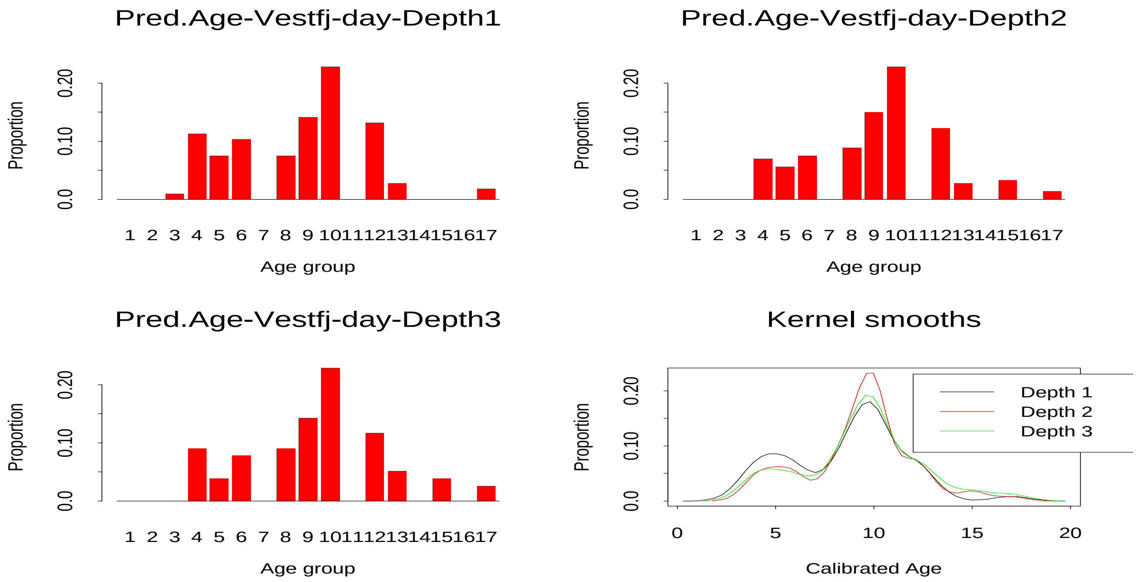

Figure 4.7: Calibrated Age by Depth in Vestfjord at day.

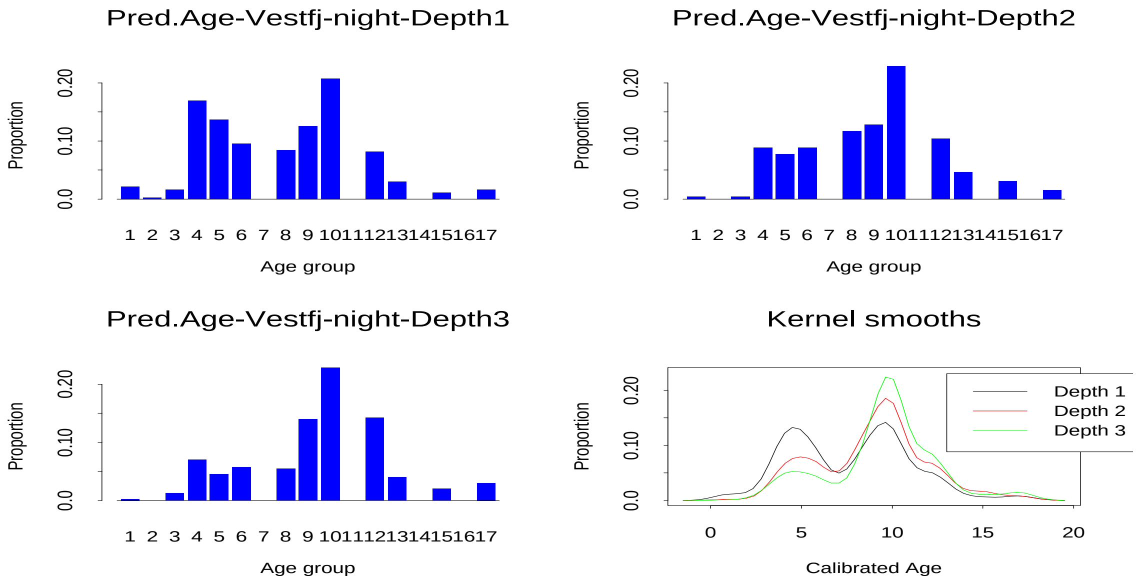

Figure 4.8: Calibrated Age by Depth in Vestfjord at dark.

4.2.3 Calibrated age distribution in Barøya by depth and time

No striking difference is apparent in the age distributions at different depths during the dark hours. However, during daylight hours there is a larger concentration of fish at deeper depth.

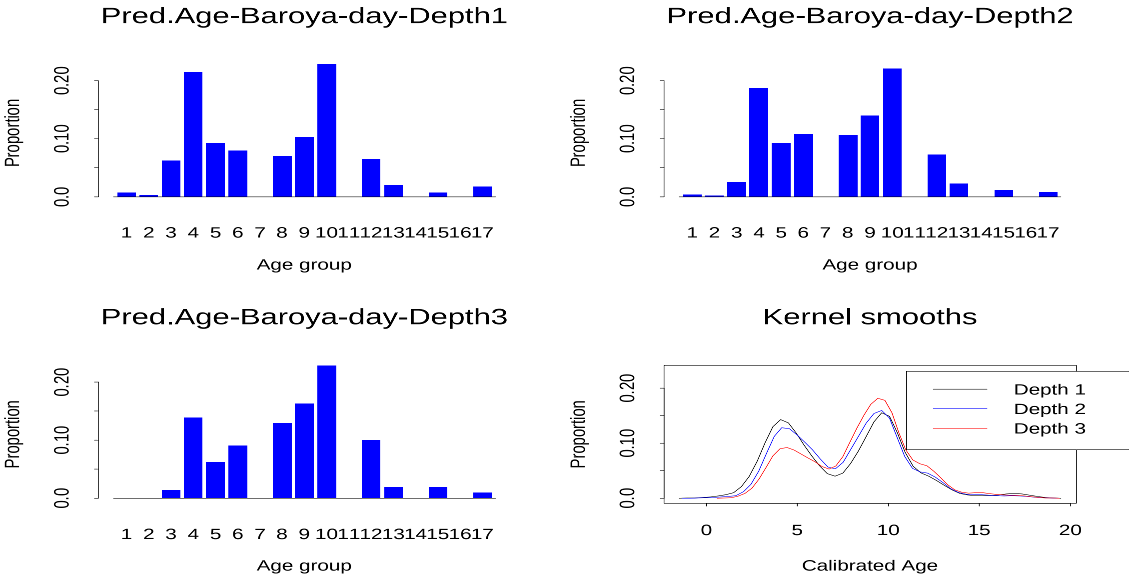

Figure 4.9: Calibrated Age by Depth in Barøya at day.

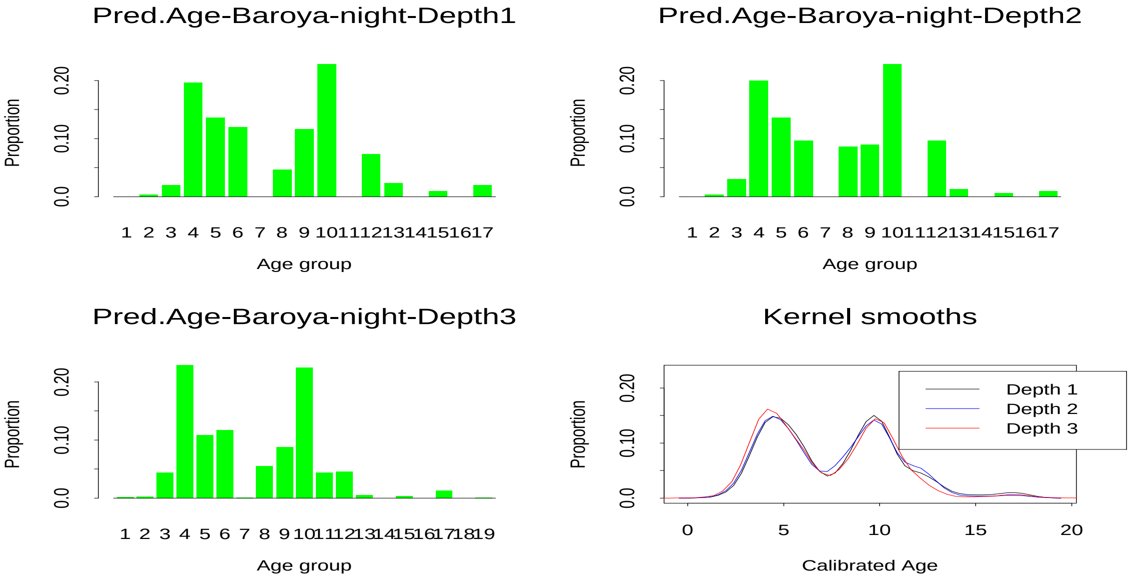

Figure 4.10: Calibrated Age by Depth in Barøya at dark.

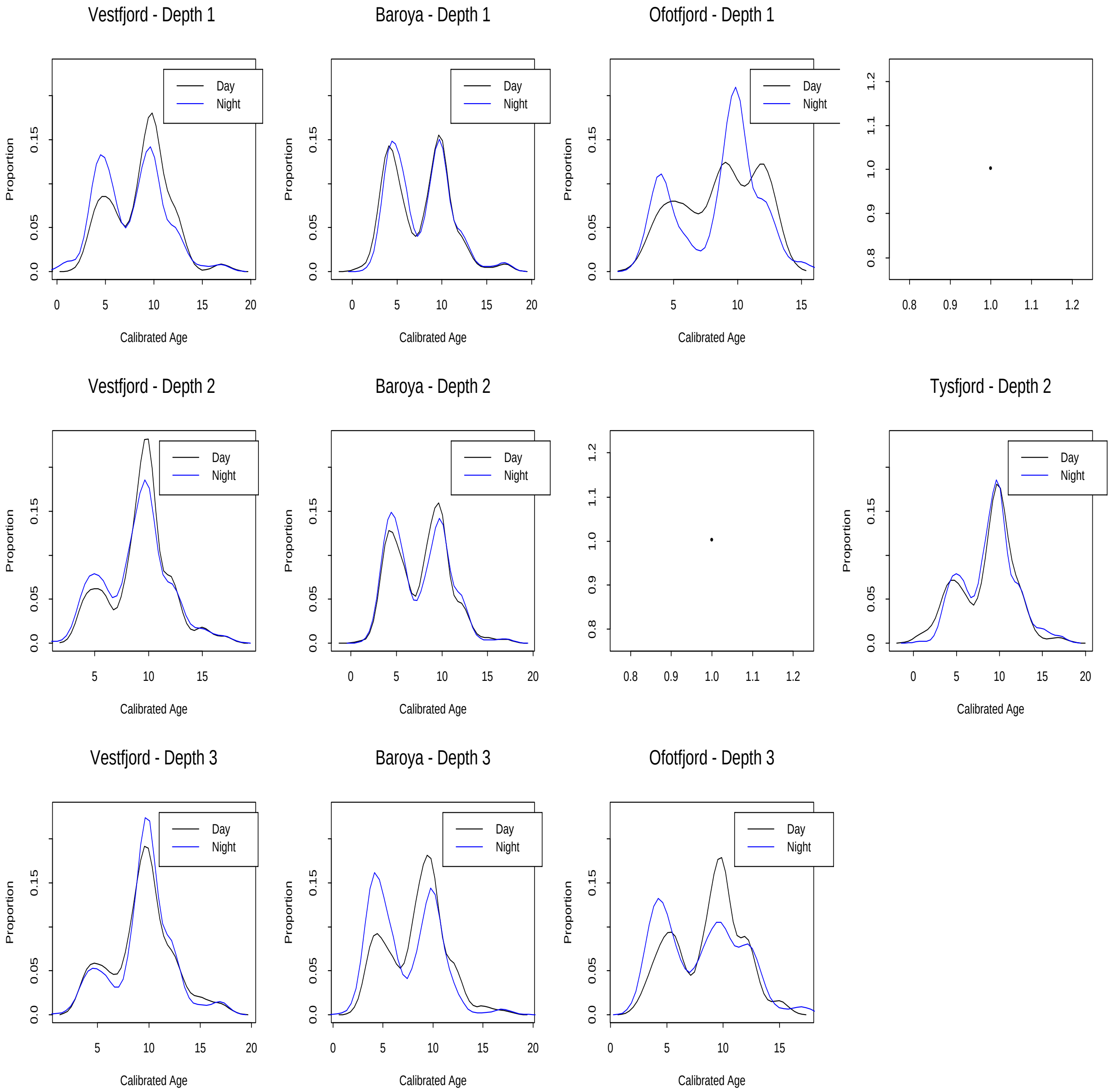

Figure 4.11: Smoothed Ages by Location, Depth and Time

Chapter 5

Advice for Sampling Experiments

If it is desired to carry out an experiment to monitor the status of abundance of herring by age in the extended Vestfjord area, then some facts should be taken into account to do this correctly.

Given that the measurement of ages is somewhat expensive and time consuming, it is almost impossible to obtain the number of age measurements that will guarantee a reasonable precision in the estimates of proportion of fish by age. We have seen that if just lengths are collected the age distribution can be obtained in quite a precise way, as given by the calibration method explained in this report. The method is very simple and it only requires the conversion of lengths into ages using the calibration table. It should be stressed that it is preferable to use a large collection of calibrated ages than to use a small set of exact ages. Usually the exact ages will not be enough to allow for an accurate description of the age distribution. Compare the age distributions obtained from exact measurements and the calibrated ones shown in this report in previous sections.

If ages are collected, then these could be used to check, for example, if the calibration table is still current and, if necessary, to update it. Ideally, the calibration table should be constructed pooling data from at least three consecutive years. We foresee that if no major changes occur in the environment of the Vestfjord (natural or caused by men) then the calibration table should be valid for quite a few years, at least ten, although this still needs to be checked by further studies.

We note that the region under study has at least two mutually exclusive subregions and that, in order to obtain a meaningful description of the age distribution, this partition of the population must be acknowledged. If it is possible to divide a heterogeneous population into subpopulations, each of which is internally homogeneous, then a gain in precision of the estimates of the whole population may be achieved. See e.g. Cochran (1977).

5.1 Sample size calculation

Assuming that stratified random sampling is carried out, with two strata in the population, the following indicates a way of how sample sizes could be chosen. One possible criterion could be the following: we would like to achieve a certain precision, as measured by the variance, in the estimate of the number of fish at an age class, whilst collecting the smallest possible number of fish. We stress that the sample sizes given are for the number of lengths measurements, or age measurements, or combination of both.

First, let us establish some notation that will be used in what follows. Also, stratum 1 represents the Vestfjord-Tysfjord region and stratum 2 represents the Barøya-Ofotfjord region.

- N_i: Size of population in stratum i, $i = 1, 2$.
- n_i: Size of sample from stratum i, $i = 1, 2$.
- $N = N_1 + N_2$: Size of the whole population.
- $n = n_1 + n_2$: Total size of sample.
- $F_{a,i}$: the number of fish of age a in stratum i, $i = 1, 2$.
- $f_{a,i}$: the number of fish of age a in stratum i in the sample, $i = 1, 2$.
- $P_{a,i}$: the proportion of fish at age class a in stratum i, $i = 1, 2$. $P_{a,i} = F_{a,i}/N_i$.

- $Q_{a,i} = 1 - P_{a,i}$
- $p_{a,i}$: the observed proportion of fish at age class a in stratum i, $i = 1, 2$. $p_{a,i} = f_{a,i}/n_i$.
- $S^2_{a,i}$: the variance of $p_{a,i}$. $S^2_{a,i} = \frac{P_{a,i}(1-P_{a,i})(N_i-n_i)}{n_i(N_i-1)}$.

It will be assumed that just the number of fish at one age category is to be determined.

Let us assume that the cost of sampling a unit, regardless of which stratum it belongs too, is a fixed number c. We also wish to estimate the total number in age category a with a fixed precision V_a and at the same time minimise the total cost of the experiment. Note that V_a represents the variance of the estimated number of fish of age a in the whole population.

Let us assume that the total cost is $C = c_0 + cn$, where c_0 represents an overhead cost. Under this assumption minimising the cost is equivalent to minimising the total number, n, of items sampled.

The variance of the population total at age a is

$$V_a = \sum_j N_j^2 S^2_{a,j}.$$

It can be easily shown that the optimal allocation, i.e. the one that will minimise the total number of units sampled for a given precision V_a is

$$n_i = \frac{N_i\sqrt{P_{a,i}Q_{a,i}}\sum_j N_j\sqrt{P_{a,j}Q_{a,j}}}{V_a + \sum_j N_j P_{a,j}Q_{a,j}}, \quad i = 1, 2.$$

In practise, $P_{i,a}$ is not known and is replaced by its unbiased estimator $p_{i,a}$. Also we have considered that $N_i/(N_i - 1) \approx 1$.

Since we have several age categories, we repeat this exercise for each of them. In our case, we may want to have age groups 1-3, 4-7, 8-10, and > 10.

It is expected that the best allocation for one age group will not in general be best for another and some compromise must be made. If reliable previous data are available, we can then compute the optimum allocation for each age group separately and see to what extent there is disagreement. For example, an average of the optimal sample sizes per stratum could be taken as a compromise allocation.

Using the abundance of Norwegian spring spawning herring estimates for 2002 provided by Havforskningsinstitutt, namely total abundance 70×10^8, with abundance in Vestfjord-Tysfjord estimated to be 28×10^8, and 42×10^8 in the Barøya-Ofotfjord area, abundance by age estimates are presented in Table 5.1. The calibrated age proportions previously presented were used.

Age (a)	$p_{a,1}$	$p_{a,2}$	$\hat{F}_{a,1}(\times 10^8)$	$\hat{F}_{a,2}(\times 10^8)$	Est.Tot.Abund.($\times 10^8$)
< 4	0.02010	0.04331	0.5628655	1.818947	2.381813
4	0.09649	0.20120	2.7017544	8.450526	11.152281
5	0.07346	0.10887	2.0570175	4.572632	6.629649
6	0.08553	0.10195	2.3947368	4.282105	6.676842
4-7	0.25548	0.41203	7.1535100	17.305264	24.458770
8	0.08516	0.08120	2.3845029	3.410526	5.795029
9	0.13267	0.11368	3.7149123	4.774737	8.489649
10	0.31067	0.23068	8.6988304	9.688421	18.387251
8-10	0.52851	0.42556	7.1535100	17.873684	32.671930
> 10	0.19591	0.11910	5.4853801	5.002105	10.487485

Table 5.1: Proportions and abundance-by-age estimates.

One possible criterion to estimate the sample size is to request that the coefficient of variation, $\sqrt{V_a}/F_a$, is equal to some small constant, say 0.05. So we wish to have an estimate of abundance in age group a which is within 5% of the true total. In the following sample size calculation we employ this criterion. Table 5.2 provides the sample sizes calculated from the estimates and formulas above. Note that since the coefficient of variation is fixed, the variance V_a will be proportional to the square of the estimate of abundance at age a. Hence, since the abundance estimates by age group are quite different, specially for the < 4 and > 10 age groups, the obtained sample sizes are rather different too. Perhaps one must exercise the compromise that the abundance in the lower ages group is estimated with less precision than in the other age groups. However, this is a matter that needs to be decided by the survey organisers.

Age	n_1	n_2	n
< 4	3458	7523	10981
4-7	269	455	724
8-10	182	271	453
> 10	999	1223	2222

Table 5.2: Optimal allocation of sample sizes.

Also note that this computation were done using the 2002 age distribution, which will change in the years to come. However, with our proposed age grouping the calculations can be used as a guidance at least for one year to come.

5.2 Other sampling considerations

Lastly, and very importantly, samples must be obtained from different depths and possibly at different times of the day to avoid bias. The evidence presented in Section 4.2 should be taken into account when conducting the survey. Perhaps a more detailed sampling design should be constructed taking into account these variables. However, this could be a subject of future studies.

Chapter 6

Summary and Conclusions

In this work we have examined the data on lengths and ages of Norwegian spring spawning herring, collected in the sampling experiment conducted in the Vestfjord system the past December 2002.

The data was collected at different geographical locations, times, and depths. The data set consists of a large amount of lengths of herring and some reduced number of age-length measurements. The objective was to characterise the population using the collected data. This is important so that future surveys are conducted in a way that they can provide an informative picture of the status of the stock.

The length measurements contain information about the main features of the population under study. However, these features are not easily and directly obtained from the length data. On the other hand, the age distribution is more helpful in capturing the features of the population.

Length is a causal effect of age and there are useful growth models available to study the relationship between age and length. This relationship is universal in the sense that, as long data from the population under study is used, it does not matter if it comes from different strata of the population. All what is needed is that the sample includes fish from all the possible ages. Also, it is desirable to include at least three years of data to better model the evolution of growth.

A comparison with the Iterated Age Length Key method is carried out. Our calibration method gives a simpler transformation of length into age than the IALK method does. Despite its simple form of ages associated with length intervals, instead of a length by age matrix of conditional distributions, our method produces substantially more dispersed age distributions for the herring stock than the IALK method.

Once the relationship between age and length has been modelled and we have verified that the model fits the age-length data well, we proceed to calibrate the ages from the lengths using a maximum likelihood method. Calibration is the statistical methodology where the independent variable is predicted rather than the dependent variable. This method applied to a range of lengths produces a calibration table. The application of the calibration table easily converts lengths into ages. In this way, from empirical length distributions we can obtain age distributions.

In the case of the population of herring in the Vestfjord system, the calibrated age distributions reveal important features. The population of herring in the Vestfjord system can be grouped into two strata: Vestfjord-Tysfjord and Barøya-Ofotfjord. These two strata are not only different geographical locations but the age distributions are significantly different in these two areas. In the Vestfjord-Tysfjord area the fish tend to be older than in Barøya-Ofotfjord. The difference seems to be related to the hypothesis that older fish apparently prefer habitats that are deeper. Also it seems that older fish tend to locate themselves deeper relative to the other fish in the school.

These observations are very important to take into account when planning a survey since the abundance at age estimates may vary significantly whether or not existing strata are considered, specially because the age distributions are definitely different in the two strata. Formulas are given to compute sample sizes.

More investigations need to be made as to the existence of more strata in this population. We have two other dimensions to consider: time and depth (relative to the school of fish). Given the dynamics of the schools of herring, these strata might not be mutually exclusive. From our exploratory analysis it seems that these two dimensions may have a significance in establishing differences between subgroups of the herring population.

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