# History matching in object models

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**Abstract**

We do history matching in a fluvial oil reservoir by using Metropolis-Hastings algorithm and a genetic algorithm.

## 1 Introduction

History matching with uncertainty quantification is a very hard problem. The goal is to sample from the posterior distribution for the reservoir characteristics, that is, sample the reservoir conditioned on prior knowledge, well data and production history. Many methods are proposed, with varying results, see for example [1], [6], [8] and [7]. By Metropolis-Hastings [5], [2], simulation it is in theory possible to sample correctly from the posterior distribution for reservoir characteristics. However, this is usually too time consuming in practice, unless the proposal function is cleverly chosen. An approximate method based on Metropolis-Hastings is proposed in [4]. In the references mentioned above, reservoir characteristics are modeled by discretized Gaussian random fields or other pixel based models. In this article we focus on object models for the reservoir characteristics. We model a fluvial reservoir consisting of high permeable channels and low permeable background. We will do history matching both by Metropolis-Hastings (MH) algorithm and a genetic algorithm.

Genetic algorithms are efficient optimization methods which can be used in optimization problems with multi-modal objective functions or where traditional analytical methods fail. The steady state genetic algorithm starts by initializing a population of $N$ individuals. In each generation a fixed fraction of the population is selected for mating using *fitness* as a selection criterion. This produces a number of offsprings which are added to the population. The population is then cut back to its original size by removing the least fit individuals. Through Darwinian 'survival of the fittest', the fitness of the population increases.

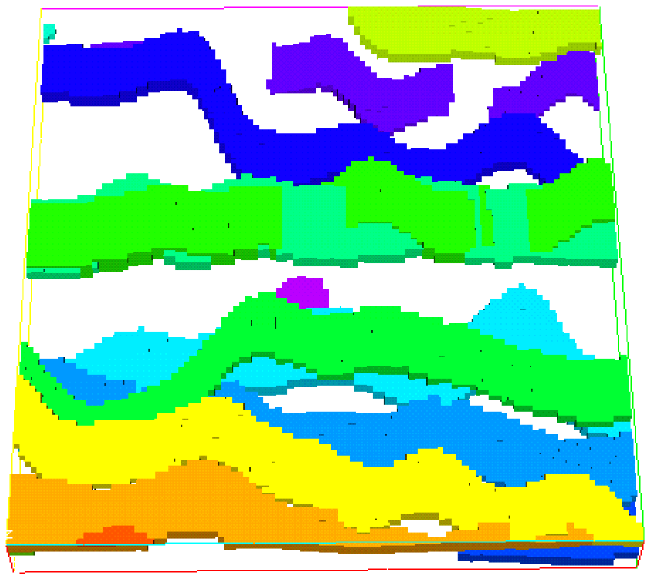

Figure 1: The channels in the true reservoir.

## 2 Description of test case

We generate a realization from the prior model for the reservoir characteristics which we will use as the "true" reservoir. Production data are generated from the "true" reservoir, and we will do history matching on these data. In the following, the prior model for reservoir characteristics and the production likelihood model are described.

We use the *Fluvial* module of *STORM* to simulate the reservoir. See [3] for a description of *Fluvial*. In *Fluvial*, the Metropolis-Hastings algorithm is used to simulate size and location of channels in a background. The reservoir consists of high permeable sand channels in a background of low permeable shale. The sand-gross is set to 0.22. The size of the reservoir is $2000 \times 2000 \times 20\ m^3$. The channels are distributed according to a Poisson model; there is no interaction between objects. The number of channels in the true reservoir is 15. The prior model gives between 8 and 15 channels in most cases. The channels of the "true" reservoir are shown in Figure 1. Permeability and porosity in channels and background are modeled as log-Gaussian and Gaussian random fields, respectively, on a $100 \times 100 \times 50$ grid. The reservoir is up-scaled to a $20 \times 20 \times 10$ grid. Reservoir simulation is performed by the *More* reservoir simulator.

Eight wells are drilled in the reservoir. There are four production wells in the east of the reservoir and four injection wells in the west. Production simulation is run for 3000 days. The production history data to be matched are production

– and injection rates and water cut on 100 equally spaced days.

The available well data are permeability and porosity values, plus information about where the channels hit the wells. Observations in different wells are said to be coupled if they belong to the same channel. Coupling between wells are unknown.

The production likelihood function is a product of Gaussian independent functions, and can be written as

$$L(d|f,p) = C \times \exp(-\sum_{i=1}^{n}(d_i - \omega(f,p)_i)^2/\sigma_i^2)$$

where $C$ is a normalizing constant, $d$ are production data, $\omega(f,p)$ is the result of the production simulator with fluvial realization $f$ and petrophysics $p$, and $\sigma_i^2$ are variances. They are estimated from 10 samples of permeability and porosity fields added to the true fluvial realization. In Figure 2 production from five of these samples are plotted. If the goal is to match total oil production, the standard deviation from wells with low production should be upweighted compared to wells with high production. The number of observations $n$ is equal to $4\times100+4\times200 = 1200$. (Four injectors where water injection is observed at 100 time steps, four producers where oil production and water cut is observed at 100 time steps.) Note that the log-likelihood can be written as a sum of components from the eight wells. In this way, wells with bad match can be identified. The negative exponent of the likelihood function is called the potential function. We have a good match to the production data if the potential function is low.

# 3 Simulation methods

We start by focusing on matching the history by placing the channels as "correct" as possible. At the beginning, we do not match with respect to petrophysics. Therefore the petrophysics is generated by kriging instead of the Gaussian model used in the "true" reservoir. Kriging is an interpolation of the well data.

In the Metropolis-Hastings algorithm, it is crucial for the speed of convergence to choose the proposal function in a clever way. We generate proposals by changing a small proportion of the channels. The channels to change are picked according to the size of the component of the likelihood function in the wells hit by the actual channel. In this way, channels in wells with bad match are proposed changed more often than channels in wells with good match. The new proposed channels are taken from a realization generated from the prior model, conditioned to well data, and with petrophysical values generated by kriging. This realization may have couplings different from the current state. Therefore channels are grouped according to well observations, and either the whole group or no members of the

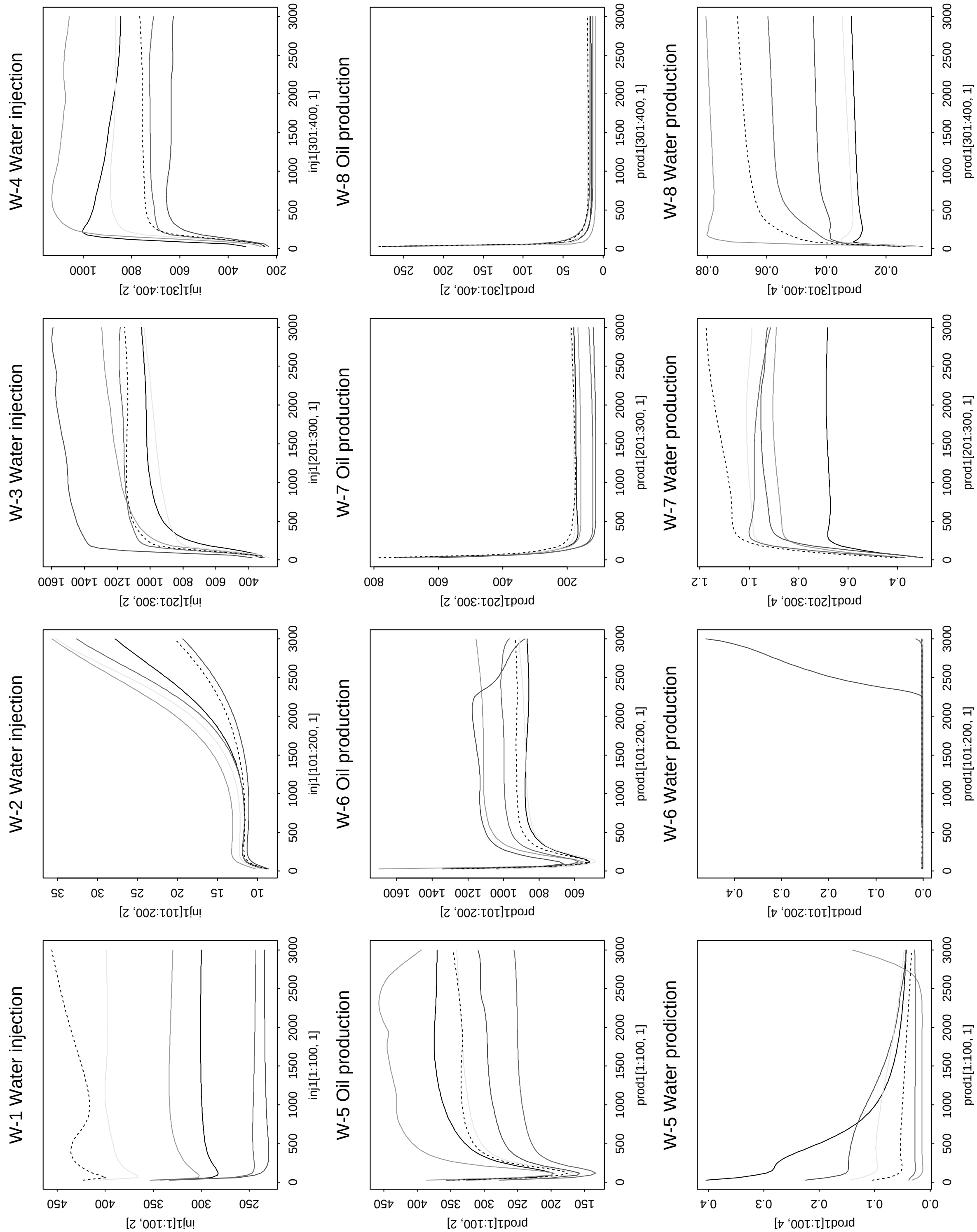


Figure 2: The true production (dashed line) and production from different realizations of petrophysics added to the "true" facies realization.

group are changed. The probability of changing a group of channels is given as

$$Q = \frac{C \sum_{i=1}^{m} p_i}{m \sum_{i=1}^{n} p_i}$$

where $C$ is a constant, $m$ is the number of channels in the group, $p_i$ is the potential for the wells hit by channel number $i$, and $n$ is the total number of channels observed. The constant $C$ is adjusted by trial and error such that the number of changed channels is not too big and not too small. After all observed channels are generated, unobserved channels are kept with probability $1-Q = 0.9$ and changed with probability $Q = 0.1$, until correct net gross is reached. The rest of the unobserved channels are removed. If net gross is too low, unobserved channels from the prior realization are added. The transition probability is given as

$$q_{ij} = \prod_{i=1}^{n_c} Q_i \prod_{i=1}^{n_u} (1 - Q_i) \qquad (1)$$

where $n_c$ is the number of changed channel groups and $n_u$ is the number of unchanged channel groups. The acceptance probability is calculated in Appendix A. A start realization is generated from the prior model conditioned to well data, and petrophysical values are generated by kriging.

The genetic algorithm (GA) is defined as follows. A population of 50 individuals is used. The start population is generated in the same way as the start realization for the MH-algorithm. To decide whether a new individual is going into the population, the potential function is used. If the potential of the new individual is less than the maximum potential of the population, it is taken into the population.

New individuals are created by choosing some channels from the mother, some from the father, and some from a mutation. The mutation has the same couplings as the mother, but the channels have different size, and the number of unobserved channels may differ from the mother. Parents are chosen from the population with probability proportional to the inverse of the potential function.

The channels are grouped according to couplings in the wells, and channels belonging to the same group are chosen from the same individual. A group is chosen from the father if the father has lowest potential in the wells hit by the group. If the mother has lowest potential, the group is chosen from the mutation with probability

$$P_{mutation} \sim U(0, 0.3)$$

and otherwise from the mother. Unobserved channels are added until the sand gross is high enough. The same probabilities as above are used, except that we use the total potential for the mother and father.

### 3.1 Matching petrophysics

We also did some tests including petrophysics. For these tests, petrophysics were generated for the true reservoir from a prior distribution. The porosity and permeability values at the well locations were then used as conditioning data.

Reservoir modeling is usually done in stages, where facies and petrophysics are modeled in two different stages. In order to get this approach to work when conditioning to production history, each step needs to be correctly conditioned to these data. Although the approach described earlier gives reasonable facies realizations, they are not from the correct distribution for a stepwise process.

Therefore, two different approaches were tried. One approach simulates both facies and petrophysics simultaneously, and should be statistically correct. In this approach, we used a burn-in phase, where only facies was changed, and the kriged petrophysics were used. Then we generated a petrophysics realization conditioned to the facies and well observations, and used this as our initial petrophysics. In subsequent iterations, we simulated new petrophysics for each channel that was changed in an iteration. MH was still used as simulation algorithm.

The other approach uses the traditional two-step approach. Although this is only an approximation, convergence should be more easily obtained, as the state space for each step is much smaller. In this approach, we first generate a facies realization as described previously. Petrophysics are then generated by changing one channel in each step. The background petrophysics are held constant. More details on the statistics are found in Appendix B.

## 4 Results and conclusions

If we take the true fluvial realization, and generate petrophysical values by kriging (conditioned to well data), the potential function (negative exponent of the likelihood function) becomes 1207. This means that we have achieved satisfactory match when the potential is of this size. If we want better match, we have to change the petrophysical values, altering channels will probably not be enough. We expect that this is best done by changing petrophysical variables. We also want to check prediction of total oil production after 5000 days for the history matched reservoirs. All results are shown in Table 1. We perform 315 iterations with both the MH and GA algorithm, and generate two samples with each of the methods in order to estimate uncertainties. Uncertainty is given as one standard deviation. The potential for the start realization in MH in the first sample is 777279. In the start population for the GA algorithm, used in both samples, the potential ranges from 8003 to $3.462760 * 10^6$. After 315 iterations, the lowest potential in the population is 1104 and the highest is 1677 in the first run. In Figure 3, the logarithm of the potential is plotted against number of iterations

| | Truth | MH | GA |
|---|---|---|---|
| Potential | 1207 | $969 \pm 43$ | $939 \pm 233$ |
| Pred. oil after 5000 days | $7.211{*}10^6 m^3$ | $6.180 \pm 0.07 * 10^6 m^3$ | $6.981 \pm 0.31 * 10^6 m^3$ |
| Number of channels | 15 | 9 | 9 |
| Tot. time spent | | 73.5h | 31.5h |
| Time spent pr it. | | 14min | 6min |

Table 1: Results from the history matching.

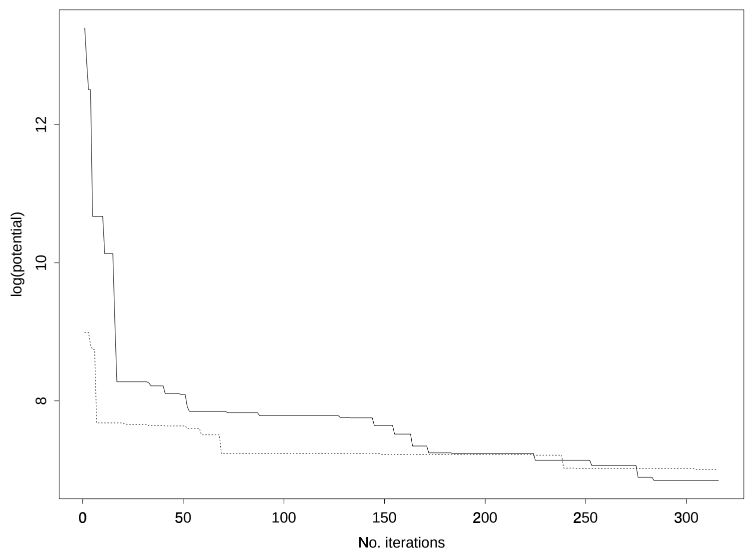


Figure 3: Logarithm of potential plotted against number of iterations for MH (solid line) and GA (dotted line).

for the two methods. For the GA we have plotted the smallest potential in the population, and therefore the start value is much smaller than for MH. In MH, the start value is based on one random sample from the prior, the minimum potential in the GA population is based on 50 samples from the prior.

The two history matched reservoirs have in common that all net-gross is contained in channels observed by wells. Therefore the number of channels in the history matched reservoirs differ considerably from the truth. This is also the case for many realizations if we generate realizations from the prior conditioned to well observations.

Although the MH algorithm is much slower than the GA, we feel that the comparison based on number of simulations is fair, because MH could be coded in a more effective way by including history matching directly into the MH loop that is used for fluvial sampling. The difference in computing time is that for each MH iteration we run *Fluvial*, which takes about 8 minutes. This is avoided for the GA algorithm because the mutation has the same couplings as the mother. Sampling petrophysical values and upscaling takes about two minutes, the reservoir simulator takes four minutes for each iteration.

In Figure 4, the production data from the true reservoir and the matched reservoirs after 315 iterations are plotted for 5000 days. Note the scale on the y-axes. Figure 5 shows the history matched reservoirs. If we compare these to the true reservoir, Figure 1, we see that they all have three groups of channels, but the number of channels in the groups differs.

There is not much difference in the history match performed with the two methods. By the MH-algorithm we sample correctly from the posterior distribution, while GA is a pure optimization technique. Therefore MH is preferable if uncertainty in prediction should be estimated.

Earlier attempts with the same methods on another test case with higher net-gross and the same expected channel-size, were not as successful as the experiment described here. A too high net-gross seems to make trouble for the history matching with our methods. With fewer channels, the localization of each channel becomes more critical.

The upscaling is critical. Using a too coarse grid gives large problems because the channels become “invisible”.

## 4.1 Matching petrophysics

An attempt was made to simulate both channels and petrophysics simultaneously while conditioning to history, as described in Section 3.1. In this case, the standard deviations in the potential function were divided by two, since these should be smaller when also petrophysics is taken into account. The residuals now corresponds to measuring errors and errors in the reservoir simulator. The new potential function is equal to four times the original potential function. After running 100 “burn in” iterations with kriging and about 400 iterations with Gaussian petrophysics, the new potential function becomes 3178. This is about the same match as we got by the earlier described attempts using kriged petrophysics.

The potential is decreasing very slowly towards the end of these iterations, indicating that no large improvements can be hoped for using this algorithm. This means that the simultaneous approach for facies and petrophysics is not feasible without better algorithms.

Instead, we tried changing only the petrophysics in channels, and held the facies realization constant. This is not statistically correct to do, see Appendix B, but the history match was improved. The potential now goes down to 1729 after

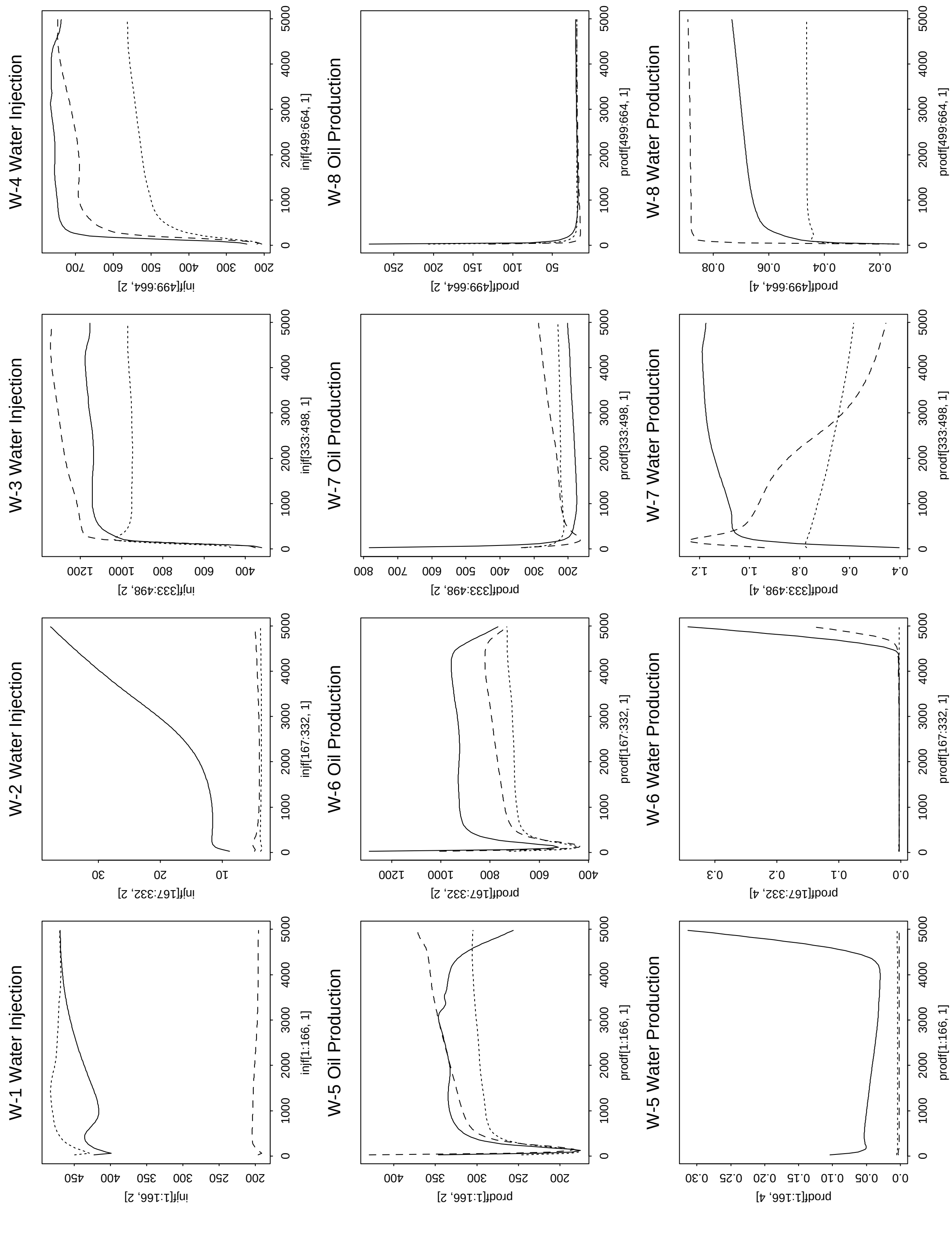


Figure 4: True production (solid line) and history matched production from MH (dotted line) and GA (dashed line).

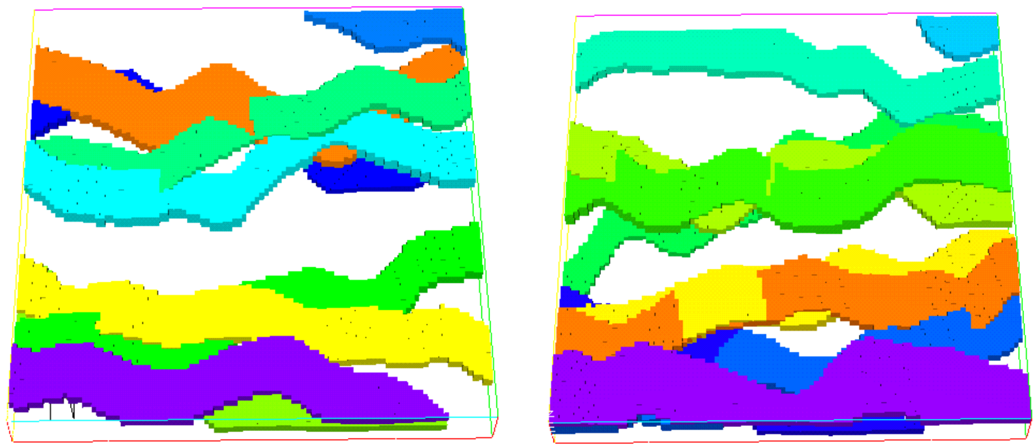

Figure 5: History matched reservoirs from MH (left) and GA (right).

another 570 iterations. The total predicted oil after 5000 days is now $6.7 * 10^6 m^3$. A plot of the predicted and true production data for 5000 days is shown in Figure 6.

## Acknowledgments

We thank Smedvig Technologies for support and for donating a license for the Modular Oil Reservoir Evaluation program (More) and Harald H. Soleng for setting up model files for the simulator. The European Commission is acknowledged for financial support through the Punq project in the Joule-III non-nuclear energy program.

## References

[1] F. J. T. Floris, M. D. Bush, M. Cuypers, F. Roggero, and A. R. Syversveen. Comparison of production forecast uncertainty quantification methods: an integrated study. *work in progress*, 1999.

[2] W.K. Hastings. Monte Carlo sampling methods using Markov chains and their applications. *Biometrika*, 57(1):97–109, 1970.

[3] L. Holden, R. Hauge, Ø Skare, and A. Skorstad. Modeling of fluvial reservoirs with object models. *Mathematical Geology*, 30(5), 1998.

[4] L. Holden, H. H. Soleng, and A. R. Syversveen. History matching with uncertainty quantification. In *Proceedings of the 1999 Annual Conference of*

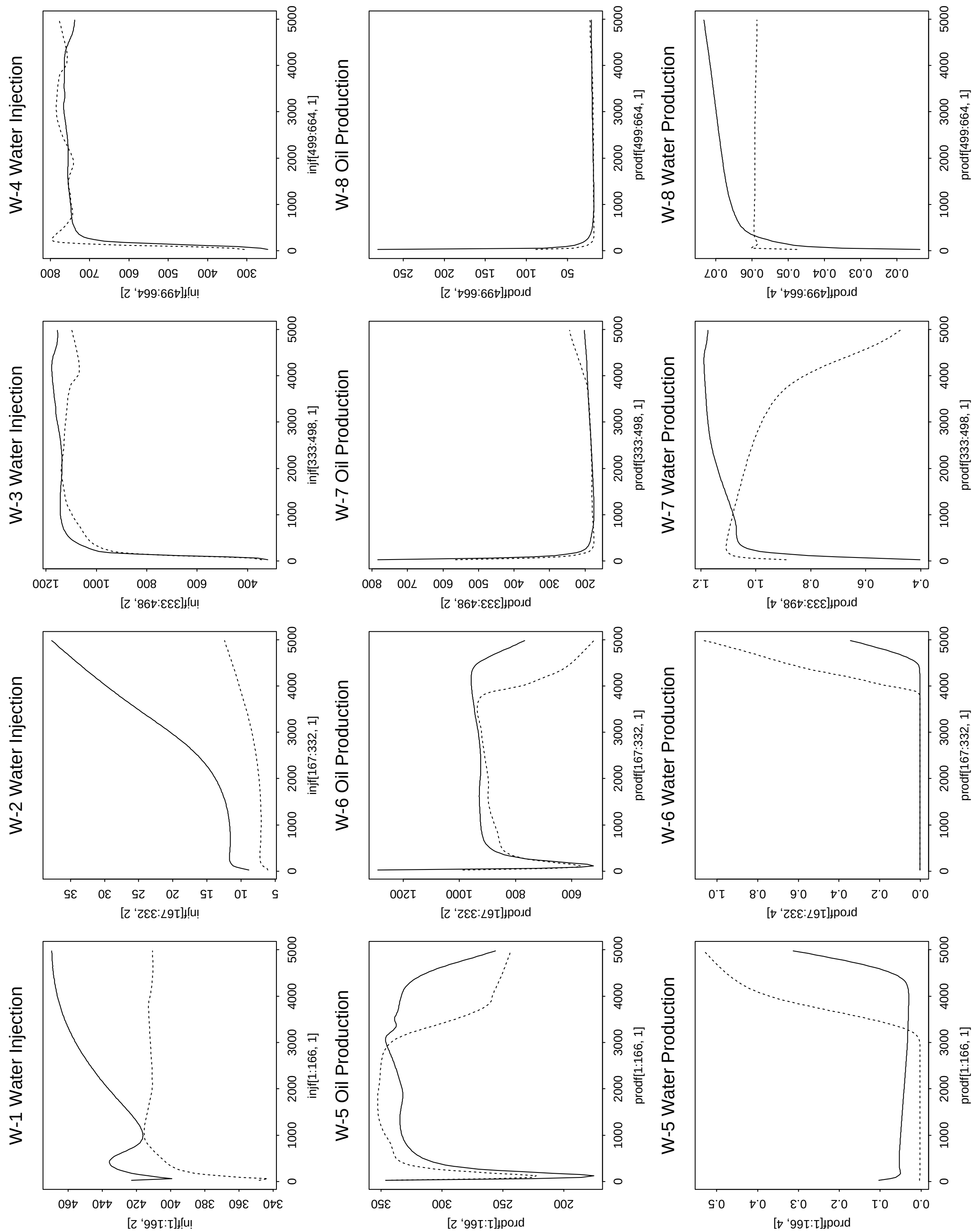


Figure 6: True production (solid line) and history matched production from MH matching both channels and petrophysics (dotted line).

*the International Association for Mathematical Geology*, 1999. Accepted for publication.

[5] N. Metropolis, A.W. Rosenbluth, M.N. Rosenbluth, A. Teller, and E. Teller. Equation of state calculation by fast computing machines. *Journal of Chemical Physics*, 21:1087–1092, 1953.

[6] D. S. Oliver, N. He, and A. C. Reynolds. Conditioning permeability fields to pressure data. In Zoltán E. Heinemann and Michael Kriebernegg, editors, *Proceedings of the Fifth European Conference on the Mathematics of Oil Recovery*, pages 259–269. Mining University Leoben, 1996.

[7] H. Omre, H. Tjelmeland, and H. T. Wist. Uncertainty in history matching—model specifications and sampling algorithms. Technical Report Statistics 6/99, Deparment of Mathematics, Norwegian University of Science and Technology, Trondheim, Norway, 1999.

[8] H. Tjelmeland. A note on the Bayesian approach to history matching of reservoir characteristics. In Vera Pawlowsky-Glahn, editor, *Proceedings of IAMG'97: The Third Annual Conference of the International Association for Mathematical Geology*, volume 2, pages 772–777, Barcelona, Spain, 1997. International Center for Numerical Methods in Engineering (CIMNE).

# A Acceptance rates for Metropolis-Hastings

Let the current state consist of the realization $i$ and a "mutation" realization $i'$ generated from the prior. The new proposed state consists of realizations $j$ and $j'$. Two alternating steps are performed:

1. Update the realization: $j$ is a combination of $i$ and $i'$. $j'$ is the channels contained in $i$ or $i'$ but not in $j$.
2. Update "mutation": $j = i$, the "mutation" $j'$ is drawn from prior.

We assume no interaction, and do not take net gross into account. Then the likelihood for a realization is the product of the likelihoods for all the channels. The acceptance probability then becomes

$$\alpha_{i,i' \to j,j'} = \frac{f(i)f(i')f(d|i)q(j,j'|i,i')}{f(j)f(j')f(d|j)q(i,i'|j,j')}$$

where $f(\cdot)$ is the prior distribution, $f(d|\cdot)$ is the likelihood function, and $q$ is the transition probability. In case one, we have that $f(i)f(i') = f(j)f(j')$, and $\alpha$ is a function of $f(d|\cdot)$ and $q$, where $q$ is given by Equation (1). In the second case, $i = j$, $q(j,j'|i,i') = f(j')$ and $q(i,i'|j,j') = f(i')$, which gives that $\alpha = 1$.

# B Simultaneous versus two-step simulation of facies and petrophysics

Let $f$ be a facies realization, $p$ a petrophysics realization, $d$ the production data and $g$ represent various likelihood functions. Our objective is to sample from the distribution $g(f,p|d)$. First, note that

$$g(f,p|d) = C \times g(p|f,d)g(f|d)$$

which gives the partial likelihoods in a two-step approach. The first term is simple; the problem lies with the second term, $g(f|d)$. Using

$$g(f,p|d) = C \times g(d|f,p)g(p|f)g(f)$$

gives

$$\begin{aligned} g(f|d) &= \int g(f,p|d)\mathrm{d}p && (2) \\ &= C \times g(f) \int g(d|f,p)g(p|f)dp && (3) \\ &= C \times g(f)\mathrm{E}_p(g(d|f,p)) && (4) \end{aligned}$$

This expectation is difficult to compute. In the two-step approach, we approximate it by computing the likelihood for the kriged petrophysics field, and compensate by using larger standard deviations. This means that the true state space is most probably a subspace of the one we get.

As it is an expectation value, it is simple to make unbiased estimators of it. However, each computation of $g(d|\cdot)$ requires a reservoir simulation, so the number of points used for estimation must be small. The simplest solution would be to use a single petrophysics; however, this would give convergence problems. If the petrophysics drawn gave a high likelihood, it would be very improbable to move out of this state, even though the state may have a low likelihood. Using a few points and averaging will not help much, as the variance is large, so the mean for small samples is the largest value divided by the sample size.