Estimation of the Mean Radar Reflectivity from a Finite Number of Correlated Samples

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Abstract— **We here compare estimators of the mean radar reflectivity on images with different spectral properties. By working on complex data rather than detected images, we can take the speckle correlation into account and thus obtain more accurate estimates. A robust and computationally efficient approximation of the optimal estimator is proposed.**

Index Terms— **Estimation, mean radar reflectivity, speckle correlation, synthetic aperture radar (SAR).**

I. Introduction

Estimation of the mean radar reflectivity is of fundamental interest in many applications and concerns various radar systems, such as Doppler weather radars, wind scatterometers, radar altimeters, and synthetic aperture radars (SARs). In general, incoherent integration is used to reduce a fading effect called speckle, which is due to the random phase fluctuations of the reflected waves by random rough surfaces or volumes. The fading effect is well modeled by a complex circular Gaussian random distribution. Let Z represent the complex amplitude produced by an imaging radar. For discrete data, the incoherent integration consists in computing the arithmetic mean of N samples of a function of $|Z|$. The usual functions and data formats are the detected envelope $A = |Z|$ (linear detector corresponding to amplitude data) with a Rayleigh distribution, the square detected envelope $I = Z^* Z$ (quadratic detector corresponding to intensity or power data) with a negative one-sided exponential distribution, and the logarithm of the square detected envelope $D = \log I$ with a Fisher-Tippett distribution.

Common estimators of the reflectivity are based on the arithmetic mean intensity (AMI), the square of the arithmetic mean amplitude (AMA), and the exponential of the arithmetic mean logarithm (AML) of N samples. The AML corresponds to integration in dB. The theoretical performances of these estimators can be established when the samples are assumed to be independent. In that case, it has been shown that the maximum likelihood (ML) estimator of the mean reflectivity is the AMI, which is unbiased and efficient, i.e., its variance reaches the Cramer-Rao lower bound [1]. All other unbiased estimators are therefore suboptimal in terms of speckle reduction.

In practice, the samples are generally correlated and the AMI is no longer the ML estimator of the mean reflectivity. For complex data and correlated samples, the ML estimator is the spatial whitening filter (SWF) [2], [3], which theoretically is unbiased and efficient. However, we show that this estimator does not work properly on oversampled data where a part of the frequency spectrum is null. We propose a solution to this problem and study the influence of the spectral properties of the speckle on the performance of the different estimators.

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II. Vectorial Probability Distribution

Let us consider a set of N complex samples, corresponding to N adjacent pixels in a complex radar image. $\mathbf{Z}$ is a signal vector containing the complex amplitudes $Z_1, Z_2, \ldots, Z_N$. If the speckle is fully developed, the probability density function of the signal vector is a circular complex Gaussian distribution

$$p(\mathbf{Z}) = \frac{1}{\pi^N |\mathbf{C_Z}|} \exp\left(-\mathbf{Z}^{t*}\mathbf{C_Z}^{-1}\mathbf{Z}\right) \quad (1)$$

where $\mathbf{C_Z}$ is the $N \times N$ complex covariance matrix corresponding to signal vector $\mathbf{Z}$.

If, furthermore, we suppose that the underlying reflectivity R is constant, $\mathbf{Z} = \sqrt{R}\mathbf{S}$, where $\mathbf{S}$ is the complex speckle vector, so that the covariance matrix of the signal vector $\mathbf{Z}$ is given by [4]

$$\mathbf{C_Z} = R \cdot \mathbf{C_S} \quad (2)$$

where $\mathbf{C_S}$ represents the covariance matrix of the speckle vector $\mathbf{S}$. As the complex speckle has zero mean and unity variance, the elements of $\mathbf{C_S}$ are the spatial correlation coefficients $\rho_S(\Delta x, \Delta y)$ of the speckle, rearranged in accordance with the vector $\mathbf{S}$. The spatial speckle correlation only depends on sensor and processor parameters. If the exact correlation coefficients cannot be obtained from the data provider, the elements of $\mathbf{C_S}$ can be estimated from the complex radar image, simply by computing the correlation coefficients of the complex amplitude Z on any part of the image where the speckle is fully developed and where the mean reflectivity is not so low that the thermal noise becomes dominant. The underlying reflectivity does not need to be constant [5]. It should be noted that $\mathbf{C_S}$ is constant in slant range images, whereas it varies slightly between near range and far range in ground range images. In the latter case, it is thus preferable to estimate and use several speckle covariance matrices when processing over the full swath.

III. Estimators of the Mean Reflectivity

It can easily be shown from (1) and (2) that the ML estimator of the radar reflectivity R is the SWF given by

$$\hat{R} = \frac{1}{N}\mathbf{Z}^{t*}\mathbf{C_S}^{-1}\mathbf{Z}. \quad (3)$$

The AMI is only a special case of (3), corresponding uncorrelated speckle (independent samples), for which $\mathbf{C_S}$ is the identity matrix:

$$\overline{I} = \frac{1}{N}\mathbf{Z}^{t*}\mathbf{Z} = \frac{1}{N}\sum_{k=1}^{N} I_k \quad (4)$$

If $\mathbf{C_S}$ is correctly computed or perfectly estimated, $\hat{R}$ is unbiased [6]. The variance of $\hat{R}$ computed on N samples is N times lower than that of the observed intensity, and $\hat{R}$ is Gamma distributed. This also applies to the AMI when the speckle is uncorrelated. However, for correlated speckle, the AMI reduces the variance by a factor inferior to N, and the output is only approximately Gamma distributed [7]–[9].

The number of multiplications per pixel for the SWF is about N^2+N, so the computational cost becomes considerable for very large windows. A practical solution is to calculate the SWF on highly overlapping smaller windows within the big window and then average the results in intensity. The computational complexity of this hybrid whitening filter (HWF) is barely higher than that of the SWF for the smaller window.

The square AMA $\overline{A}^2$ and the exponential of the AML $\exp(\overline{D})$ are both biased estimators of R. However, they can be made

TABLE I
CORRELATION COEFFICIENTS OF THE CRITICALLY SAMPLED SIMULATED COMPLEX SPECKLE.

$\|\rho_S\|$	Δx=0	Δx=1	Δx=2
Δy=0	1.00	0.29	0.02
Δy=1	0.42	0.12	0.01
Δy=2	0.05	0.02	0.01

TABLE II
CORRELATION COEFFICIENTS OF THE OVERSAMPLED SIMULATED COMPLEX SPECKLE.

$\|\rho_S\|$	Δx=0	Δx=1	Δx=2
Δy=0	1.00	0.48	0.01
Δy=1	0.60	0.29	0.00
Δy=2	0.12	0.06	0.00

unbiased through multiplication with appropriate factors. Analytic expressions for these factors as a function of N for uncorrelated speckle are given in [1].

IV. Experimental Results

The performance of the above mentioned estimators in terms of bias and variance has been studied experimentally on three simulated SAR images (512 × 512 pixels) with constant reflectivity but different spectral properties.

The first image represents the idealized case of uncorrelated speckle, corresponding to complex circular white noise with the experimental amplitude spectra in azimuth and range illustrated by Fig. 1.

In the second image, the speckle is correlated due to the non-constant transfer function of the radar system (antenna gain pattern and/or raw data prefiltering). The experimental amplitude spectrum in azimuth is shown in Fig. 2. The spectral properties in range are only slightly different. The correlation coefficients up to a two pixel lag in both directions are given in Table I.

The third image is similar to the second one, except that it has been oversampled in the spectral domain so that about 20% of the frequency components are null in azimuth as well as in range. The azimuth spectrum is represented in Fig. 3. The first correlation coefficients are given in Table II. The spectral properties of this image are very close to those of the speckle in ERS single look complex (SLC) SAR images. The second image, with the azimuth spectrum shown in Fig. 2, is therefore representative for a resampled ERS SLC image, where the unused frequency components have been eliminated.

The equivalent number of independent looks (ENIL)

$$L' = E^2[\hat{R}]/Var[\hat{R}] \qquad (5)$$

is used to quantify the reduction of the variance of the estimate $\hat{R}$ as a function of the number of samples N. It should be noted that the ENIL is the same for a biased estimator as for its unbiased counterpart, obtained by multiplying the former by a constant factor.

Let us first consider the case of uncorrelated speckle, where the SWF reduces to the AMI. Fig. 4 presents the ENIL for the AMI and for the unbiased estimators based on the AMA and the AML. As predicted, the variance of the estimate is reduced by a factor $L' = N$ when the AMI is computed on N independent samples. For the estimators based on the AMA and the AML, $L' < N$ by approximately 7% and 39%, respectively, for N large. We note in particular the strong performance loss for the estimator that effectuates the incoherent summation on the logarithm of the detected image.

For the critically sampled correlated speckle, the performances of the estimators working on detected data are substantially poorer than that of the optimal estimator, as can be seen from Fig. 5. The ENIL of the AMI is here 35% lower than that of the SWF, and the ENIL of the estimator based on the AML is 55% lower. The performance loss of the HWF, based on the SWF computed on maximally overlapping 3 × 3 windows, compared to the true SWF, is only 20%.

Even though we use estimated speckle covariance matrices, the bias measured for the SWF (and the HWF) is less than 0.5%, which is close to that of the AMI and thus insignificant. The speckle correlation does not change the bias of the square AMA and of the exponential of the AML much, but a small bias (of the order of 1%) may remain if we use the theoretical debiasing factors for uncorrelated samples given in [1].

For the image where a part of spectrum is null, as in ERS SLC images, the SWF encounters serious problems in terms of variance reduction and bias when the number of samples exceeds about 50. For a 3 × 3 window, neither of the problems occur. The HWF based on the 3 × 3 SWF is practically unbiased, and the ENIL, represented in Fig. 6, is only 38% below N. In fact, this corresponds approximately to the percentage of the two-dimensional spectrum which is null. Adding null frequency components obviously does not add information. The maximum number of uncorrelated samples of the image in the spatial domain is equal to the number of non-null frequency components. Moreover, the frequency components that are null cannot, in principle, be raised to the same level as the others through a whitening process without deteriorating the signal. Nevertheless, our experiments indicate that it is possible to locally decorrelate a small number of pixels (3 × 3) with the SWF, despite the null frequency components. As the correlation is stronger than for the critically sampled image, the performances of the estimators based on the AMI, AMA and AML are further reduced. The relative differences between the performances of these three estimators are, however, not significantly altered by the speckle correlation.

V. Discussion

Complex radar data can be given any of the spectral forms shown in Figs. 1–3 without loss of information. Each representation has its advantages and disadvantages.

Complex SAR images from sensors such as ERS and Radarsat are generally oversampled with spectra similar to the one shown in Fig. 3. The speckle is strongly correlated, but the side-lobes of strong scatterers in the detected image are moderate. The necessary bandwidth doubles when transforming the image into intensity, so the detected image will actually be somewhat undersampled. Due to the null frequency components, we cannot use the optimal SWF estimator, but the HWF can be used as an approximation.

Critically sampled complex data, as shown in Fig. 2, seems to be the ideal representation for automatic analysis tasks, as it permits the use of the SWF. This estimator is, however, computationally complex, and it is necessary to oversample such images before detection and visualization.

The advantage of critically sampled and whitened complex radar images, illustrated by Fig. 1, is that the optimal estimator reduces to the AMI, which only demands one multiplication per pixel within the analysis window. Unfortunately, the side-lobes

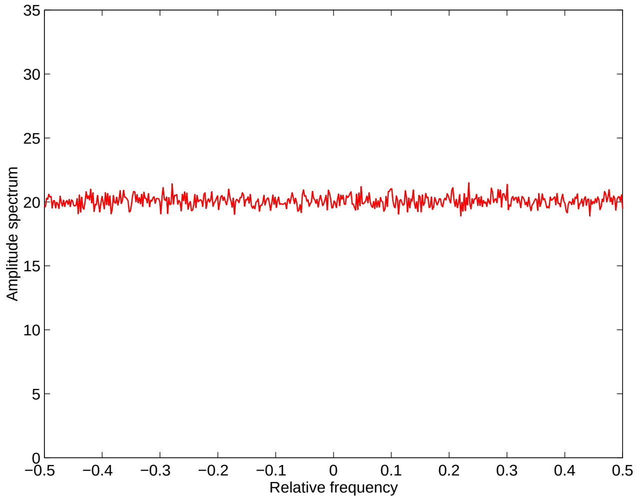

Fig. 1. Spectrum of the uncorrelated complex speckle.

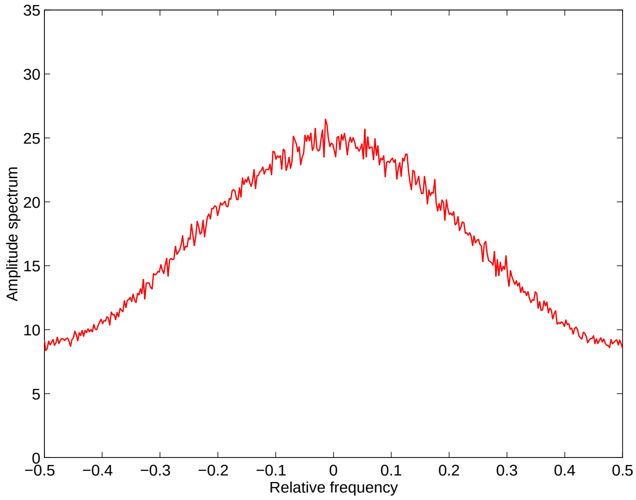

Fig. 2. Spectrum of the critically sampled correlated complex speckle.

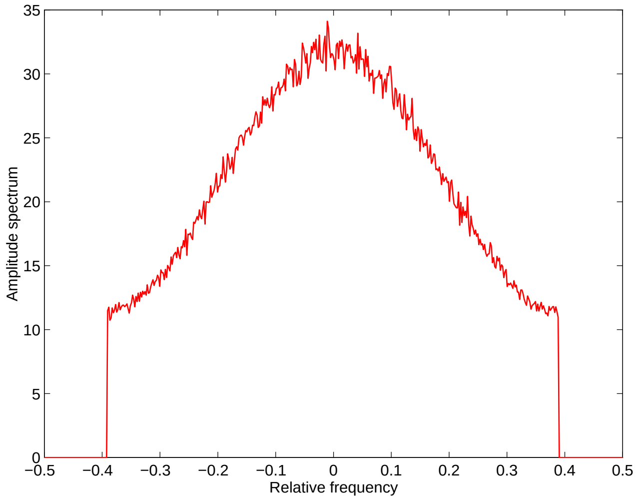

Fig. 3. Spectrum of the oversampled correlated complex speckle.

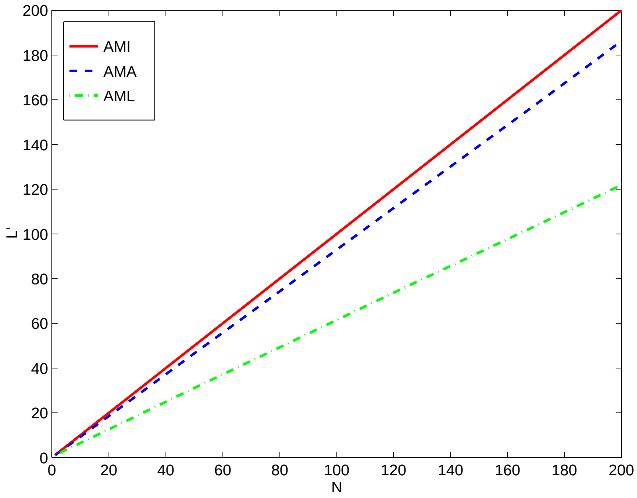

Fig. 4. ENIL for the uncorrelated complex speckle.

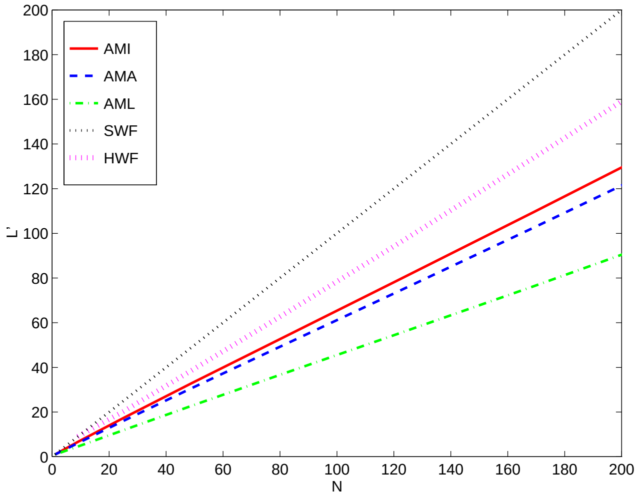

Fig. 5. ENIL for the critically sampled correlated complex speckle.

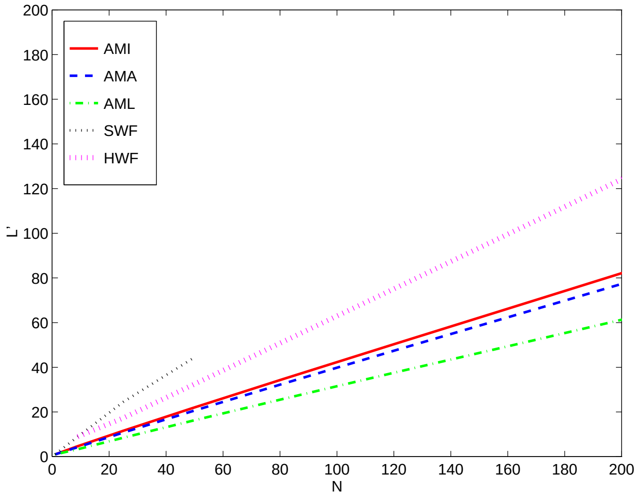

Fig. 6. ENIL for the oversampled correlated complex speckle.

of strong scatterers become very pronounced and will generally complicate the analysis of the image.

VI. Conclusion

The experimental results clearly demonstrate that it is advantageous to estimate the mean radar reflectivity from complex data when the samples are correlated. This is particularly important for the analysis of SAR images. However, complex SAR images are usually oversampled, with a significant portion of null frequency components (more than 35% in the case of ERS SLC images). This poses a problem for the use of the optimal ML estimator, which is the SWF. One solution is to eliminate the superfluous frequency components and thus reduce the size of the image before applying the SWF. Alternatively, a new hybrid filter combining the SWF computed on a small number of samples and incoherent summation can be applied to the initial complex image. The usual estimators, based on averaging in intensity, amplitude or dB (logarithm of the intensity), have substantially poorer performance. In particular, the use of logarithmic summation should be avoided. We have recently demonstrated the improvement brought by the SWF in point target detection [3], edge detection and SAR image segmentation [10], [11].

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