BAYESIAN MODELLING OF SEQUENCE STRATIGRAPHIC BOUNDING SURFACES

Simulation from a Marked Point Process Model using Sampling/Importance Resampling

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Abstract. The adoption of sequence stratigraphy concepts to reservoir description involves correlation of bounding surfaces between wells to produce a high resolution reservoir zonation. Where significant uncertainty is associated with the geometry of these surfaces stochastic modelling provides a correct basis for field development decisions.

2D Gaussian fields have been used to model the bounding surfaces, while incised valleys in the expectation of the bounding surfaces are modelled by a marked point process. In this process the marks are expected direction of the valley and correlated Gaussian random fields defining size and shape of the valley along the direction line.

A Bayesian approach has been used in the modelling. Prior distributions describing general geologic knowledge and uncertainty associated with this knowledge are combined with reservoir specific observations into posterior distributions. The model is too complex to allow analytic calculations of the posterior model, hence the parameters are implicitly estimated while simulating from the model using the Sampling/Importance Resampling algorithm.

1. Introduction

In recent years geological interpretation techniques known as sequence stratigraphy techniques have been increasingly applied to provide high resolution reservoir descriptions (e.g. Van Wagoner, Mitchum, Campion & Rahmanian (1990)). The techniques place emphasis on correlation of surfaces from well to well as opposed to traditional techniques which focus on correlation of similar lithologies between wells.

A stochastic model has been developed to describe different types of sequence stratigraphic bounding surfaces. In particular we have focussed on modelling surfaces with incised valleys. Such surfaces are typically complex and their geometries between wells are generally associated with significant uncertainties. In fluvial reservoirs the surface geometries are important as they may control the lateral extent of shale barriers in high net/gross intervals and the spatial distribution of reservoir sandstone in low net/gross intervals.

2. The Sequence Stratigraphy Model

A sequential approach is used when defining the sequence stratigraphy model. The reservoir is constructed by simulating a number of surfaces from the base of the reservoir and upwards. Each surface is modelled as a 2D Gaussian field

$$S_i(x) = \mu_i(x) + \epsilon_i(x) \tag{1}$$

where $\mu_i(x)$ is the expectation of surface i and $\epsilon_i(x)$ a Gaussian field with zero expectation and a specified covariance function.

The surfaces are conditioned on observed depths in wells. Standard geostatistical techniques are used for simulation (Omre, Sølna & Tjelmeland 1993) and conditioning (Journel & Huijbregts 1978) of the Gaussian fields. As a surface may be eroded by subsequent surfaces, conditioning on inequality constraints is included using standard multi-normal theory and rejection sampling (Ripley 1987).

An example of a reservoir zonation is given in Figure 1. Here two surfaces having a simple geometry with a constant expectation and two complex surfaces having an expectation containing incised valleys are illustrated.

3. Model for Expected Surface Geometry

The expectation surface $\mu(x)$ is assumed to consist of a certain number of valleys giving the depth of a 2D surface. A marked point process model (Ripley 1981) has been used to model the valleys utilizing experience from modelling of fluvial channel sand-stones (Skare, Skorstad, Hauge & Holden 1996). A realistic geometry of the valleys are emphasised in the model. It is

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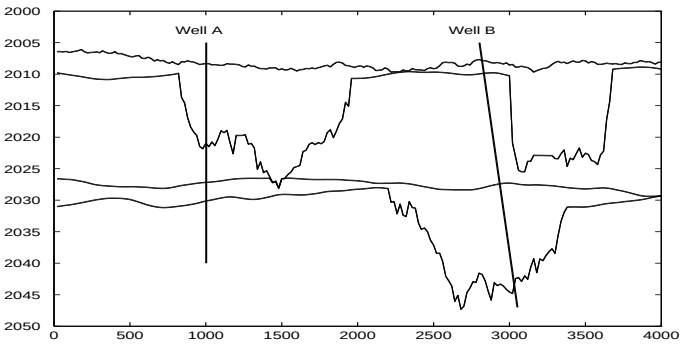

Figure 1. Cross-section of sequence stratigraphy model with 4 bounding surfaces. Two surfaces are containing incised valleys.

also important that it is possible to simulate from the model conditioned on all available data like observed depths, well patterns and well correlations (wells penetrating the same valley).

3.1. PRIOR MODEL

The expectation surface is completely described by direction lines l^i, shape parameters ρ^i and corresponding vectors $V^i(s)$ describing the valley geometry along the lines, $i = 1 \ldots n$, where n is number of valleys. The direction line l^i can be parametrized by i.e. a reference point and an angle with specified distributions. $V^i(s)$, defining position and size of the valley, is a correlated Gaussian function with a one-dimensional reference along l^i:

$$V^i(s) = \{U_D(s), U_W(s), U_T(s), U_{DP}(s)\} \tag{2}$$

with $U_D(s)$ being deviation from direction line, $U_W(s)$ valley width, $U_T(s)$ valley erosion thickness and $U_{DP}(s)$ deviation of deepest point from center of valley in position s along l^i. The univariate properties are defined by an expectation and a spatial covariance function. The cross-sectional form of a valley is defined by $c_0 r^\rho + c_1$ where r is measured normal to the line, with $r = 0$ in the position of the deepest point. Two set of constants (c_0^-, c_1^-) and (c_0^+, c_1^+), for $r < 0$ and $r > 0$ respectively, are computed from the vector $V^i(s)$ to define an asymmetric valley cross-section. The model is illustrated in Figure 2 and 3.

To simplify notation in the following sections the parameters l^i, V^i and ρ^i describing a valley are denoted θ^i, distributed according to $\pi(\theta)$. The prior model for an expectation surface $\mu(x)$ containing n valleys is hence given by

$$\pi(\mu(x)) = \pi(n) \prod_{i=1}^{n} \pi(\theta^i) \tag{3}$$

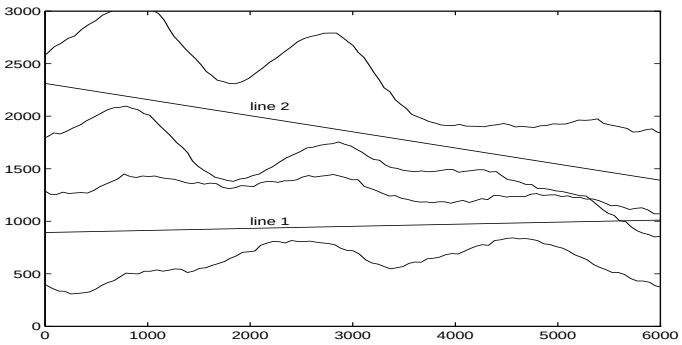

Figure 2. Direction lines and valley margins for expectation surface with 2 valleys.

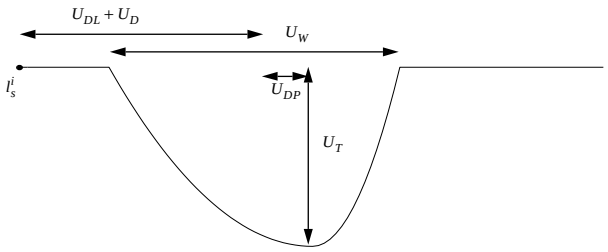

Figure 3. Cross-section of the reservoir perpendicular to the principal direction of the valley. A parabolic shape parameter ($\rho = 2$) is used.

where $\pi(n)$ is the prior distribution for number of valleys in the expectation surface.

In order to represent the uncertainty associated with the general geological knowledge, prior distributions for the parameters in the distributions in equation (3) is used, i.e.:

$$\pi(\theta) = \int_{\Omega_\phi} \pi(\theta|\phi)\pi(\phi)d\phi \tag{4}$$

with ϕ being distribution parameters in prior distribution for θ.

3.2. POSTERIOR MODEL

The posterior model for the expectation surface $\mu(x)$ is conditioned on observed locations and depths of the surface in wells and soft information derived from i.e. well tests or geochemical analyses. With $\boldsymbol{w}$ being all avail-

able information from M wells the posterior model is given by

$$\begin{aligned}\pi(\mu(x)|\boldsymbol{w}) &= \pi(n)\prod_{i=1}^{n}\pi(\theta^i, C|\boldsymbol{w}) \\ &= \pi(n)\prod_{i=1}^{n}\pi(\theta^i|C,\boldsymbol{w})\pi(C|\boldsymbol{w}) \qquad (5)\end{aligned}$$

where C is the actual well configuration and $\pi(C|\boldsymbol{w})$ a function modelling the probability of C given soft information from pairs of wells (w_l, w_m), $l, m = 1, \ldots M$:

$$\pi(C|,\boldsymbol{w}) = \prod_{i=1}^{n}\left\{\prod_{w_l, w_m \in \theta^i} c_{lm} \prod_{w_l \in \theta^i, w_m \notin \theta^i}(1 - c_{lm})\right\} \qquad (6)$$

where $0 \leq c_{lm} \leq 1$ is specified using other information than observed depths. The parameter $c_{lm} = 0.5$ corresponds to no knowledge about well correlations.

4. Simulation from Posterior Model

Using the distribution $\pi(\mu(x)|\boldsymbol{w})$ in (5) in a simulation algorithm two problems arise. First, studying the function $\mu(x)$, there are complicated nonlinear relations between the variables in this function in order to satisfy the well observations due to the nonuniform thickness of the valleys. Secondly, the density is also highly multi-modal due to thin valley observations located on either side of the valley and not in the middle of the valley, and the many different combinations of same valley observed in different wells. Hence traditional Markov Chain Monte Carlo (MCMC) algorithms with small changes is not suited.

4.1. SAMPLING/IMPORTANCE RESAMPLING

Let $\pi(x) = \pi(\mu|\boldsymbol{w})$, and assume that a fast generation algorithm $q(x)$ which satisfies $q(x) \approx \pi(x)$ can be made. This generation function could be used in a MCMC algorithm, but the Sampling/Importance Resampling (SIR) proposed by Rubin (1987) is preferred. The SIR algorithm $Q(x)$ is

- *Generate N independent realizations from $q(x)$.*
- *Choose one or more of these realizations proportional with the importance ratio $\frac{\pi(x)}{q(x)}$.*

Both Rubin (1987) and Gelfand & Smith (1990) show that $Q(x) \to \pi(x)$ when $N \to \infty$. See also Rubin (1988). We will find the rate of convergence and find out what is critical for the convergence.

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Let the generated realizations be $x_1, x_2, \ldots, x_N$. The probability density for drawing these realizations and choosing one particular realization (where we assume $x = x_1$) is then

$$N \frac{\frac{\pi(x)}{q(x)}}{\sum_{i=1}^{N} \frac{\pi(x_i)}{q(x_i)}} \prod_{i=1}^{N} q(x_i). \tag{7}$$

The probability distribution for $Q(x)$ is hence given by

$$Q(x) = \int N \frac{\frac{\pi(x)}{q(x)}}{\sum_{i=1}^{N} \frac{\pi(x_i)}{q(x_i)}} \prod_{i=1}^{N} q(x_i) dx_2 \ldots dx_N = \pi(x) \int \frac{\prod_{i=2}^{N} q(x_i)}{\frac{1}{N} \sum_{i=1}^{N} \frac{\pi(x_i)}{q(x_i)}} dx_2 \ldots dx_N. \tag{8}$$

Consider the integral in (8). The numerator $h(y)$, $y = (x_2, \ldots, x_N)$, has the property $\int h(y)dy = 1$. The denominator can be split into $f(x) = \frac{\pi(x)}{Nq(x)}$ and $g(y) = \frac{1}{N} \sum_{i=2}^{N} \frac{\pi(x_i)}{q(x_i)}$ which satisfies $E_q g(y) = \frac{N-1}{N}$ and $Var_q\{g(y)\} = \frac{N-1}{N^2}\sigma^2$, where $\sigma^2 = Var_q\{\frac{\pi(x)}{q(x)}\}$. Assume $A > \frac{\pi(x)}{q(x)} > B^{-1} > 0$ for all values of x, and let R be the range of $g(y)$ and R_0 be the area where $g(y) > 0.5$. Using the law of large numbers it can easily be shown that the probability for $y \in R - R_0$ is less than $\sigma(2\pi)^{-\frac{1}{2}} e^{-N/8\sigma^2}$ for large N. This gives

$$\begin{aligned}
|\frac{Q(x)}{\pi(x)} - 1| &= |\int_R \frac{h(y)}{f(x) + g(y)} dy - 1| \qquad (9)\\
&\leq \int_R h(y) |\frac{\frac{N}{N-1}(f(x) + g(y)) - 1}{f(x) + g(y)}| dy + \frac{1}{N}\\
&\leq 2 \int_{R_0} h(y) |\frac{N}{N-1}(f(x) + g(x)) - 1| dy + \frac{1}{N} + \sigma N^{-\frac{1}{2}} B e^{-N/8\sigma^2}\\
&\leq 2\frac{N}{N-1} A \frac{1}{N} + 2\frac{N}{N-1} (\frac{2}{\pi})^{\frac{1}{2}} \sigma N^{-\frac{1}{2}} + \frac{1}{N} + \sigma N^{-\frac{1}{2}} B e^{-N/8\sigma^2}\\
&\leq N^{-\frac{1}{2}} (2\sigma + 1 + 3AN^{-\frac{1}{2}} + \sigma B e^{-N/8\sigma^2})
\end{aligned}$$

which proves the following theorem:

Theorem *For a fixed $\pi(x)$ and $q(x)$ which satisfies the inequalities $A > \frac{\pi(x)}{q(x)} > B^{-1} > 0$, we have for sufficient large N that*

$$|\frac{Q(x)}{\pi(x)} - 1| \leq N^{-\frac{1}{2}} (2\sigma + 1 + 3AN^{-\frac{1}{2}} + \sigma B e^{-N/8\sigma^2}) \tag{10}$$

This theorem clearly shows the importance of bounding the upper limit A and σ. The lower limit B has only minor importance.

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This can also be seen intuitively since if $q(x) \ll \pi(x)$ for x in an area D, the probability that $q(x)$ generates realizations in D is too small which implies that $Q(x) < \pi(x)$ in the same area.

In some application it is only possible to estimate $r(x) = c\frac{q(x)}{\pi(x)}$ for an unknown constant c. A and σ may be estimated using $E_Q \frac{q(x)}{\pi(x)} \to 1$ when $N \to \infty$. It is intuitively seen that $E_Q \frac{q(x)}{\pi(x)}$ decreases when N increases. N should be chosen such that the expected value of the ratio $r(x)$ for the chosen realization has reached a constant level.

In our application $r(x)$ varies by many orders of magnitude between the different realizations. We believe however that both A and σ are reasonable small since the ratio $r(x)$ for the chosen realization stops decreasing for N moderate large.

4.2. SIMULATION ALGORITHM

The problem of sampling from $\pi(\mu(x)|\boldsymbol{w})$ in (5) is split into 1) finding a well configuration C according to $\pi(C|\boldsymbol{w})$, i.e. which wells are penetrating the same valley and 2) sampling from $\pi(\theta|C, \boldsymbol{w})$. The SIR algorithm is used for both.

N well configurations $\{C_j\}_{j=1}^N$ is generated by the following algorithm:

1. While there are observations not conditioned on
   (a) Draw distribution parameters $\phi_j^i \sim \pi(\phi)$.
   (b) Draw an observation not yet conditioned on and a line l_j^i close to it.
   (c) Simulate θ_j^i in all well positions along the line.
   (d) Include new wells in the configuration if located inside the valley.
2. Compute importance ratio for configuration C_j

$$r_j = q \cdot \pi(C_j|\boldsymbol{w})$$

   where q = 1 if $n_j \leq n_m$ and $\pi(n_j)/\pi(n_m)$ if $n_j > n_m$. Here n_j is number of generated valleys, and n_m the mode.

A configuration C_k with n_k valleys is chosen proportional to r_k. N realizations $\{\theta_j\}_{j=1}^N$ consisting of valleys θ_j^i, $i = 1 \ldots n_k$ is then generated from $q(\theta|C_k, \boldsymbol{w})$ by the following scheme:

1. Draw distribution parameters $\phi_j^i \sim \pi(\phi)$.
2. Draw the shape parameter ρ_j^i and a direction line l_j^i in the area of interest.
3. Given ρ_j^i, l_j^i, $\{w_l\}_{l=1}^M$, M being number of wells, and well configuration C_k, draw $V_j^i(s)$ in each well position $s = s_{w_l}$ by sequential simulation.

In each cross-section penetrated by a well K possible vectors $V_j^i(s)$ are drawn and the 2 closest are interpolated to match the observed depth.

4. Calculate the importance ratio $p_j^i \approx \pi(\theta_j^i|C_k, \boldsymbol{w})/q(\theta_j^i|C_k, \boldsymbol{w})$.

A realization of one valley $\theta^i = \theta_k^i$ is chosen proportional to p_k^i. For this realization the 1D Gaussian fields defining valley width, thickness, etc. are drawn on a dense grid along l_k^i conditioned on $\{V_k^i(s_{w_l})\}_{l=1}^M$ using model parameters ϕ_k^i.

5. Examples

The example is intended to illustrate 1) the combined influence of prior distributions and well patterns on the posterior distribution of valley directions and valley correlations and 2) the number of realizations generated in the SIR algorithm in order to define a stable posterior model.

5.1. UPDATING OF MODEL PARAMETERS

The sequence boundary is interpreted to consist of a single valley which is observed in three of the six wells. The minimum distance between two wells is approximately 1000 m. In the prior model the expected valley direction is parallel to the x-axis with a moderate uncertainty in the angle $\sim N(0, 30°)$ and distance of line from x-axis $\sim N(0, 250)$. In the base case $U_W \sim N(500, 100)$ and $U_D \sim N(0, 250)$.

By varying valley width and well correlations, the ability of the simulation algorithm to generate realizations from the posterior model is investigated. Typical realizations are shown in Figure 4 and 5.

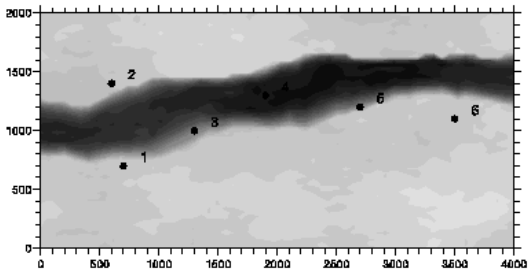

Figure 4. Realization of valley with expected valley width $\mu_W = 500$.

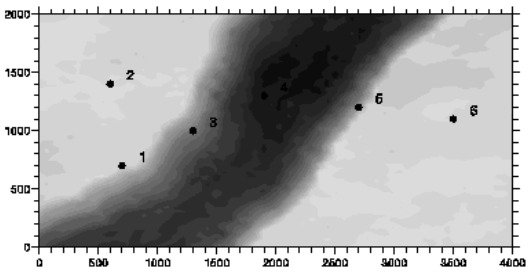

Figure 5. Realization of valley with expected valley width $\mu_W = 1500$.

Figure 6 shows the chosen direction lines for 10 different realizations from 4 prior models. Where one narrow valley is assumed, the resultant valley direction lines must be oriented approximately E-W in order to honour the prior model and the well observations, whereas a prior model with wider valleys changes the resultant orientation to more SE-NW and NE-SW. In the case of unknown well correlations (a uniform prior for 1, 2 or 3 valleys) and narrow valleys, this allows, as expected, more SE-NW and NE-SW orientation than in the base case. Also more valleys are generated (20) than in the case of wide valleys (16) because wells are correlated easier in the latter case.

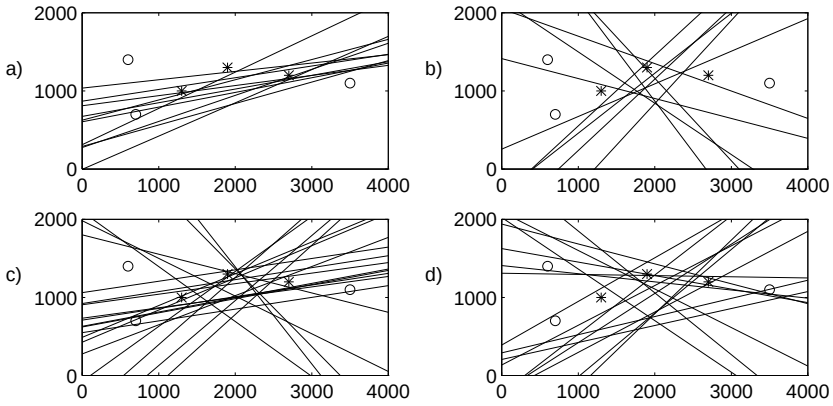

Figure 6. The chosen direction lines for 10 realizations of the sequence boundary. Wells within valleys are marked with an asterisk ($*$), while wells outside with an open circle. In a) the lines have been chosen based on a prior model where $E\{U_W\} = 500$ m and in b) based on a prior model with $E\{U_W\} = 1500$ m. The models in c) and d) are the same as above except that no well correlations and a uniform prior for 1 2 or 3 valleys was specified.

The complex, but physical sensible, interdependence between parameters and observations seen in the example above is an important aspect of

the posterior model. It is not possible to estimate the model parameters from observations except using simulation.

5.2. CONVERGENCE OF SIMULATION ALGORITHM

Recall from Section 4 that N realizations are generated in order simulate one valley from the posterior model. To illustrate the convergence of the algorithm the ratio p_k^i for the chosen line has been inspected as a function of N and with the same start seed for the random number generator. The base case model from above has been used.

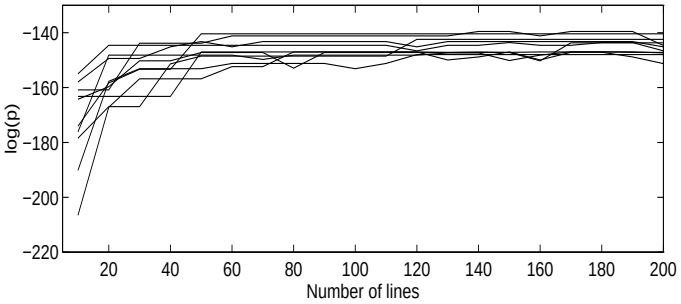

Figure 7. Logarithm of probability density for chosen realization as a function of number of generated realizations N. 10 sequences with different start seeds.

Figure 7 shows the logarithm of the ratio p_k^i of the chosen line as a function of N for 10 different start seeds. The figure shows that the probability of the chosen line increases with increasing N up to a plateau for $N \approx 50$. To assure convergence of the algorithm N should hence be set at least equal to 50 in this example.

6. Closing remarks

In response to the increasing application of sequence stratigraphy as a geological interpretation technique, a stochastic model has been developed to describe the geometry of sequence stratigraphic bounding surfaces. Gaussian fields have appeared to be well suited for modelling of the complex geometry in these surfaces because of the large degree of flexibility associated with the trend surfaces.

Using Bayesian updating, combining field specific observations and general geological information and the Sampling/Importance Resampling technique, realizations from a posterior model have been produced. The model meets the challenge of a complex conditioning scheme including well-to-well correlation and non-linear conditioning. The use of prior distributions

containing significant uncertainty in the model parameters are important as the exact values of the parameters cannot be estimated.

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