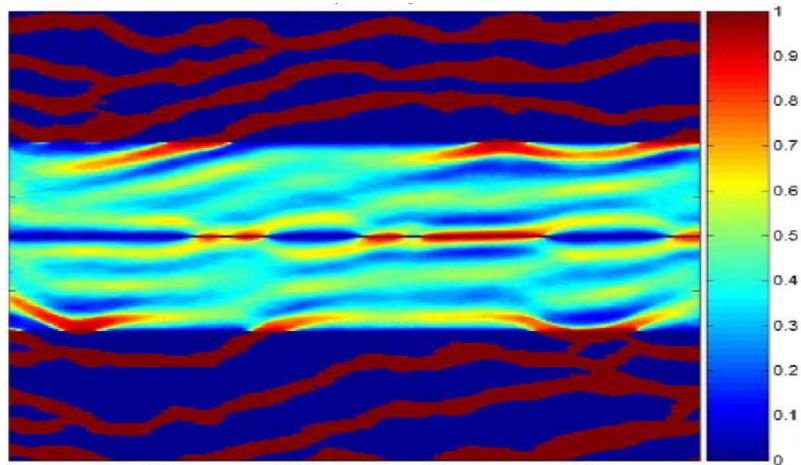


Main Results of the Multipoint Project



Note no

NR/SAND/2008/13

Authors

H. Kjøsberg, O. Kolbjørnsen, M. Stien, P. Abrahamsen

Date

November 11, 2008

About the authors

H. Kjøsberg, O. Kolbjørnsen and M. Stien are research scientists and P. Abrahamsen is research director at the Norwegian Computing Center. They work on petroleum reservoir modelling and spatial statistics, and have the last three years participated in the project Multipoint Methods for Improved Reservoir Models.

Norsk Regnesentral

Norsk Regnesentral (Norwegian Computing Center, NR) is a private, independent, non-profit foundation established in 1952. NR carries out contract research and development projects in the areas of information and communication technology and applied statistical modeling. The clients are a broad range of industrial, commercial and public service organizations in the national as well as the international market. Our scientific and technical capabilities are further developed in co-operation with The Research Council of Norway and key customers. The results of our projects may take the form of reports, software, prototypes, and short courses. A proof of the confidence and appreciation our clients have for us is given by the fact that most of our new contracts are signed with previous customers.

Title	Main Results of the Multipoint Project
Authors	H. Kjøsberg, O. Kolbjørnsen, M.Stien, P. Abrahamsen
Date	November 11
Year	2008
Publication number	NR/SAND/2008/13

Abstract

The aim of this project is to **develop new and improved methods for modelling geological facies** by combining the efficiency of multipoint methods with the robustness and consistency of Markov random field methods. It is a goal to develop software tools and test new methods on real cases and data. The project started in 2006 and is planned to finish in December 2008. The project is a mutually beneficial collaboration between the Norwegian Computing Center, the Norwegian University of Science and Technology and Stanford University. It is sponsored by the Research Council of Norway, ENI and StatoilHydro. The industry partners have contributed with valuable input for the discussions as well as useful guidelines for the choice of research topics.

In this document we first provide a short summary of the main achievements in the project. This is followed by an introduction to multiple point methods and a somewhat more detailed account of each theme of the project.

Keywords	Multipoint, Markov mesh, SNESIM, D-vines, reservoir modelling
Target group	Project participants
Availability	Open
Project number	808002
Research field	Reservoir modelling
Number of pages	15
© Copyright	Norsk Regnesentral/ Norwegian Computing Center

Contents

1	Executive summary	7
2	Multipoint methods, background.....	8
3	Markov mesh models	8
4	Modified SNESIM	11
5	Markov random field models.....	13
6	Pattern comparison tool	13
7	D-vine creation of non-Gaussian random fields	14

List of figures

Figure 1. Training images and realizations, MMM..	9
Figure 2. Training image and realizations, SNESIM and MMM.....	10
Figure 3. Well conditioning, MMM.....	10
Figure 4. Local update, MMM.....	11
Figure 5. Seismic conditioning, MMM..	11
Figure 6. Modified SNESIM.....	12
Figure 7. Pattern comparison.	14
Figure 8. D-vine random fields..	15

1 Executive summary

The main result in this project is that we have established Markov mesh models as a viable tool for pixel based reservoir modelling. We have developed methodology for parameter estimation, seismic conditioning, well conditioning, and local updating in Markov mesh models. The body of this work is documented in Stien, Kjøsberg, Kolbjørnsen, Abrahamsen (2008), and is planned for publication. The work is implemented in a computer program which is available for project sponsors.

- The method used for seismic conditioning in Markov mesh models is an adaption of the method used in SNESIM.
- The method used for well conditioning in Markov mesh models is novel and is presented in Kjøsberg and Kolbjørnsen (2008).
- The well conditioning approach developed in this project is also suitable for solving the well conditioning problem that arises in SNESIM when using multigrids and subgrids.
- The approach of local updating, which also is presented in Kjøsberg and Kolbjørnsen (2008), can be used to alter the model in some regions while keeping it fixed in other regions.

Through the project we have gained insight to the algorithm SNESIM. By analyzing the simulation algorithm of SNESIM through the eyes of a statistician we have been able to identify weaknesses and make proposals on how this algorithm can be improved to provide better simulation results. This work has been published in Stien, Kolbjørnsen, Hauge, Abrahamsen (2007).

We have tested the pseudolikelihood estimator for Markov random fields (MRFs). We have concluded that this estimator is inefficient for estimation in MRF models. Other existing methods for parameter estimation in MRF models are currently too time consuming to be considered for reservoir models. Research to formulate efficient models and estimation procedures for Markov random fields is currently carried out by a project-sponsored PhD student at the Norwegian University of Science and Technology

Traditionally, the evaluation of multipoint methods is done by visually comparing facies simulations to the training image. We have developed a novel approach for comparing the facies simulations to the training image. This work has been published in Soleng, Syversveen, Kolbjørnsen (2006), and a program is available for project sponsors.

Multipoint methods have since their origin focused on discrete variables, i.e. the variable is considered to be in a given class, e.g. sand or shale. We have developed novel methodology which extends the multipoint approach to continuous variables, e.g. porosity, permeability. This work has been published in Kolbjørnsen and Stien (2008).

2 Multipoint methods, background

Multipoint methods are a set of pixel based simulation techniques. The term *multipoint* is used to stress that higher order statistics is used to capture the patterns seen in nature. This is in contrast to the variogram based techniques such as truncated Gaussian random fields and indicator kriging. In this sense, all methods we are investigating in this project are multipoint methods - including the Markov random field and Markov mesh models.

Simulating a discrete pixel based random field can be looked upon as a two step approach:

1. **Estimation:** In the estimation step statistics from a training image (TI) is extracted. E.g. SNESIM stores the frequency of occurrence of all patterns in the TI within a predefined template. For Markov random field and Markov mesh models a probability distribution is adapted to the TI using maximum likelihood estimation (MLE) or approximations to MLE.
2. **Simulation, unconditioned or conditioned:** Based on the estimates, a simulation algorithm will generate samples that aim at having the statistical properties of the TI. Furthermore, we want the simulated samples to honour well data and seismic data.

3 Markov mesh models

Markov mesh models are a subclass of Markov random fields, distinguished by the property that they can be formulated in terms of a finite, sequential neighbourhood. This makes both parameter estimation and simulation a lot easier and faster. For parameter estimation the maximum likelihood can be applied directly, without the need for approximations. In addition, simulation can be done sequentially and there is no need for iterations. In some sense the Markov mesh models provide a combination of traditional MRFs and the SNESIM approach: A probabilistic model is fitted to the training image, and simulation is performed sequentially, without iterations.

A main challenge for Markov mesh models is to choose a parameterized probability distribution. We have tested a wide range of choices within the Multipoint project. The present choice of parametric model works well for several different training images in both 2D and 3D. The parameterization is designed with a view to connectivity in the three spatial directions. In addition we let the training image have direct impact on the choice of model by using principles of data reduction in the parameter estimation.

Another challenge is data conditioning. When using sequential simulation, the update of a cell value is conditioned on past visited cells, not on what might happen in the future, for instance in terms of data points along the future path. This problem can be dealt with using approximate methods or by using iterations. The Multipoint project has successfully studied both approaches. The approximate method uses indicator kriging to take into account future data points. The iterative method uses the approximate method to suggest new states in a Markov chain Monte Carlo simulation, then specifies the Metropolis-Hastings accept probability such that the samples are drawn from the correctly conditioned model.

In the Multipoint project we have implemented a computer program (Stien, Kjøsberg, Kolbjørnsen, Abrahamsen (2008), Kjøsberg and Kolbjørnsen (2008)) that can:

- Perform parameter estimation by use of the maximum likelihood method to fit the model to the training image.
- Generate unconditional samples from the model.
- Generate samples conditioned on hard data (wells) and soft data (seismic), by
 - using an approximate method and sequential simulation,
 - using an exact method based on iterations.
- Perform local update of an existing realization to fit new data.

In the following we refer to this computer program as MMM (Markov Mesh Model). A few examples of the MMM program's abilities are presented in the following.

Figure 1 shows two examples of 2D binary training images (left) and samples generated by the MMM program. The main features are captured well by MMM.

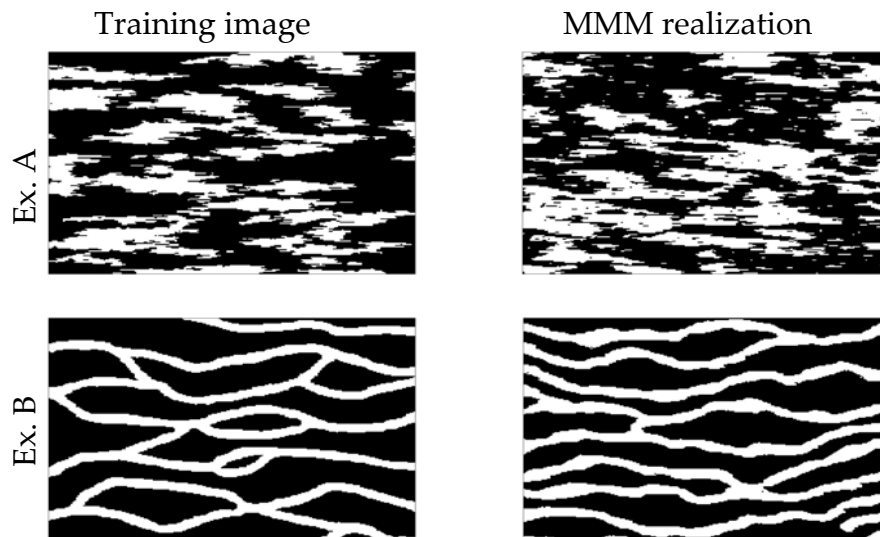


Figure 1. Training images (left) and realizations generated by MMM (right).

Figure 2 shows an example with three facies types. The figures show the training image (left), a sample obtained using MMM (middle), and a sample made by SNESIM (right). The simulated samples have a lot in common with the training image, although they are generally more rugged.

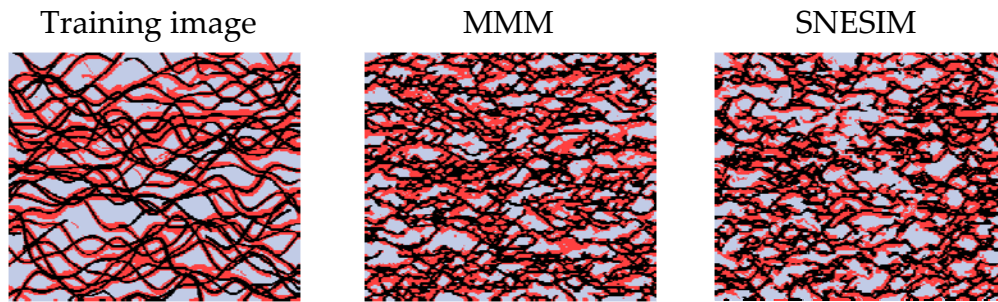


Figure 2. Training image (left), realization generated by MMM (middle), and realization generated by SNESIM (right).

Hard data conditioning using the approximate method has been tested in 2D. Figure 3 shows the cell-wise sand probability for a correctly conditioned model (left), obtained by rejection sampling, and the sand probability found using the approximate method (right). In both cases conditioning is done with respect to an isolated sand well in the middle of the figure, and the Markov mesh model for which conditioning is done corresponds to the sample in the lower right pane of Figure 1. The degree of similarity between the two figures in Figure 3 illustrates that the method gives a good match.

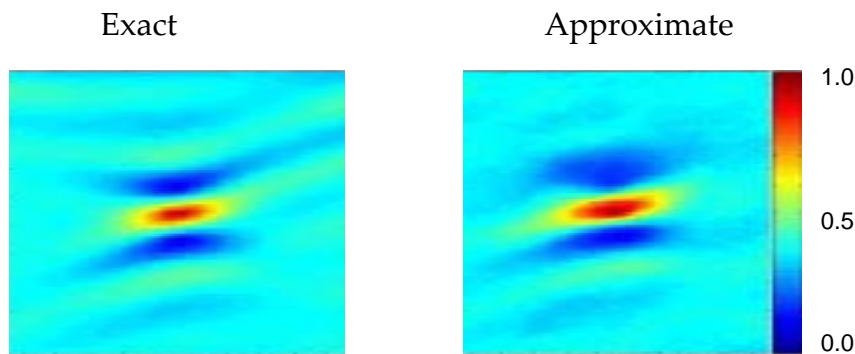


Figure 3. Well conditioning using MMM. An isolated sand well is located in the middle of the grid. Figures show the probability for sand. Left: Exact conditioning obtained by rejection sampling. Right: Conditioning using the approximate method in MMM.

Local update is illustrated in Figure 4. A horizontal well was introduced along a straight line in the middle of the figure. The facies of the well changes as we follow the well left to right. The figure shows the sand probability obtained by re-simulating via iterations an area extending in both directions away from the well. Conditioning is done with respect to the well and to the part of the realization that is outside the rectangle of local update. The figure illustrates that local update in this case conditions correctly both on the well and on the fixed part of the realization.

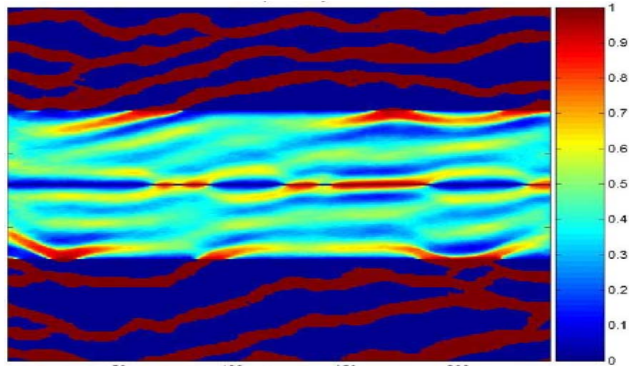


Figure 4. Local update using MMM's iterative method. Conditioning is done relative to a horizontal well line in the center of the figure. The figure shows the probability for sand.

The MMM program has also been tested in 3D. Based on a training image received from one of the partners of the Multipoint project, we have estimated the parameters of a Markov mesh model for this case. Simulation from the model gives results that are comparable in quality to the best results of other pixel based methods.

Seismic conditioning has also been performed for this case. Figure 5 shows a comparison between the seismic probability cube (left), showing the sand probability, and the cell-wise sand probability obtained from 1000 samples generated by using conditioned MMM (right). The close similarity of the two figures is evident.

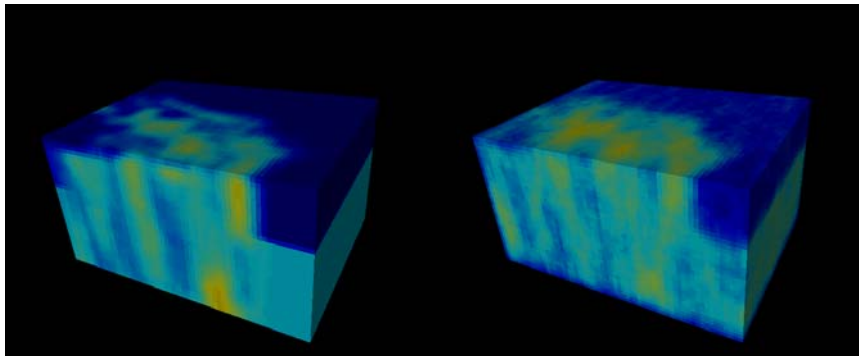


Figure 5. Seismic conditioning using MMM. Left: seismic probability cube for sand. Right: cell-wise sand probabilities obtained using the MMM program.

The Markov mesh study carried out in the Multipoint project has been successful, and has established Markov mesh models as a viable tool for pixel based reservoir modelling. The methods implemented in the MMM program can be improved and expanded, and more tests and case studies should be carried out to investigate the robustness of the approach. The Multipoint project has provided a proof of concept for this methodology.

4 Modified SNESIM

Single Normal Equation Simulation (SNESIM), developed at Stanford University, is the most well known and widely used multipoint simulation technique. SNESIM is based on a simple idea: Count all pattern frequencies found in the training image. Then draw a random path

through all grid cells in the simulation grid. Follow the random path sequentially, and at each cell draw the current cell value according to the pattern frequencies found in the training image. The considered training image patterns are those that match the pattern of the previously simulated cells.

The biggest challenge in the SNESIM approach is the following: As the sequential simulation proceeds along the random path, cell values are drawn based on pattern frequencies in the training image. However, SNESIM only considers patterns that match cell values that have been drawn in the past of the simulation, and not patterns that take into account what *might* be drawn in the future. As a consequence the simulation gets stuck into partial patterns that do not exist in the training image. When this happens, the simulation has to start over again. If the training image is small, this may happen frequently. As a first attempt to resolve this, SNESIM includes a method that ignores some of the previously simulated cell values when comparing patterns to the training image. This ensures that matches with the training image can actually be found. The effect is that the simulation itself does not so easily get stuck, but the final samples contain artefacts such as isolated cell values or object shapes not found in the training image.

The introduction of artefacts could be avoided by marginalizing the probabilities, i.e. take into account all possible future draws in the simulation and compare them to the training image as well. However, this is computationally too expensive and is probably not even a good solution for finite size training images. We have in the Multipoint project tested an approach where we allow previously simulated cell values to be deleted so that they have to be re-drawn (Stien, Kolbjørnsen, Hauge, Abrahamsen (2007)). Several criteria for selecting cells for re-simulation have been tested and evaluated. Figure 6 displays a training image (left), a sample using SNESIM (middle), and a sample using our modified SNESIM (right) where we re-draw some of the cell values. It is quite clear that the modified algorithm avoids creating dead ends like those seen in the middle picture, and that the sample to the right is visually closer to the training image. This is confirmed by a statistical analysis of the image properties of the modified algorithm, the original algorithm, and the training image.

Well data conditioning is also tested for the modified algorithm, and for the training image in Figure 1 we can draw the same conclusions as without well conditioning. Also these results are confirmed by statistical analysis. We thus find good evidence that the modified simulation algorithm represents an improvement of the original SNESIM.

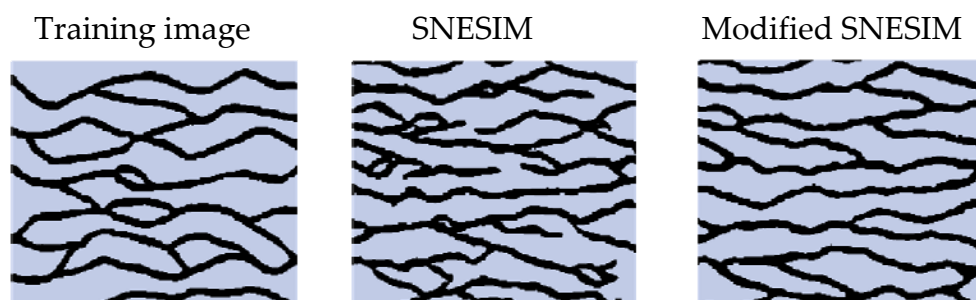


Figure 6. Modified SNESIM compared to original SNESIM. Left: Training image; middle: ordinary SNESIM; right: modified SNESIM.

5 Markov random field models

Markov random fields (MRFs) are a class of probability distributions on a grid. A key property is that the probability for an arbitrary but specific cell value only depends on the cell values in a neighbourhood, not on all other cells in the grid. The neighbourhood is symmetric around the cell, and its size determines the possible complexity of the model. It is in general not useful to use the full complexity allowed by a large neighbourhood, but rather select a subset of the possible neighbourhood cell configurations to define the model.

In order to use the MRFs for modelling geological facies it is necessary to:

1. Choose a parameterized probability distribution.
2. Adapt this probability distribution to the training image using standard estimation techniques.
3. Simulate from the adapted probability distribution.

A main challenge here is the parameter estimation. The maximum likelihood estimation is in general very time consuming for realistic MRF models. It is therefore of importance to establish the applicability of approximations to this method, as well as means to speed it up. During the first phase of the Multipoint project we made an effort to investigate the usefulness of the pseudolikelihood estimation. For a wide range of parameterized probability distributions the pseudolikelihood estimation was performed, and simulations of the resulting model carried out. Satisfactory replication of patterns in the training image proved to be difficult, even if some features were captured. It is in general difficult to know whether absence of pattern reproduction is to be blamed on the choice of parameterized probability distribution, the estimation method, or the simulation algorithm. But based on convergence tests for the simulations and experience with successfully choosing parameterized probability distributions for Markov mesh models, we strongly believe that the pseudolikelihood method is the main problem.

The maximum likelihood estimation for MRFs is presently being pursued by the Multipoint project participants located at the Norwegian University of Science and Technology in Trondheim. The aim is to find efficient ways of doing the maximum likelihood. This work is part of a PhD project, and will extend until 2010.

6 Pattern comparison tool

In order to obtain objective measures for how good a particular method is in reproducing the properties of a given training image, we have implemented a body detector and analyzer program (Soleng, Syversveen, Kolbjørnsen (2006)). This program quantifies image properties such as global facies fractions; the number of objects; object volumes, surface areas and extensions in the various directions. These data are extracted from both the realizations and the training image and represented by box plots as in Figure 7. The straight vertical line represents the training image, and the box plots contain data from 1000 realizations generated using the modified SNESIM algorithm (left) and the MMM program (right). In each case the model was fitted to the channel training image in Figure 1. Figure 7 illustrates that the pattern comparison

tool can be used to compare various multipoint methods. This comparison complements the use of visual inspection of the realizations.

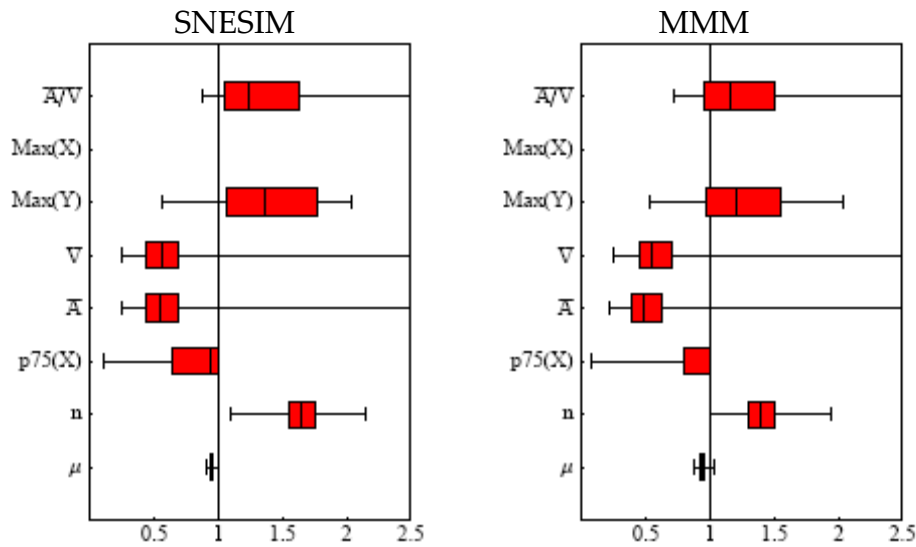


Figure 7. Pattern comparison. Box-plots of measures obtained from the body detector and analyzer program. Left: SNESIM, right: MMM.

7 D-vine creation of non-Gaussian random fields

The D-vine creation of non-Gaussian random fields is a novel approach to introduce nonlinear relations for petrophysical properties such as porosity and permeability.

In current practice spatial nonlinear behaviour in petrophysical properties is introduced through facies models such as the multipoint methods investigated in the current project. Within each facies class the standard methodology for simulating petrophysical properties is based on linear geostatistical methods. Traditional nonlinear geostatistical methods, such as disjunctive kriging and indicator kriging, can be used also for nonlinear models such as the mosaic model, but simulation based techniques do not cover these model types. In the Multipoint project we formulate a framework that includes the linear models, but also allows for nonlinear relations between variables (Kolbjørnsen and Stien (2008)). In addition we have developed a methodology for non-parametric estimation of the model parameters. By introducing nonlinear behaviour we are able to capture a much larger class of models. Figure 8 shows, in the top row, a training image generated by the mosaic model (left) and a realization from the estimated model (right). The bottom row shows the joint distribution of the cell values for two horizontally neighbouring cells. The line in the left scatter plot is a singularity. This singularity was not taken into account in the estimation procedure and has therefore been smoothed out in the estimated model. The resulting distribution displays distinct nonlinear features.

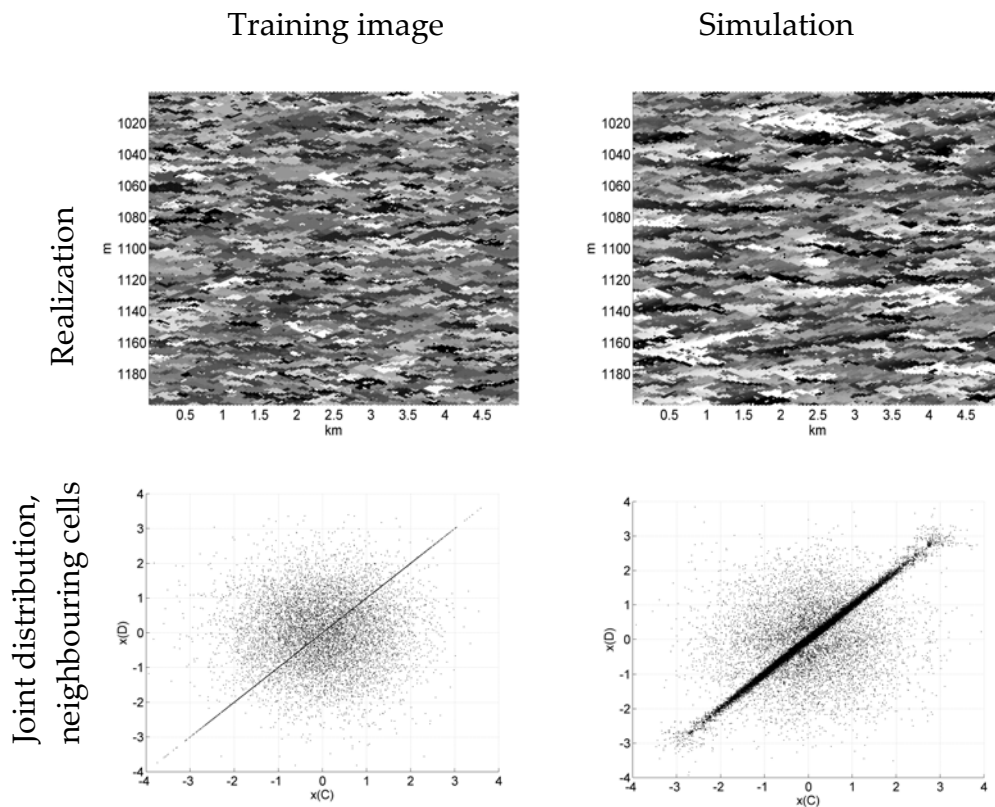


Figure 8. D-vine random fields. Top row shows training image to the left and a simulation from the estimated model to the right. Bottom row shows scatter plot of cell values for two horizontally neighbouring cells, C and D referring to a template numbering of the neighbouring cells. The left panel is from the training image, and the right panel is from the simulation.

References

- H. Kjærnsberg and O. Kolbjørnsen (2008). Markov Mesh Simulations with Data Conditioning through Indicator Kriging. In Proceedings of Geostats 08, Chile. 2008
- O. Kolbjørnsen and M. Stien (2008). The D-Vine Creation of non-Gaussian Random Fields. In Proceedings of Geostats 08, Chile. 2008
- H. H. Soleng, A. R. Syversveen, and O. Kolbjørnsen (2006). Comparing Facies Realizations: Defining Metrics on Realization Space. In Ecmor X. Proceedings of the 10th European Conference in the Mathematics of Oil Recovery, pages A014. European Association of Geoscientists & Engineers. 2006
- M. Stien, H. Kjærnsberg, O. Kolbjørnsen, and P. Abrahamsen (2008). An implementation of conditional Markov mesh simulation with parameter estimation. Overview and user's manual for version 0.1. Technical report, Norwegian Computing Center/SAND/08/04,2008.
- M. Stien, O. Kolbjørnsen, R. Hauge, and P. Abrahamsen (2007). Modification of the SNESIM algorithm, EAGE Petroleum Geostatistics, Cascais, Portugal.