

**Errors in Calibration**  
**Addendum to NR Report No.992**

Notat SAMBA/12/2003

Sonia Mazzi  
mazzi@nr.no

Norwegian Computing Centre/Norsk Regnesentral  
P.O.Box 114, Blindern  
0314 Oslo, Norway

June 2003

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# 1 Introduction

In NR Technical Report No.992 (Mazzi and Høst, 2003), a statistical calibration method to obtain ages from length data of herring in the Vestfjord system was presented.

The method consisted on modeling length, related with age, using a Von Bertalanffy-type growth model. From this model the joint likelihood of lengths, for certain observed ages, can be obtained. If one of the ages is unknown, then this age, which enters as a parameter in the model, can be estimated via maximum likelihood using the joint likelihood.

As expected, in many cases the calibrated ages do not coincide with the true ages of the fish. A summary of some precision measures of the calibration method used is given in Tables 3.2, 3.3, 3.4, and 3.5 of Report No.992. The summary statistics presented in these tables were obtained calibrating the ages of the calibration (observed) sample of lengths and ages and then comparing the observed and calibrated ages.

In this note we give a correction method that can be used when obtaining the age distributions using the calibration method proposed in Report No.992. This correction is applied to take into account the calibration errors.

## 2 Errors in calibration

Let  $a$  denote the true age of a fish and  $a^*$  be its calibrated age. Then,

$$a = a^* + e_{a^*},$$

where  $e_{a^*}$  can take on values in a subset of the integer numbers,  $\mathcal{E}$ . For the particular data we have at hand,  $\mathcal{E} = \{-9, -8, \dots, 0, \dots, 6\}$ .

Let  $p_{a^*}(i)$  denote the proportion of ages calibrated to be  $a^*$  such that  $a - a^* = i$ . If  $e_{a^*}$  is thought of as a random variable then  $p_{a^*}(i) = P(e_{a^*} = i)$ .

If  $n_a$  is the true number of ages equal to  $a$  in the sample, and  $n_{a^*}$  is the number of ages calibrated to be  $a^*$ , it is easily seen that

$$n_a = n_1^* p_1(a-1) + n_2^* p_2(a-2) + n_3^* p_3(a-3) + \dots = \sum_{i=1}^{a-m} n_i^* p_i(a-i), \quad (1)$$

where  $m = \min(\mathcal{E})$ .

### 2.1 Estimation of the calibration error distribution

We propose to estimate the quantities  $p_{a^*}(i)$  from the calibration sample, using the empirical distribution.

The comparison of the ages in the calibration sample and the calibrated ages, obtained from the lengths in the calibration sample, yields the quantities in Table 1. These quantities make (1) hold exactly for the calibration sample. The entries in Table 1 will be the estimates of  $p_{a^*}(i)$ .

For example, 56% of the fish in the calibration sample aged 1 had their age calibrated correctly and we estimate that 56% of the ages calibrated to be 1 are exact for other samples, i.e.  $\hat{p}_1(0) = 0.56$ . Or, 10% of the fish in the calibration sample with age calibrated to be 4 are actually 5 years old and this is our estimate of  $p_4(1)$ , etc.

$a - a^* (i)$	Calibrated Age ( $a^*$ )												
	1	2	3	4	5	6	8	9	10	12	13	15	17
-9	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.06
-8	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
-7	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.19
-6	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.12
-5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.02	0.00	0.22	0.19
-4	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.04	0.03	0.07	0.39	0.06
-3	0.00	0.00	0.00	0.00	0.00	0.01	0.04	0.09	0.00	0.08	0.36	0.33	0.00
-2	0.00	0.00	0.00	0.00	0.02	0.07	0.26	0.00	0.04	0.48	0.39	0.00	0.00
-1	0.00	0.00	0.01	0.08	0.37	0.21	0.00	0.07	0.11	0.32	0.14	0.00	0.00
0	0.56	0.26	0.49	0.80	0.30	0.44	0.09	0.11	0.50	0.05	0.00	0.00	0.32
1	0.44	0.74	0.48	0.10	0.26	0.01	0.15	0.47	0.26	0.00	0.00	0.00	0.00
2	0.00	0.00	0.02	0.02	0.02	0.06	0.32	0.25	0.04	0.00	0.04	0.00	0.06
3	0.00	0.00	0.00	0.00	0.00	0.05	0.10	0.00	0.01	0.00	0.00	0.00	0.00
4	0.00	0.00	0.00	0.00	0.00	0.08	0.02	0.00	0.00	0.00	0.00	0.06	0.00
5	0.00	0.00	0.00	0.00	0.02	0.06	0.00	0.00	0.00	0.00	0.00	0.00	0.00
6	0.00	0.00	0.00	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 1: Distribution of the calibration errors for the observed sample (estimates of  $p_{a^*}(i)$ ).

### 3 Corrected calibrated age distributions

Following, some corrected calibrated age distributions are presented. The correction method is the one described above.

#### 3.1 Calibrated and corrected age distributions for all the length data

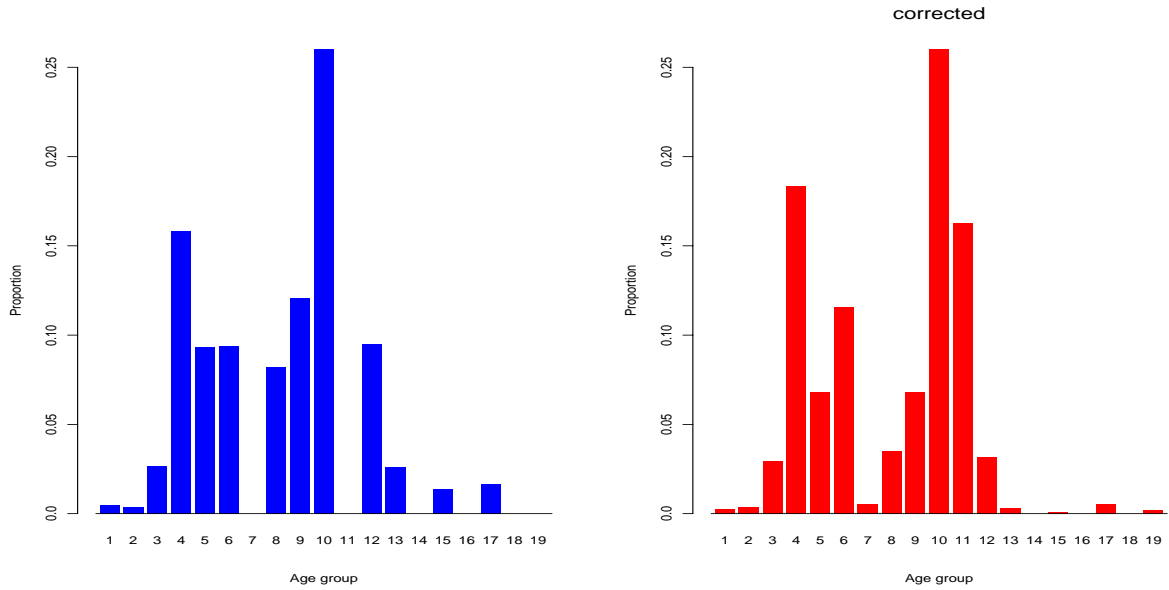


Figure 1: Calibrated and corrected age distributions of all data in the sample.

Age	1	2	3	4	5	6	7	8	9	10
Calibrated	0.0050	0.0034	0.0268	0.1583	0.0933	0.0938	0.0000	0.0823	0.1208	0.2644
Corrected	0.0028	0.0034	0.0295	0.1837	0.0682	0.1159	0.0055	0.0352	0.0678	0.2823

Age	11	12	13	14	15	16	17	18	19
Calibrated	0.0000	0.0951	0.0263	0.0000	0.0140	0.0000	0.0166	0.0000	0.0000
Corrected	0.1630	0.0315	0.0032	0.0000	0.0010	0.0000	0.0052	0.0000	0.0018

Table 2: Calibrated and corrected age distributions of all data in the sample.

### 3.2 Calibrated and corrected age distributions by location

Applying the correction to the calibrated age distributions does not change the conclusions in Report No.992, §4.1, namely that the distributions in the four subregions studied are statistically significantly different. Also, the age distributions in the subregions Vestfjord and Tysfjord appear to be similar. The same applies to the subregions Barøya and Ofotfjord. These results are arrived at more directly using the corrections.

Age	Vestfjord		Tysfjord		Barøya		Ofotfjord	
	Calibr	Corr	Calibr	Corr	Calibr	Corr	Calibr	Corr
1	0.007	0.004	0.009	0.005	0.002	0.001	0.000	0.000
2	0.001	0.003	0.005	0.006	0.003	0.002	0.000	0.000
3	0.009	0.014	0.012	0.018	0.039	0.039	0.034	0.033
4	0.102	0.124	0.089	0.112	0.204	0.233	0.182	0.207
5	0.078	0.055	0.067	0.053	0.111	0.080	0.096	0.070
6	0.081	0.109	0.092	0.110	0.104	0.124	0.089	0.108
7	0.000	0.006	0.000	0.006	0.000	0.005	0.000	0.006
8	0.087	0.038	0.083	0.039	0.084	0.033	0.059	0.032
9	0.135	0.076	0.129	0.077	0.113	0.061	0.116	0.064
10	0.308	0.326	0.315	0.331	0.230	0.247	0.236	0.275
11	0.000	0.191	0.000	0.192	0.000	0.139	0.000	0.165
12	0.113	0.040	0.130	0.038	0.070	0.025	0.123	0.033
13	0.038	0.004	0.035	0.004	0.016	0.003	0.037	0.003
14	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
15	0.022	0.001	0.012	0.001	0.010	0.001	0.017	0.001
16	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
17	0.020	0.006	0.022	0.007	0.014	0.004	0.010	0.003
18	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
19	0.000	0.002	0.000	0.002	0.000	0.001	0.000	0.002

Table 3: Calibrated and corrected age distributions by location.

Area		Age									
		< 4	4	5	6	7	8	9	10	11	> 11
Vestfj.	Cal.count	34	200	89	176	10	61	123	526	308	86
	Cal.prop.	0.021	0.124	0.055	0.109	0.006	0.038	0.076	0.326	0.191	0.053
Tysfj.	Cal.count	33	126	59	123	7	44	86	371	215	58
	Cal.prop.	0.029	0.112	0.053	0.110	0.006	0.039	0.077	0.331	0.192	0.052
Barø	Cal.count	123	680	234	362	15	96	178	721	406	99
	Cal.prop.	0.042	0.233	0.080	0.124	0.005	0.033	0.061	0.247	0.139	0.034
Ofotfj.	Cal.count	13	84	28	44	2	13	26	112	67	17
	Cal.prop.	0.033	0.207	0.070	0.108	0.006	0.032	0.064	0.275	0.165	0.042

Table 4: Cross-tabulation of calibrated (and corrected) number of fish at different Age classes, Tysfjord and Vestfjord.

Comparison	$\chi^2$ statistic	d. of f.	p-value
T-V-B-O	209.80	27	0.000
T-V	2.83	9	0.971
B-O	6.80	9	0.658

Table 5:  $\chi^2$  test results using corrected calibrated proportions.

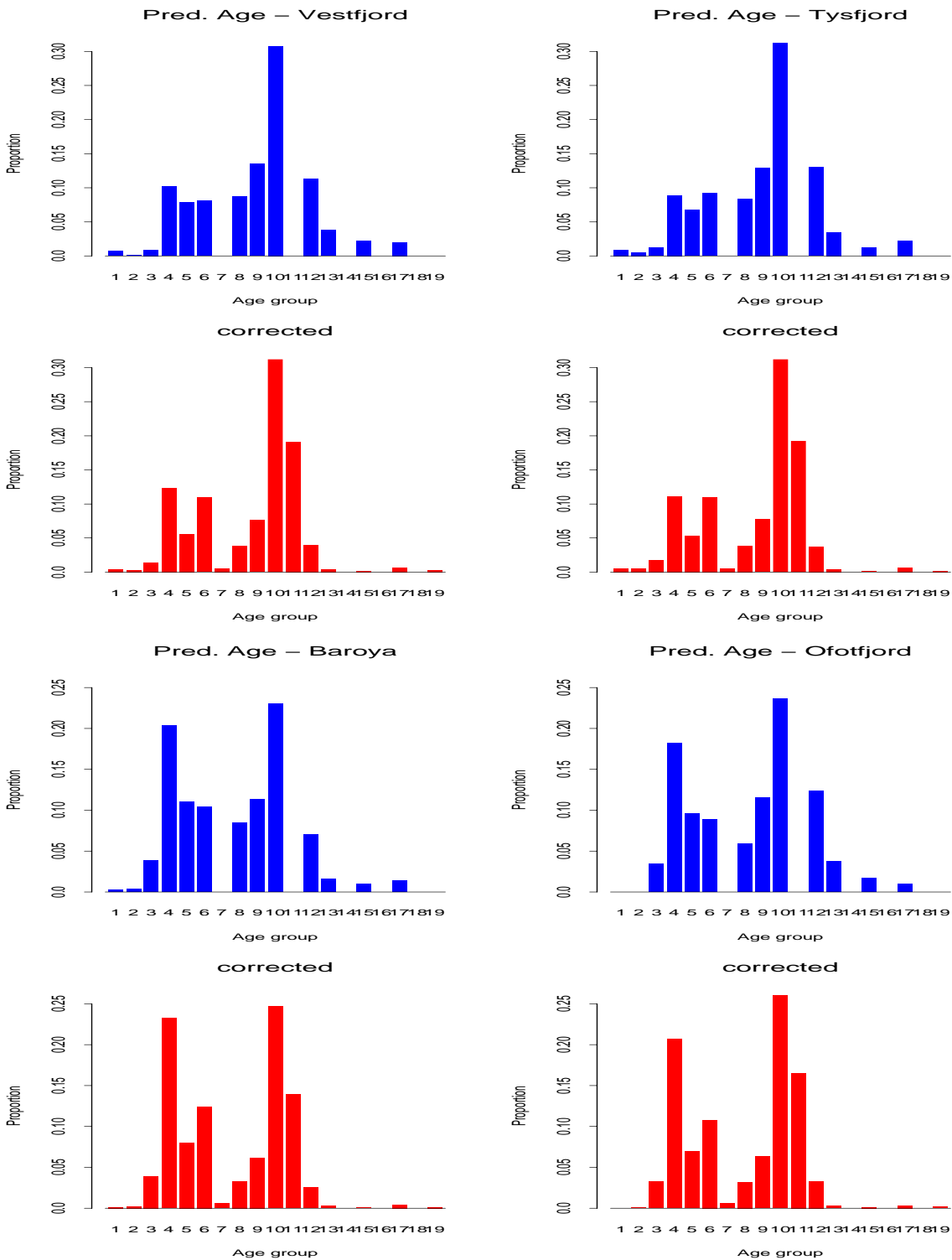
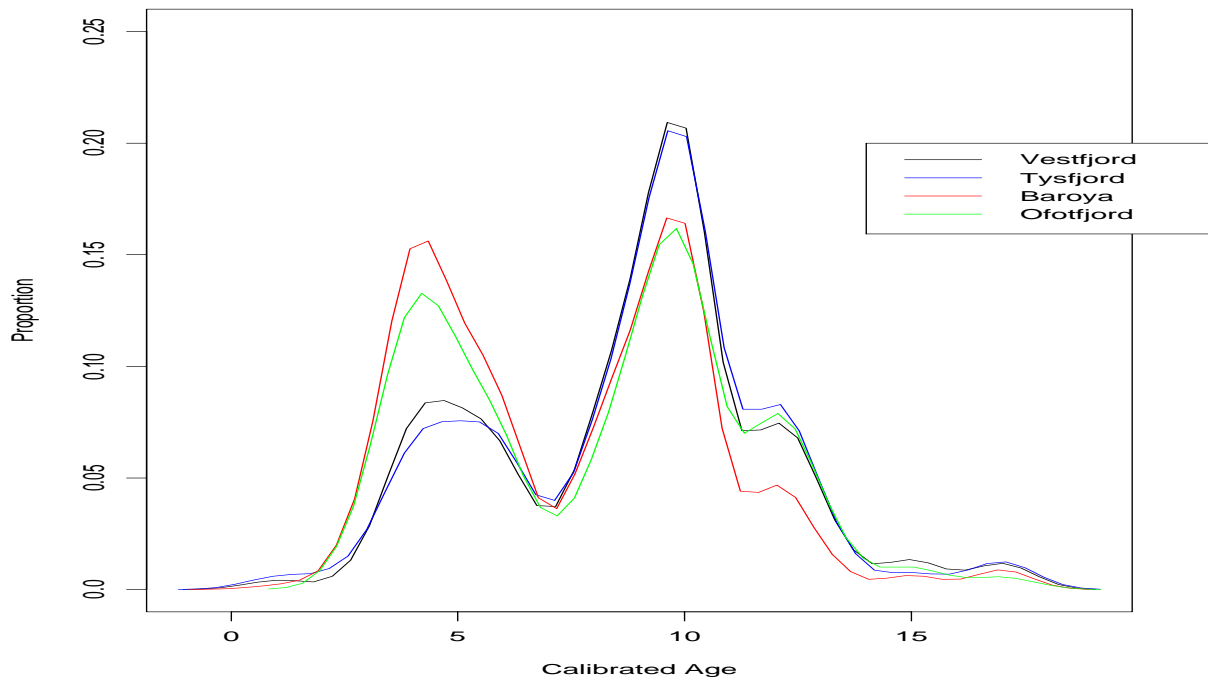
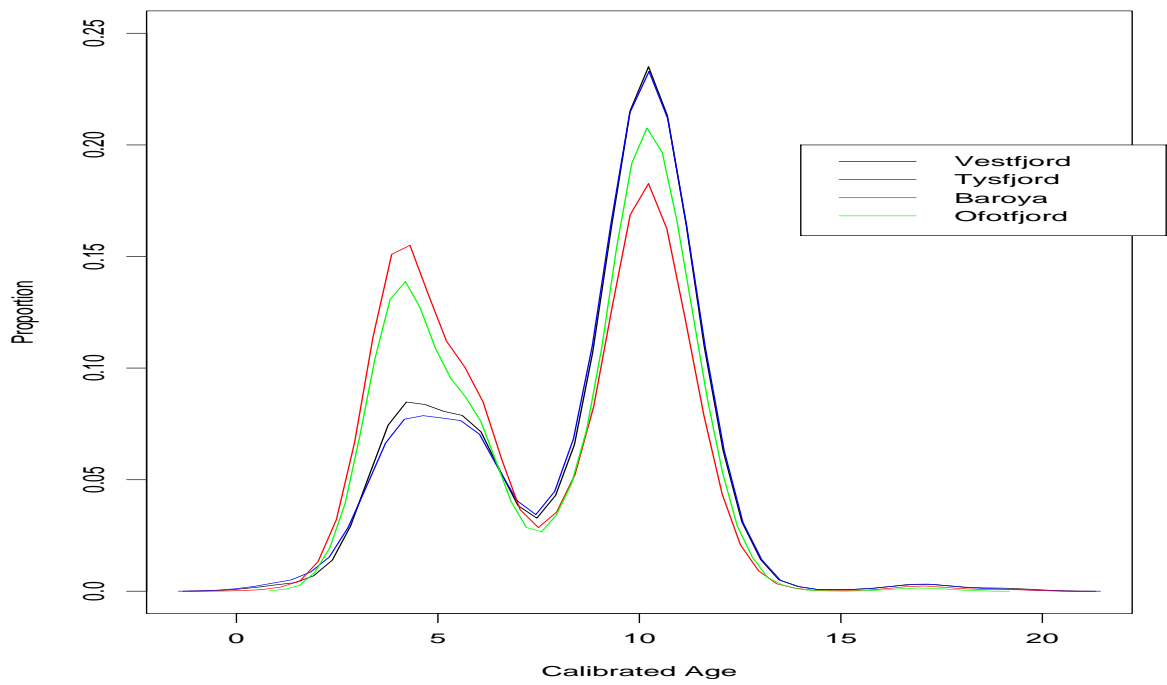


Figure 2: Calibrated and corrected age distributions by location.





corrected



### 3.3 Calibrated and corrected age distributions by depth

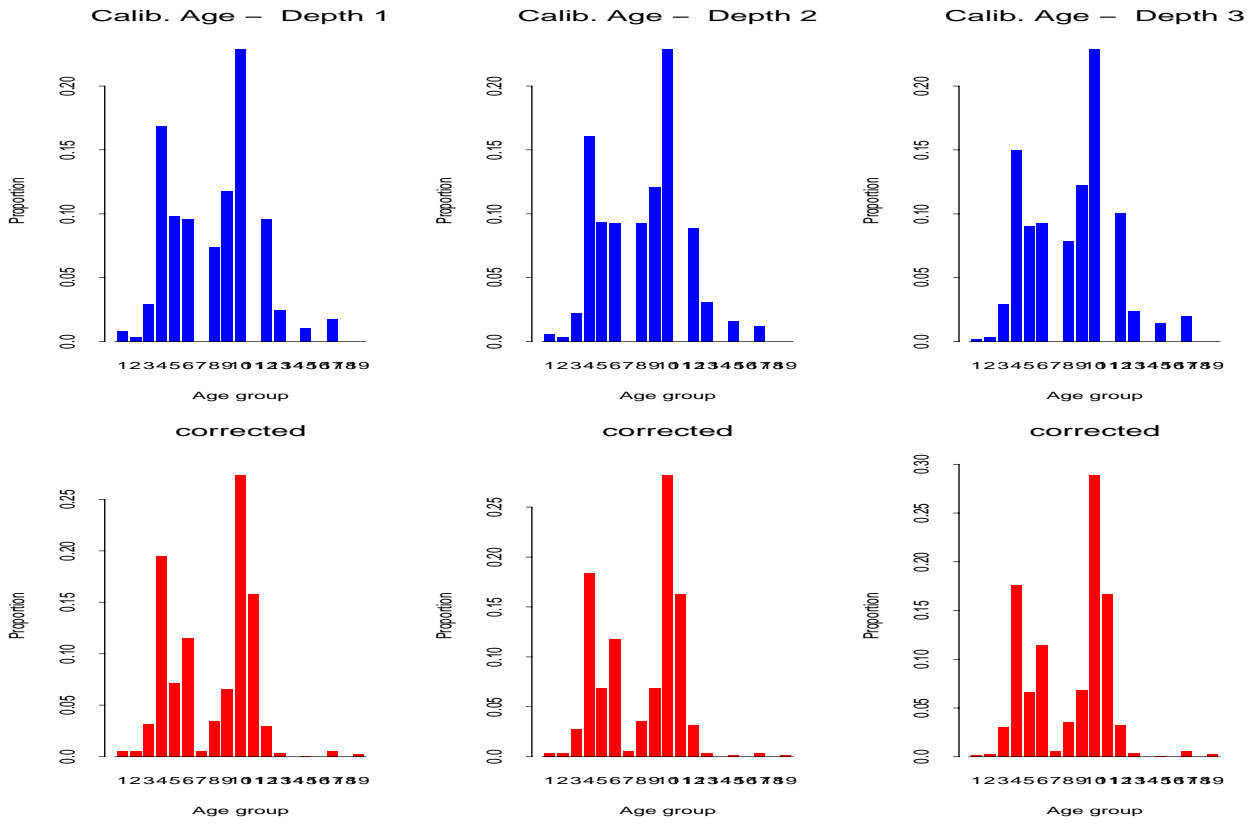


Figure 3: Calibrated and corrected age distributions by depth.

## 4 References

Mazzi, S., Høst, G. (2003). A study of the length and age distribution of herring in the Vestfjord system using a December 2002 survey data. Norwegian Computing Centre Technical Report no.992.