Quantifying uncertainty in HBV runoff forecasts by stochastic simulations

Øyvind Langsrud, Gudmund Høst, Turid Follestad,
Arnoldo Frigessi and David Hirst

December 2, 1998

Abstract

The Swedish HBV-model for rainfall-runoff is extensively used as a flood forecasting tool in the Scandinavian countries. To develop a statistical method for assessing the uncertainty in runoff forecasts, we combine a model for the HBV-model error and models for the uncertainty of the weather forecasts. An algorithm for simulating runoff values have been developed and empirical distribution of simulated values form the basis for probability calculations. The results for two Norwegian catchments seem promising.

Contents

1 Introduction

Floods may threaten human lives and inflict damage on nature, the infrastructure and the economy. Because of their destructive potential, they must be carefully forecasted well in advance. During the spring flood of 1995 in the Glomma River Basin in Eastern Norway, the total cost of the damage was estimated to be NOK 1.8 billion (NOU 1996:16). At that time, daily flood forecasts were made independently by Glommen and Laagen's Water Management Association (GLB) and by the Norwegian Water Resources and Energy Administration (NVE). The GLB and NVE forecasts were often different.

The uncertainty associated with a flood forecast is important for risk assessment and should be taken into account in the decision making process. For example, the probabilities of exceeding certain critical levels may be more informative for a decision maker than the precise expected level of such a flood. Therefore, it is useful to quantify this uncertainty and incorporate it as part of the forecasting and flood warning routine. Based on a study of the flood in Glomma in 1995, Lundquist (1997) lists the following elements as important sources for this uncertainty: meteorological forecasts, the rainfall-runoff model, initial conditions of the rainfall-runoff model, transport time, temporary loss of water and discharge rating curves.

As part of the HYDRA program, the Norwegian Computing Center (NR) is engaged in the project " Quantification of uncertainty in runoff forecasts". The main purpose of this project is to quantify the uncertainty in these forecasts due both to errors in precipitation and temperature forecasts and to approximations performed by the specific rainfall-runoff model. The rainfall-runoff model we consider in this study is the Swedish HBV-model, which is widely used in Scandinavia. However the methods developed to assess the uncertainties could be applied to other models as well.

This report is the final of a series of three. The first report (Langsrud *et al.*, 1997) presented a statistical method for assessing the uncertainty in the rainfall-runoff model. The second report (Follestad and Høst, 1998) considered the uncertainties in the meteorological forecasts. In this report the methodology from the first two reports will be combined to quantify the uncertainty in runoff forecasts.

As in the two first reports we apply the methodology to the two catchments Røykenes (Western Norway) and Knappom (Eastern Norway). Section 2 presents the actual data for these catchments. The statistical method is described in section 3 and the results for Røykenes and Knappom are given in section 4.

2 The data for Knappom and Røykenes

The catchment of Røykenes in Western Norway has an area of $50 km^2$. The annual mean flood (runoff expected to be exceeded once per year) used by NVE for this catchment is 51 m^3/s. Large runoffs at Røykenes often occur during fall or winter, due to heavy rain. The catchment of Knappom has an area of $1625 km^2$ and an annual mean flood of 178 m^3/s. Large flows at Knappom often result from a combination of snowmelt and rain

Tabell 2.1 Notasjon

Table 2.1 Notation

T_t	Temperature at day t
$S_t^{(j)}$	Forecast for temperature at day t, forecasted at day $t-j$.
R_t	Precipitation at day t.
$P_t^{(j)}$	Forecast for precipitation at day t, forecasted at day $t-j$.
$Q_{\text{OBS}}(t)$	Measured runoff at day t.
$Q_{\text{SIM}}(t)$	Runoff at day t calculated by the HBV-model.
$Q_{\text{FOR}}^{(j)}(t)$	Forecast for runoff for day t, forecasted at day $t-j$.
$\text{SWE}(t)$	Snow water equivalent estimated by the HBV-model.

during spring. Daily measurements have been taken at the catchments of Knappom and Røykenes since 1957. We denote by $Q_{\text{OBS}}(t)$ the time series of measured runoffs, where t counts days. The unit of measurement is m^3/s. In the sequel we shall always make clear which catchment is under consideration.

With $Q_{\text{SIM}}(t)$ we denote the runoff predicted with the HBV-model for day t, where actual meteorological conditions are input to the calculations. Hence $Q_{\text{SIM}}(t)$ can be considered as a function of today's and the historical meteorological conditions:

$$Q_{\text{SIM}}(t) = \text{HBV}(T_t, T_{t-1}, T_{t-2}, \ldots, R_t, R_{t-1}, R_{t-2}, \ldots) \tag{1}$$

where T_t denotes the temperature at day t and R_t denotes the precipitation at day t. Note that the two single values T_t and R_t represent the temperature and the precipitation for the whole catchment. They are calculated based on observations at specific locations.

To forecast future runoff the HBV-model is run forward with forecasted temperature and precipitation as input:

$$\begin{aligned} Q_{\text{FOR}}^{(j)}(t) \quad = \quad & \text{HBV}(S_t^{(j)}, S_{t-1}^{(j-1)}, \ldots, S_{t-j+1}^{(1)}, T_{t-j}, T_{t-j-1}, T_{t-j-2}, \ldots \\ & P_t^{(j)}, P_{t-1}^{(j-1)}, \ldots, P_{t-j+1}^{(1)}, R_{t-j}, R_{t-j-1}, R_{t-j-2}, \ldots) \end{aligned} \tag{2}$$

Here $Q_{\text{FOR}}^{(j)}(t)$ is the runoff for day t forecasted at day $t-j$. Similarly $S_t^{(j)}$ is forecasted temperature and $P_t^{(j)}$ is forecasted precipitation.

An overview of the notation is given in Table 2.1. This table also includes $\text{SWE}(t)$ which is the snow water equivalent estimated by the HBV-model. This means the $\text{SWE}(t)$ is also a function of today's and the historical meteorological conditions:

$$\text{SWE}(t) = \text{HBV}_{SWE}(T_t, T_{t-1}, T_{t-2}, \ldots, R_t, R_{t-1}, R_{t-2}, \ldots) \tag{3}$$

By using meteorological forecasts it is also possible to forecast the SWE. In this study we consider forecasts from one to six days ahead ($j = 1, \ldots, 6$).

The data considered in this study are for the period 8/6-95 to 26/8-97. To illustrate these data Figure 2.1 (page 13-14) shows the observed (Q_{OBS}), the HBV predicted (Q_{SIM}) and the forecasted (Q_{FOR}) runoff for Røykenes in 1996. Figure 2.2 (page 15-16) shows the

Tabell 2.2 *Andel av variansen til observert vannføring som forklares av den simulerte og den prognoserte vannføringen.*

Table 2.2 *The amount of the variation in observed runoff that is explained by the predicted and the forecasted runoff.*

	explained variance			
	original data		log-transformed data	
	Røykenes	Knappom	Røykenes	Knappom
Predicted (Q_{SIM})	0.80	0.83	0.76	0.65
Forecast, step 1	0.32	0.69	0.65	0.64
Forecast, step 2	0.32	0.67	0.56	0.63
Forecast, step 3	0.27	0.71	0.55	0.64
Forecast, step 4	0.26	0.58	0.50	0.63
Forecast, step 5	0.18	0.49	0.44	0.61
Forecast, step 6	0.07	0.48	0.37	0.60

same plots for Knappom. For Røykenes the scatterplots of the observed values (Q_{OBS}) versus the HBV predicted (Q_{SIM}) and forecasted (Q_{FOR}) are given in Figure 2.3 (page 17-18). It can be seen that the predicted values are closer to the true values than any of the forecasts. Among the forecasts, step 1 is best and step 6 is worst. This is, of course, as expected. The scatterplots for Knappom are given in Figure 2.4 (page 19-20). The difference between the predicted values and the different forecasts are in this case not so clear as for Røykenes. This indicates that the HBV-model error part is more important for Knappom than it is for Røykenes. In contrast, the meteorological forecasts seem to be a major source of uncertainty for several-days-ahead forecasts at Røykenes.

More formally we evaluate how much of the total variance of the observed runoff is explained by the predicted and the forecasted runoffs. Table 2.2 shows such explained variances calculated as

$$1 - \frac{\sum_t (Q_{\mathrm{OBS}}(t) - Q_{\mathrm{SIM}}(t))^2}{\sum_t (Q_{\mathrm{OBS}}(t) - \overline{Q_{\mathrm{OBS}}})^2} \tag{4}$$

where $\overline{Q_{\mathrm{OBS}}}$ denotes the sample mean. For the forecasts Q_{SIM} is replaced by $Q_{\mathrm{FOR}}^{(j)}$ ($j = 1, \ldots, 6$). Since the explained variances are influenced very much by a few large values, Table 2.2 also presents explained variances calculated after all data have been log transformed. As indicated earlier, for Knappom there is not much difference between the explained variances for predictions and forecasts (especially after log-transformation).

Note that it is possible to calculate how much of the uncertainty is caused by the model error. For the log-transformed data at Røykenes the forecasts one day ahead explain 65% of the variation. The uncertainty is then represented by 35% and we know that $(100 - 76) = 24\%$ represents the model error. Hence, we can say that $24/35 = 69\%$ of the uncertainty is caused by the model error.

3 The simulation algorithm

The aim of this study is to quantify the uncertainty of the runoff forecasts, $Q_{\text{FOR}}^{(j)}(t)$. This means that we are interested in the distribution of the error $Q_{\text{OBS}}(t) - Q_{\text{FOR}}^{(j)}(t)$ for $j = 1, \ldots, 6$. We decompose the error into two parts:

$$\left(Q_{\text{OBS}}(t) - Q_{\text{FOR}}^{(j)}(t)\right) = \left(Q_{\text{OBS}}(t) - Q_{\text{SIM}}(t)\right) + \left(Q_{\text{SIM}}(t) - Q_{\text{FOR}}^{(j)}(t)\right) \tag{5}$$

The first term, $(Q_{\text{OBS}}(t) - Q_{\text{SIM}}(t))$ is the HBV-model error. A statistical model for this term is developed by Langsrud *et al.* (1997). The second term, $\left(Q_{\text{SIM}}(t) - Q_{\text{FOR}}^{(j)}(t)\right)$, is error due to the uncertainty of the meteorological forecasts. Statistical models for the distribution of temperature and precipitation given their forecasts are developed by Follestad and Høst (1998). Rather than modelling $\left(Q_{\text{SIM}}(t) - Q_{\text{FOR}}^{(j)}(t)\right)$ directly we will use their results to develop a simulation algorithm.

In modern statistics stochastic simulations are often used when the distributions are too complex for analytical calculations. Such methods are often much more precise than methods based on simplifying analytical approximations. Here we are interested in the distribution of $Q_{\text{OBS}}(t) - Q_{\text{FOR}}^{(j)}(t)$, or alternatively in the distribution of $Q_{\text{OBS}}(t)$ given all that was known when the forecast $Q_{\text{FOR}}^{(t)}(j)$ was made (the forecast and the observations of temperature, precipitation and runoff up to that day). Below we will formulate an algorithm for simulating values of $Q_{\text{OBS}}(t)$. The empirical distribution of the simulated values will form the basis for probability calculations. An uncertainty interval will be constructed to cover 95% of the simulated values. The probability of exceeding a certain threshold value will be estimated as the proportion of simulated values that exceeds this threshold. The accuracy will depend on how many values are simulated.

Consider any day k for which all relevant present and past weather observations exist, and on which forecasts have been made for the next 6 days. The algorithm for simulating the runoff values for these six following days, $Q_{\text{OBS}}^*(k+1), \ldots, Q_{\text{OBS}}^*(k+6)$, consists of the following three steps:

1. Simulate the weather for day $k+1, \ldots, k+6$ given the observed weather up to day k and the forecasts made on that day. That is, generate synthetic values for temperature and precipitation: $T_{k+1}^*, \ldots, T_{k+6}^*, R_{k+1}^*, \ldots, R_{k+6}^*$. The values are drawn according to Follestad and Høst (1998). A detailed description of this is given in the appendix.

2. Treat the simulated values of temperature and precipitation as real and calculate future values of Q_{SIM} using the HBV-model. These simulated values are denoted as $Q_{\text{SIM}}^*(k+1), \ldots, Q_{\text{SIM}}^*(k+6)$. In the appendix this is described in more detail.

3. Treat the simulated values of temperature, precipitation and Q_{SIM} as real data and draw HBV-model errors according to Langsrud *et al.* (1997). With these errors added to the simulated values of Q_{SIM}, we now have a set of simulated values for Q_{OBS}: $Q_{\text{OBS}}^*(k+1), \ldots, Q_{\text{OBS}}^*(k+6)$.

This algorithm produces one value for Q_{OBS} for each of the six days after day k. It can be repeated to produce a large number of values for each of these days. Thus we obtain the distribution of actual runoffs for each of the six days after day k, given the conditions and forecasts on that day. This can be repeated for any other day for which the relevant data is available.

As an example consider a single day, say 20.07.95. The temperature and rainfall on this day were 13.73C and 6.84mm respectively. The forecasts for the next 6 days for rainfall were 17.78, 4.43, 9.55, 5.26, 6.85 and 1.45. Those for temperature were 14.23, 11.53, 10.58, 14.66, 16.54 and 17.34. One set of simulations of rainfall based on the conditions on the day and the forecasts was 16.84, 14.24, 2.85, 0.53, 0.28 and 1.33. The corresponding simulations of temperature were 14.72, 11.59, 9.43, 9.59, 8.77 and 12.39. Running the HBV model on these values gives values of Q_{SIM} of 5.74, 8.5, 3.87, 1.03, 0.96 and 0.91. After adding model errors the simulated Q_{OBS} values are 6.80, 7.64, 2.86, 0.64, 0.56 and 0.29. This procedure was repeated 100 times to obtain the distribution of Q_{OBS} for each of the 6 days after 20.07.95, given the conditions and forecasts on that day.

4 Results

With the current implementation of the algorithm, it takes about ten minutes to generate 1000 sets of 6 simulations. Therefore, the simulations have only been performed for a period of 100 days during summer 1995. This means there are 95 days for which we have an observed runoff, 6 HBV forecasts (1 day ahead up to 6 days ahead) and 6 simulated distributions. These simulated distributions form the basis for the probability calculations below.

Together with the HBV-forecasted runoffs we can now calculate uncertainty intervals. Figure 4.1 (page 21-23) (Røykenes) and Figure 4.2 (page 24-26) (Knappom) show such 95% intervals together with the HBV forecasts and the observed runoffs. We can see that the observed runoffs are rarely outside the intervals. As expected, the intervals get wider as the forecast time increases up to six days. Note that the increased uncertainty is not only caused by errors in the weather forecasts but also by increasing HBV model errors. This is because the model error for a j day ahead forecast depends on the errors in all of the shorter range forecasts. This is most important for Knappom since the values of α_t (14) are relatively close to one.

One may also use the empirical distribution of 1000 simulated Q_{OBS}-values to calculate the probability of exceeding certain threshold values. Based on our historical data for Røykenes we expect Q_{OBS} to exceed the threshold 51 m^3/s once a year, and 73 m^3/s once every ten years. The corresponding values for Knappom are 178 m^3/s and 264 m^3/s. One may therefore be interested in the probabilities that these values will be exceeded given a particular forecast. Figure 4.3 (page 27-29) (Røykenes) and Figure 4.4 (page 30-32) (Knappom) present the result of such calculations. We see that the probabilities of exceedence for the period under study are quite low, usually below 5%.

One way to check the performance of the probability calculations is to verify that the probabilities for the real observations, $P(Q_{\text{OBS}} >$ "real Q_{OBS}") are distributed Uniform(0,1). For Røykenes these probabilities are plotted in Figure 4.5 (page 33-35). A uniform distribution looks plausible, although there is some autocorrelation among the probabilities. However, autocorrelation is expected for step 2 to step 6 (most for step 6). The corresponding probabilities for Knappom are plotted in Figure 4.6 (page 36-38). In this case there seems to be some deviation from the uniform distribution. The number of extreme observations (outside the 95% interval) seems OK, but the distribution elsewhere seems skewed. To find out whether this is a general tendency one should check the simulation algorithm on other periods. However, the skewness could be explained by the fact that the model error is a very important part of the forecast uncertainty for Knappom. It is possible to improve the model error model by using a more complex model including heavy tailed distributions (Langsrud *et al.*, 1997).

5 Conclusions and recommendations

In this report we have combined a model for the HBV-model error and models for the uncertainty of the weather forecasts to quantify the uncertainty of the HBV runoff forecasts. An algorithm for simulating runoff values has been developed and empirical distributions of simulated values form the basis for probability calculations.

The methodology has been applied to the two catchments Røykenes (Western Norway) and Knappom (Eastern Norway). For Røykenes, we have seen that the uncertainty of the weather forecast (precipitation and temperature) is most important for the uncertainty of the runoff forecasts. On the other hand, for Knappom, the HBV model error part is in fact most important. Note that we have built in a correction for the HBV model error. The latest observed model error is used to predict future model errors. Such a correction could also be used to improve the HBV forecasts directly.

Overall, the results of the probability calculations seem reasonable. The results for Knappom indicate, however, that improvements are possible. Since the model error is so important, an more accurate model error model may be useful.

Finally, we recommend that one start using the presented methodology routinely to gain experience. Note that, in addition to what we have described, it is possible to use the method to find other types of information. One may, for example, calculate the probability of exceeding a threshold value within the next six days (not only the probability for a single day).

Appendix: The simulation algorithm in detail

Simulation of temperature and precipitation

Simulation of temperature

The temperature model described by Follestad and Høst (1998) is for each forecasting step:

$$\begin{aligned} T_t &= \alpha_0^{(j)} I_{[S_t^{(j)}>=0]} + \alpha_1^{(j)} I_{[S_t^{(j)}<0]} + \alpha_2^{(j)} S_t^{(j)} I_{[S_t^{(j)}>=0]} + \alpha_3^{(j)} S_t^{(j)} I_{[S_t^{(j)}<0]} \\ &+ \alpha_4^{(j)} (T_{t-1} - S_{t-1}^{(j)}) I_{[S_t^{(j)}>=0]} + \alpha_5^{(j)} (T_{t-1} - S_{t-1}^{(j)}) I_{[S_t^{(j)}<0]} + \epsilon_t^{(j)}. \end{aligned} \tag{6}$$

Here T_t is the actual temperature on day t, $S_t^{(j)}$ is the j step forecast for this temperature and the superscript (j) on the alpha parameters indicates that they are different for different steps $(j = 1, \ldots, 6)$. $I_{[S_t^{(j)}<0]}$ and $I_{[S_t^{(j)}>=0]}$ are indicator variables taking the values 1 if $S_t^{(j)} < 0$ ($S_t^{(j)} >= 0$) or 0 if $S_t^{(j)} >= 0$ ($S_t^{(j)} < 0$), and the $\epsilon_t^{(j)}$'s are i.i.d Gaussian variables with zero mean and standard deviation $\sigma_t^{(j)}$ where

$$\sigma_t^{(j)} = \begin{cases} \sigma_{pos}^{(j)} & \text{if } S_t^{(j)} >= 0 \\ \sigma_{neg}^{(j)} & \text{if } S_t^{(j)} < 0 \end{cases} \tag{7}$$

In total we have 48 temperature parameters. For each of the six steps $(j = 1, \ldots, 6)$ we have the eight parameters: $\alpha_0^{(j)}, \ldots, \alpha_5^{(j)}, \sigma_{pos}^{(j)}, \sigma_{neg}^{(j)}$. All parameters are estimated based on historical data.

The simulation of temperature values is done as follows:

- Draw $\epsilon_{t+1}^{(1)}$ from a normal distribution with zero mean and standard deviation $\sigma_{t+1}^{(1)}$. The simulated temperature value, T_{t+1}^*, is calculated by setting $j = 1$ in (6)).
- Draw $\epsilon_{t+2}^{(2)}$ from a normal distribution with zero mean and standard deviation $\sigma_{t+2}^{(2)}$. T_{t+2}^*, is calculated from the step-two model ($j = 2$ in (6)) replacing T_{t+1} by T_{t+1}^*.
- The values T_{t+3}^*, T_{t+4}^*, T_{t+5}^* and T_{t+6}^* are generated in the same way as T_{t+2}^*.

Simulation of precipitation

Let R_t be the volume of rain on day t, and $P_t^{(j)}$ be its j step forecast. Let $p_t^{(j)}$ be the probability that $R_t > 0$ given this forecast.

Follestad and Høst (1998) model $p_t^{(j)}$ by a generalized linear model, using a binomial distribution with *logit* link. The model is

$$\log\left(\frac{p_t^{(j)}}{1 - p_t^{(j)}}\right) = \beta_0^{(j)} + \beta_1^{(j)} I_{[P_t^{(j)}>0]} + \beta_2^{(j)} \sqrt{P_t^{(j)}} + \beta_3^{(j)} I_{[P_{t-1}^{(j)}>0 \cap R_{t-1}=0]} \tag{8}$$

Given that R_t is greater than zero, its square root is assumed to be Gamma distributed, with an expected value at time t depending only on the forecasted precipitation at time t. Thus,

$$\sqrt{R_t}\ |(R_t > 0) \sim Gamma(\mu_t^{(j)}, \nu^{(j)}) \tag{9}$$

Here, $\nu^{(j)}$ is the inverse of the dispersion parameter of the Gamma distribution. The mean value, $\mu_t^{(j)}$, is modeled as

$$\mu_t^{(j)} = \gamma_0^{(j)} + \gamma_1^{(j)}\sqrt{P_t^{(j)}} \tag{10}$$

In total we have 42 precipitation parameters. For each of the six steps ($j = 1, \ldots, 6$) we have the seven parameters: $\beta_0^{(j)}, \ldots, \beta_3^{(j)}, \gamma_0^{(j)}, \gamma_1^{(j)}, \nu^{(j)}$ All parameters are estimated based on historical data. The simulation of precipitation values is done as follows:

- Calculate the probability $p_{t+1}^{(1)}$ of precipitation at day $t+1$ ($j = 1$ in (8)).
- According to this probability draw whether the precipitation will be zero or positive.
  - If the result is zero set $R_{t+1}^* = 0$.
  - If the result is positive draw a random number from the appropriate Gamma distribution ($j = 1$ in (9) and (10)). The square of the result is the simulated precipitation R_{t+1}^*.
- Precipitation for day $t+2$, R_{t+2}^*, is generated similarly ($j = 2$ in (8), (9) and (10)) replacing R_{t+1} by R_{t+1}^* in the expression for $p_{t+2}^{(1)}$.
- The values R_{t+3}^*, R_{t+4}^*, R_{t+5}^* and R_{t+6}^* are generated in a similar manner.

Finding simulated Q_{SIM}-values

$Q_{\text{SIM}}^*(t+1), \ldots, Q_{\text{SIM}}^*(t+6)$ are calculated by

$$\begin{aligned} Q_{\text{SIM}}^*(t+1) &= \text{HBV}(T_{t+1}^*, T_t, T_{t-1}, \ldots, R_{t+1}^*, R_t, R_{t-1}, \ldots) \\ &\vdots \\ Q_{\text{SIM}}^*(t+6) &= \text{HBV}(T_{t+6}^*, \ldots, T_{t+1}^*, T_t, T_{t-1}, \ldots R_{t+6}^*, \ldots, R_{t+1}^*, R_t, R_{t-1}, \ldots) \end{aligned} \tag{11}$$

Final simulation of Q_{OBS}-values

Langsrud *et al.* (1997) used a logarithmic transformation to model the error of the HBV-model. Following their notation:

$$d_t = \log(Q_{\text{OBS}}(t)) - \log(Q_{\text{SIM}}(t)) \tag{12}$$

The following dynamic model is used:

$$d_t = \alpha_t d_{t-1} + \sigma_t u_t \tag{13}$$

Tabell 3.1 *Kategoriene som brukes til parameter-estimeringen.*
Table 3.1 *The chosen categories for the estimation.*

$i(t)$	Røykenes	Knappom
1	$T_t \leq 0$	$T_t \leq 0$
2	$T_t > 0$ AND SWE$(t) = 0$	$T_t > 0$ AND SWE$(t) = 0$ AND $R_t = 0$
3	$T_t > 0$ AND SWE$(t) > 0$	$T_t > 0$ AND SWE$(t) > 0$ AND $R_t = 0$
4		$T_t > 0$ AND SWE$(t) = 0$ AND $R_t > 0$
5		$T_t > 0$ AND SWE$(t) > 0$ AND $R_t > 0$

where u_t are i.i.d. with standard normal distribution $N(0,1)$, and

$$\begin{aligned} \alpha_t &= I_{[Q_{\text{SIM}}(t) \geq Q_{\text{THR}}]} \left(a_{i(t)} + b \log(Q_{\text{SIM}}(t))\right) + \\ &\quad I_{[Q_{\text{SIM}}(t) < Q_{\text{THR}}]} \left(\tilde{a}_{i(t)} + \tilde{b} \log(Q_{\text{SIM}}(t))\right) \quad (14) \\ \log(\sigma_t) &= I_{[Q_{\text{SIM}}(t) \geq Q_{\text{THR}}]} \left(A_{i(t)} + B \log(Q_{\text{SIM}}(t))\right) + \\ &\quad I_{[Q_{\text{SIM}}(t) < Q_{\text{THR}}]} \left(\tilde{A}_{i(t)} + \tilde{B} \log(Q_{\text{SIM}}(t))\right) \quad (15) \end{aligned}$$

The subscript $i(t)$ indicates that the parameters vary with the meteorological regime at time t. For the catchments in this study, the regimes fall into the categories in Table 3.1; three for Røykenes and five for Knappom. Note that $I_{[Q_{\text{SIM}}(t) \geq Q_{\text{THR}}]}$ and $I_{[Q_{\text{SIM}}(t) < Q_{\text{THR}}]}$ are indicator variables taking the values 1 or 0. This means that there are separate parameters for small and large values of Q_{SIM} (See Langsrud *et al.* (1997) for an explanation). In our cases the threshold value, Q_{THR}, is chosen so that this value is exceeded in 25% of the days.

For the specific catchment, Røykenes, we have a total of 17 parameters (25 for Knappom): $Q_{\text{THR}}, a_1, a_2, a_3, b, A_1, A_2, A_3, B, \tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{b}, \tilde{A}_1, \tilde{A}_2, \tilde{A}_3, \tilde{B}$. All parameters are estimated based on historical data.

The simulation of the Q_{OBS}-values is as follows:

- Find $i^*(t+1)$ by replacing $T(t+1)$, $R(t+1)$ and SWE$(t+1)$ by $T^*(t+1)$, $R^*(t+1)$ and SWE$^*(t+1)$ in table 3.1. Here SWE$^*(t+1)$ is calculated from (3) as: SWE$^*(t+1) =$ HBV$_{SWE}(T^*_{t+1}, T_t, T_{t-1}, \ldots, R^*_{t+1}, R_t, R_{t-1}, \ldots)$
- Calculate α_{t+1} and σ_{t+1} from (14) and (15), replacing $Q_{\text{SIM}}(t+1)$ by $Q^*_{\text{SIM}}(t+1)$ and $i(t+1)$ by $i^*(t+1)$
- Draw $ut+1$ from N(0,1) and calculate d^*_{t+1} from (13)
- Calculate $Q^*_{\text{OBS}}(t+1) = \exp\left(\log(Q^*_{\text{SIM}}(t+1)) + d^*_{t+1}\right)$.
- $Q^*_{\text{OBS}}(t+1)$ can be generated similarly, replacing d_{t+1} by d^*_{t+1} in (13)).
- The values $Q^*_{\text{OBS}}(t+3)$, $Q^*_{\text{OBS}}(t+4)$, $Q^*_{\text{OBS}}(t+5)$ and $Q^*_{\text{OBS}}(t+6)$ are generated in a similar manner to $Q^*_{\text{OBS}}(t+2)$.

References

Follestad, T. and Høst, G. (1998), A statistical model for the uncertainty of meteorological forecasts with application to the Knappom and Røykenes catchments. HYDRA note, Available from the Norwegian Water Resources and Energy Administration (NVE), Oslo.

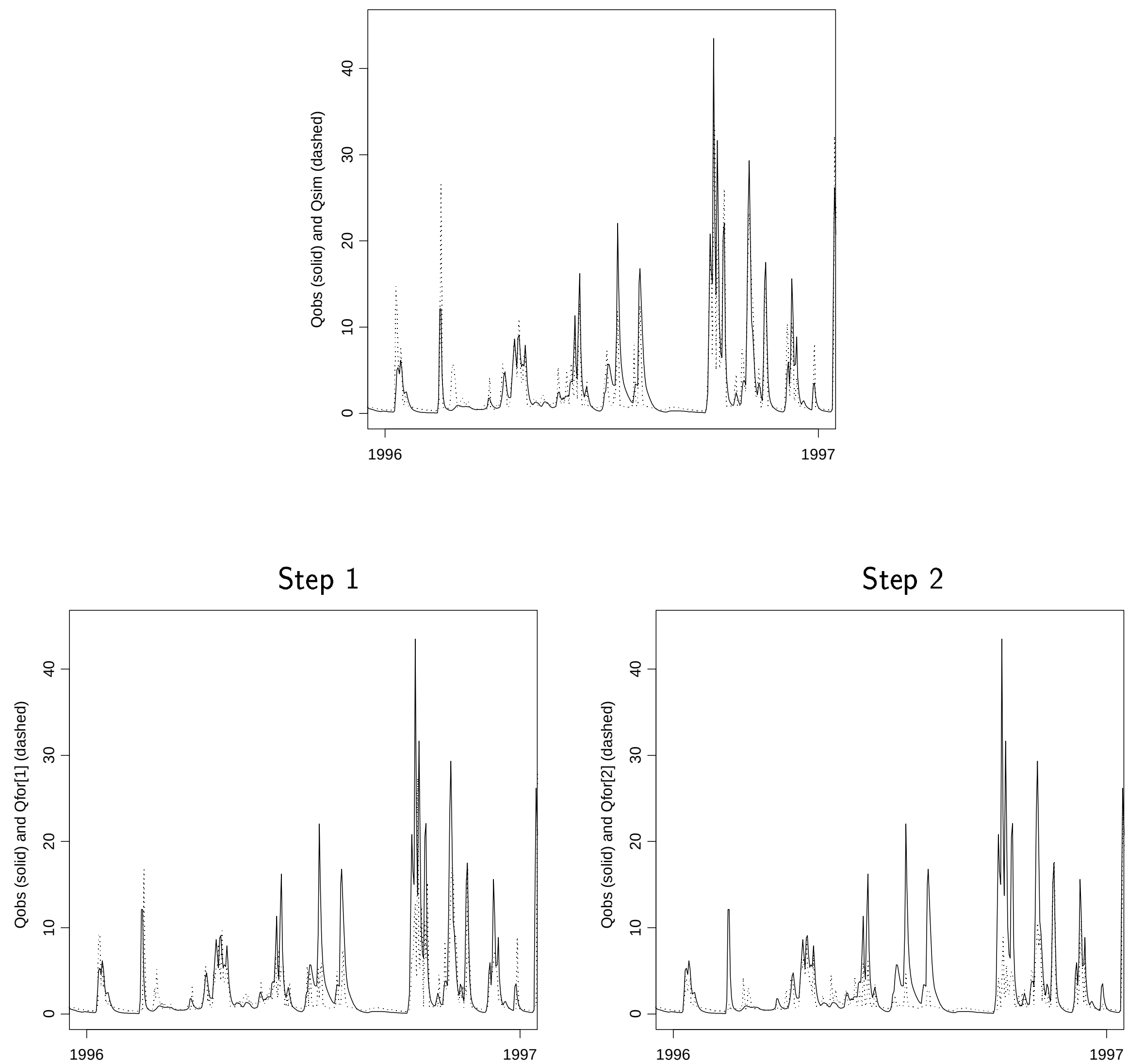
Langsrud, Û., Frigessi, A. and Høst, G. (1998), HBV-model error prediction, HYDRA note, Available from the Norwegian Water Resources and Energy Administration (NVE), Oslo.

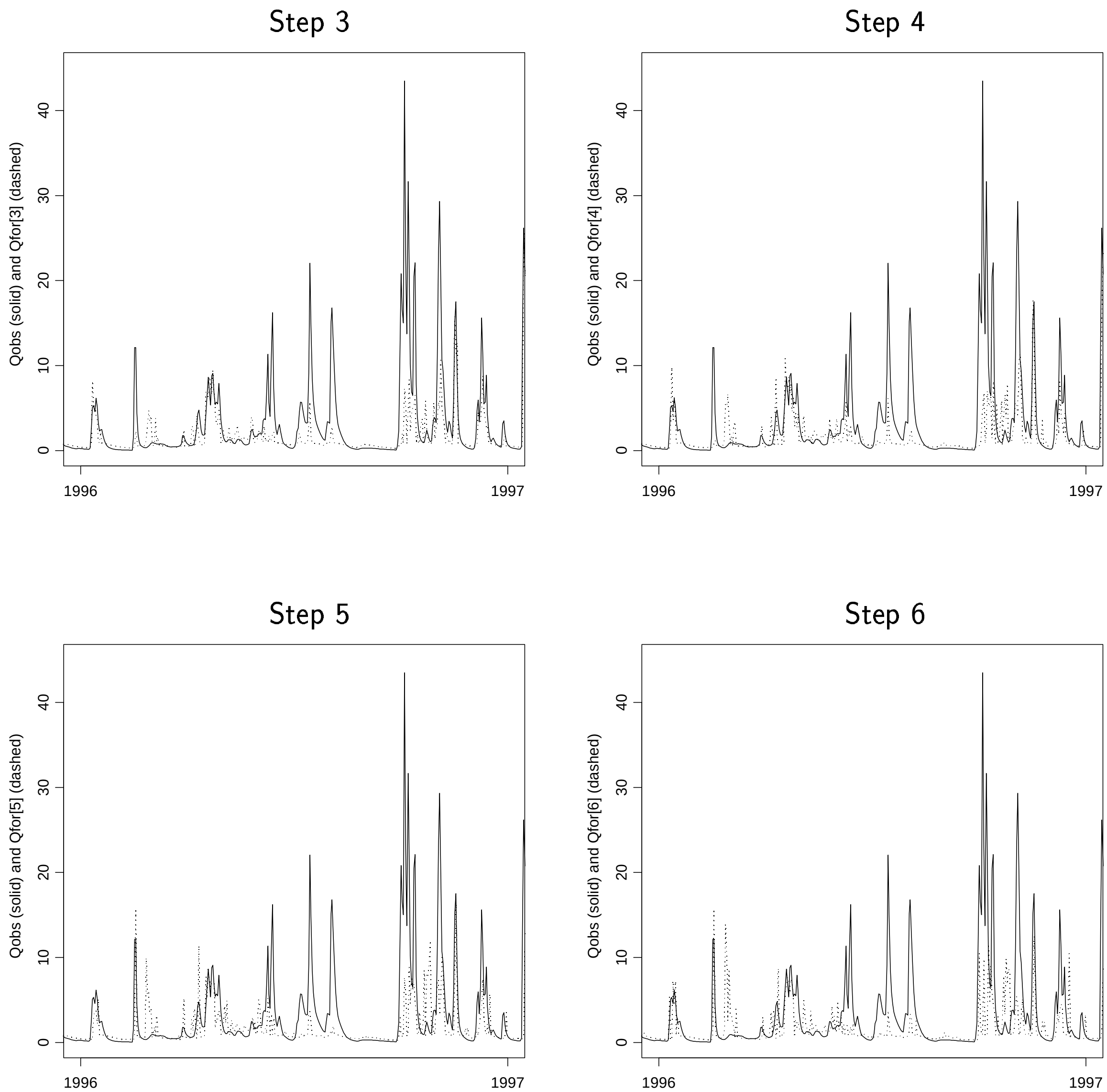
Lundquist, D. (1997), Flood forecasting in practice, Paper presented at European Geophysical Society, XXII General Assembly, Vienna, Austria, 21,-25. April, 1997.

NOU 1996:16 (1996), Flood Protection Measures (in Norwegian), Norges Offentlige Utredninger, Statens forvaltningstjeneste, Statens trykning, Oslo.

Figures

Røykenes

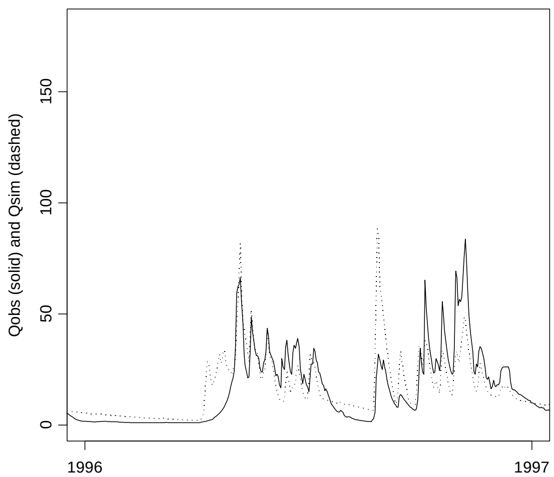




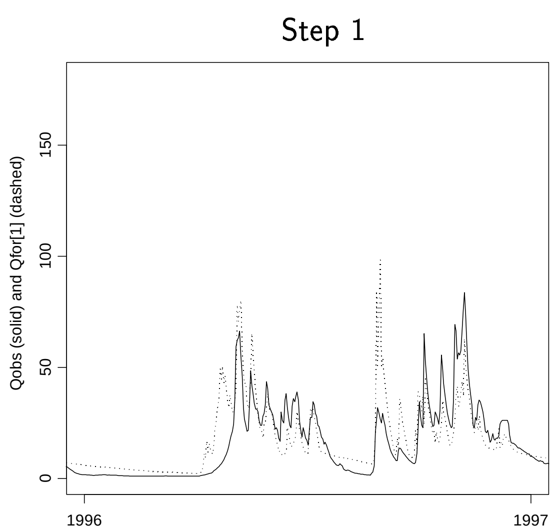
Figur 2.1 *Observert (Q_{OBS}) (heltrukket), simulert (Q_{SIM}) (øverst)*
(to sider) *og prognosert (Q_{FOR}) vannføring for Røykenes i 1996.*

Figure 2.1 *Observed (Q_{OBS}) (solid), HBV predicted (Q_{SIM}) (upper panel)*
(two pages) *and forecasted (Q_{FOR}) runoff for Røykenes in 1996.*

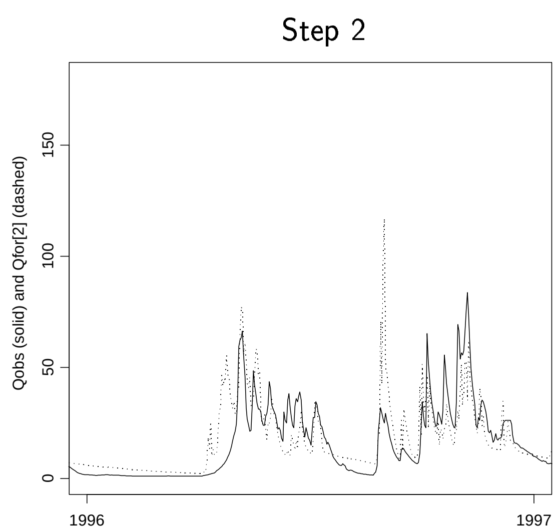
Knappom



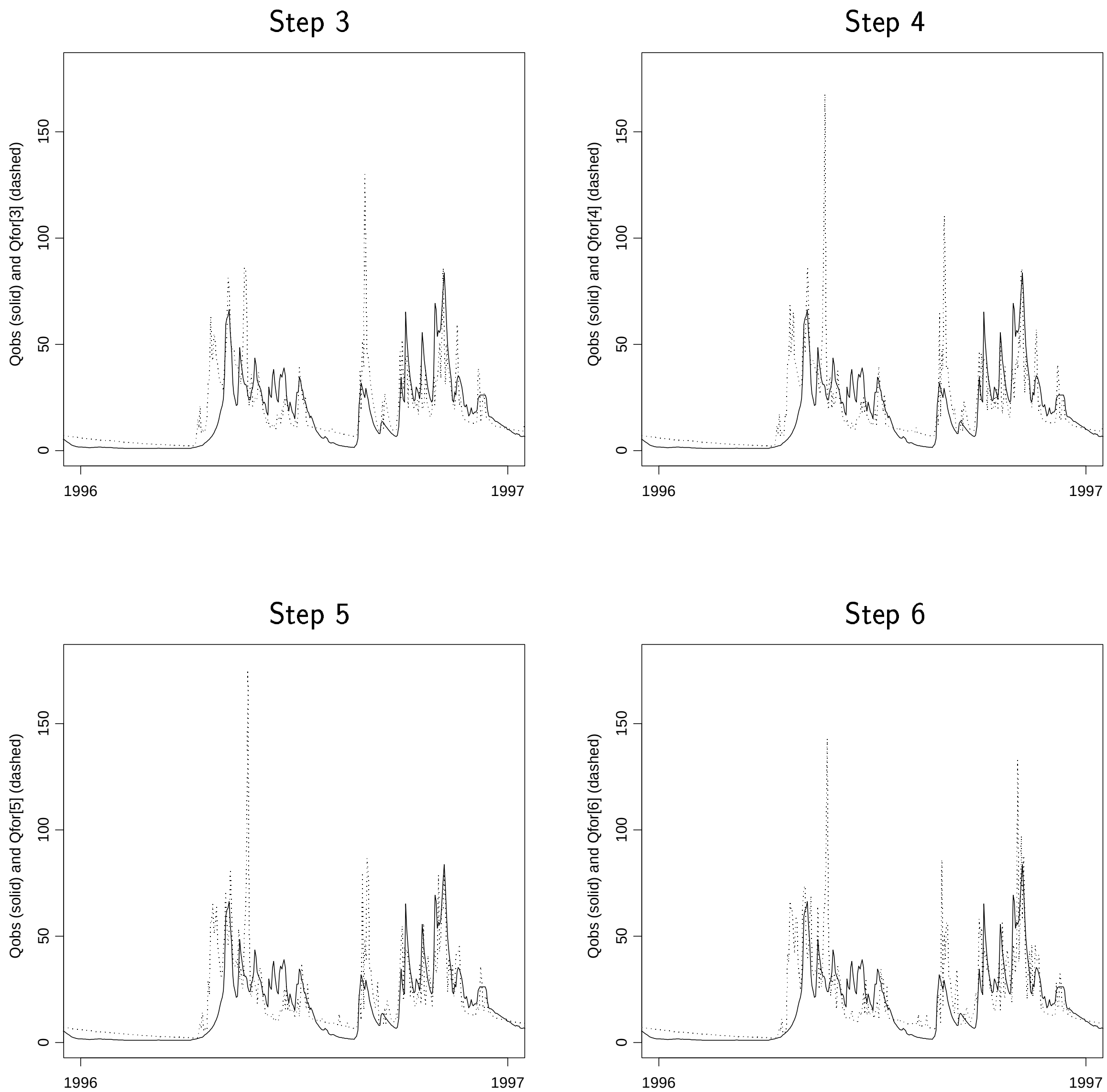
Qobs (solid) and Qsim (dashed)
0
50
100
150
1996
1997



Step 1
Qobs (solid) and Qfor[1] (dashed)
0
50
100
150
1996
1997



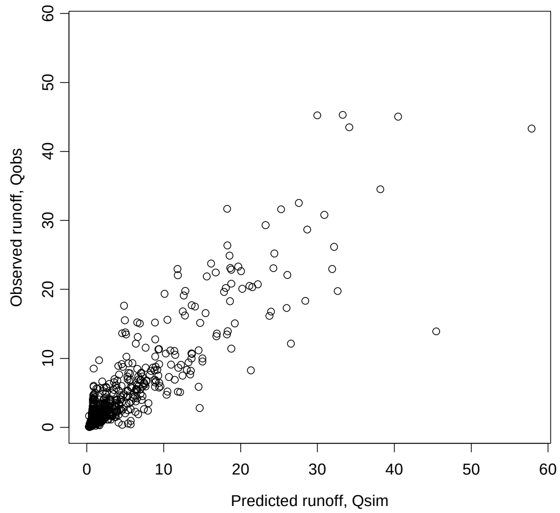
Step 2
Qobs (solid) and Qfor[2] (dashed)
0
50
100
150
1996
1997



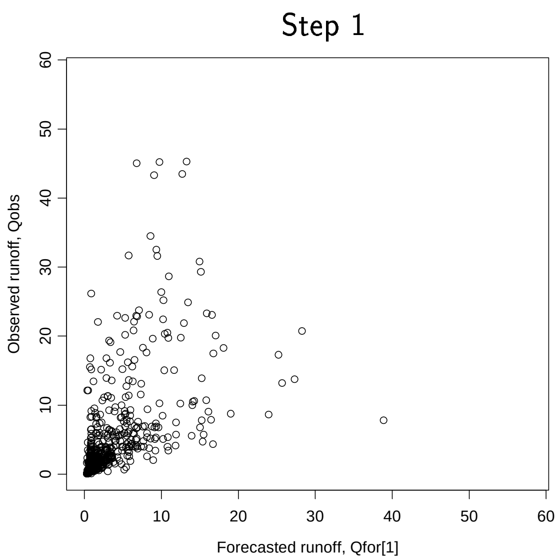
Figur 2.2 (to sider) — *Observert (Q_{OBS}) (heltrukket), simulert (Q_{SIM}) (øverst) og prognosert (Q_{FOR}) vannføring for Knappom i 1996.*

Figure 2.2 (two pages) — *Observed (Q_{OBS}) (solid), HBV predicted (Q_{SIM}) (upper panel) and forecasted (Q_{FOR}) runoff for Knappom in 1996.*

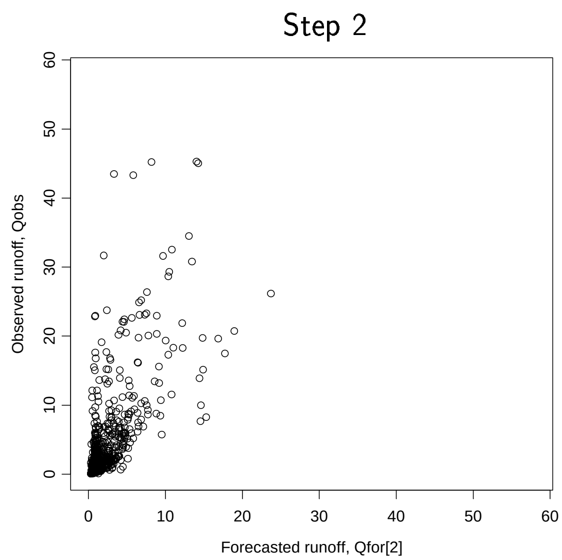
Røykenes



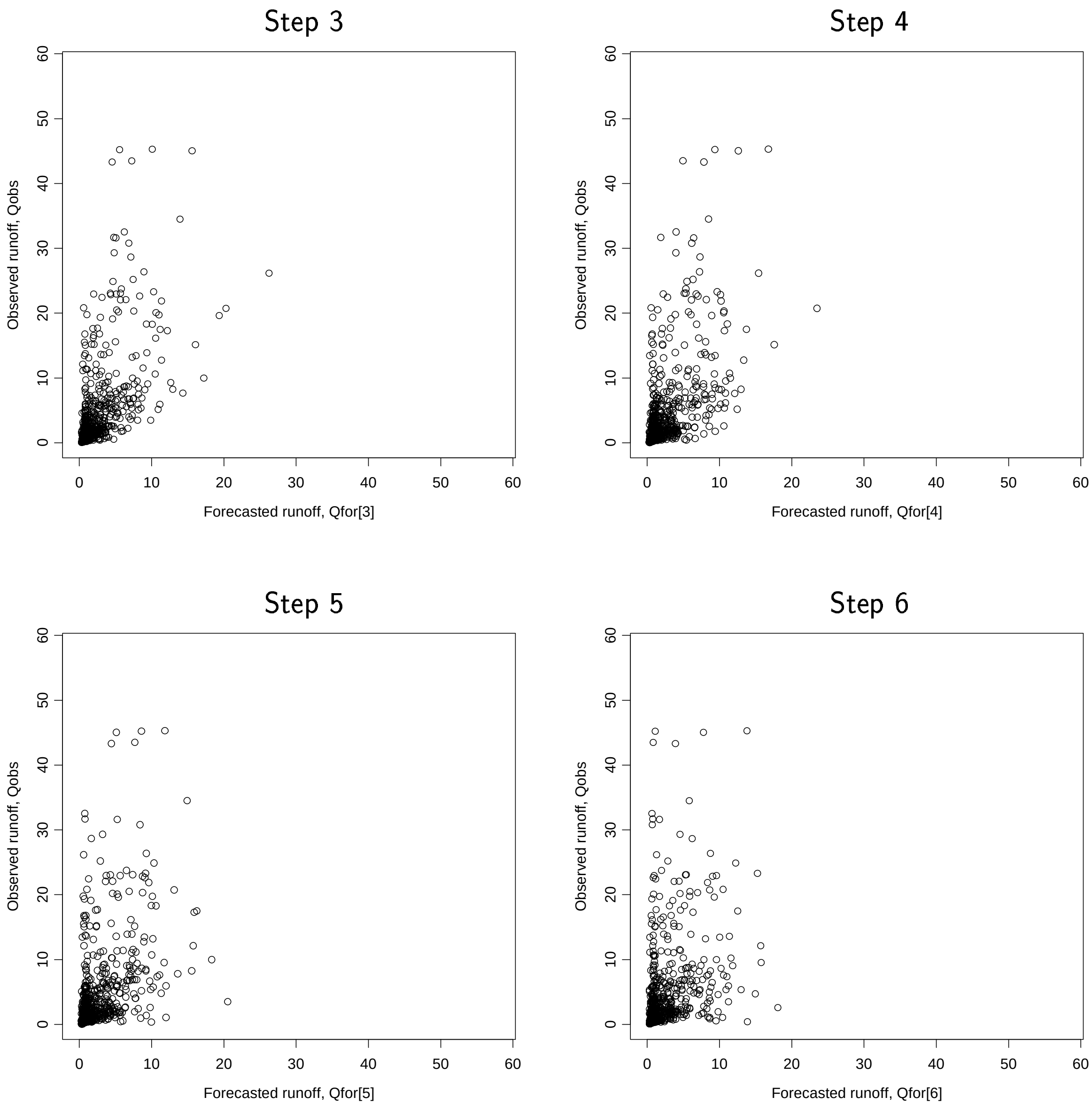
Observed runoff, Qobs
Predicted runoff, Qsim



Step 1
Observed runoff, Qobs
Forecasted runoff, Qfor[1]



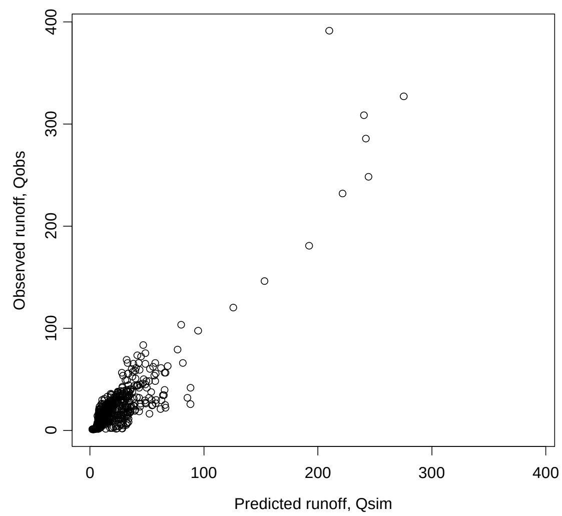
Step 2
Observed runoff, Qobs
Forecasted runoff, Qfor[2]



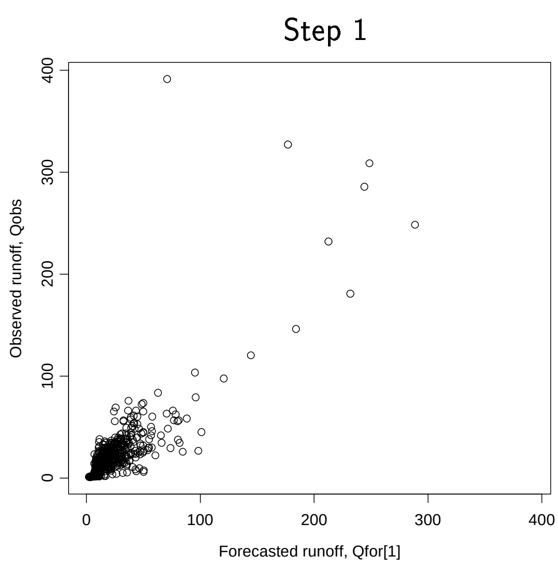
Figur 2.3 (to sider) *Observert (Q_{OBS}) mot simulert (Q_{SIM}) og prognosert (Q_{FOR}) vannføring for Røykenes.*

Figure 2.3 (two pages) *Observed (Q_{OBS}) vs. HBV predicted (Q_{SIM}) and forecasted (Q_{FOR}) runoff for Røykenes.*

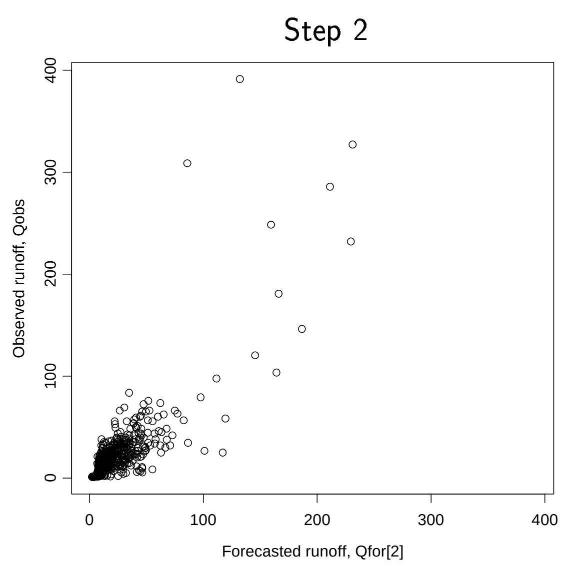
Knappom



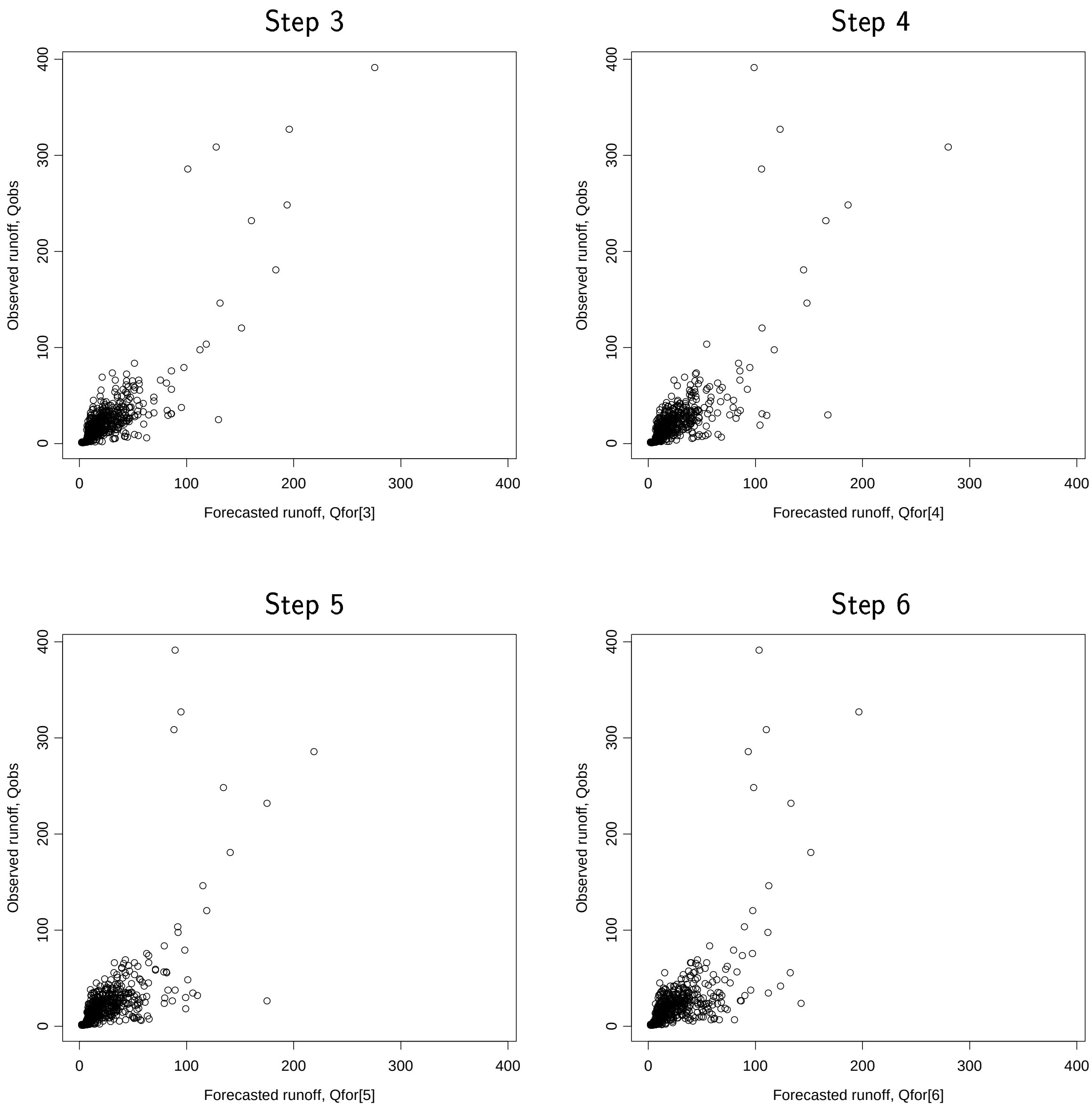
Observed runoff, Qobs
Predicted runoff, Qsim
0
100
200
300
400



Step 1
Observed runoff, Qobs
Forecasted runoff, Qfor[1]
0
100
200
300
400



Step 2
Observed runoff, Qobs
Forecasted runoff, Qfor[2]
0
100
200
300
400

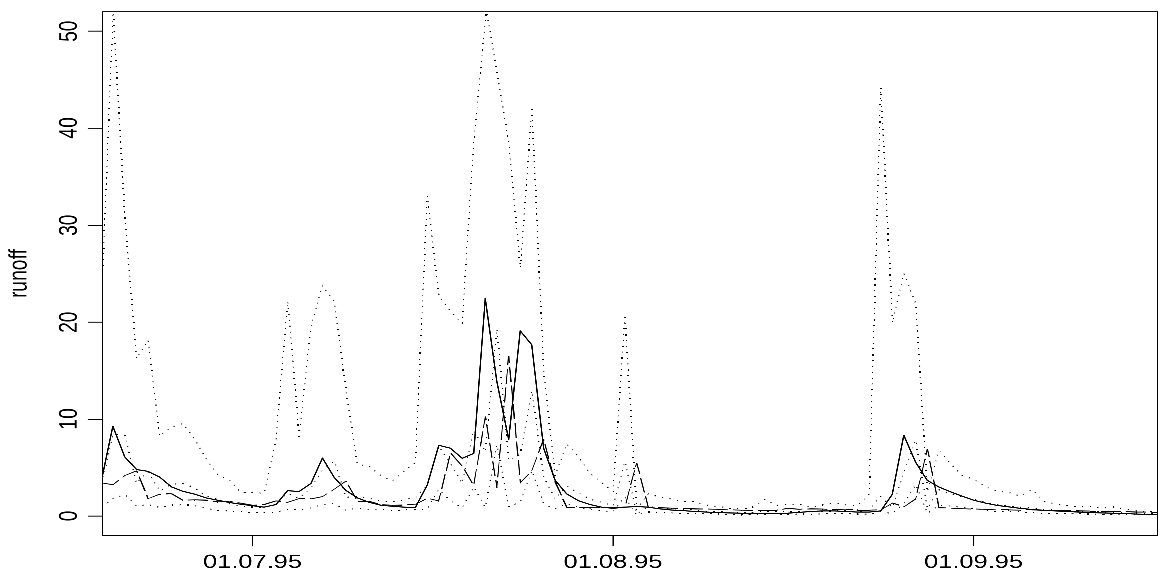


Figur 2.4 (to sider) — *Observert (Q_{OBS}) mot simulert (Q_{SIM}) og prognosert (Q_{FOR}) vannføring for Knappom.*

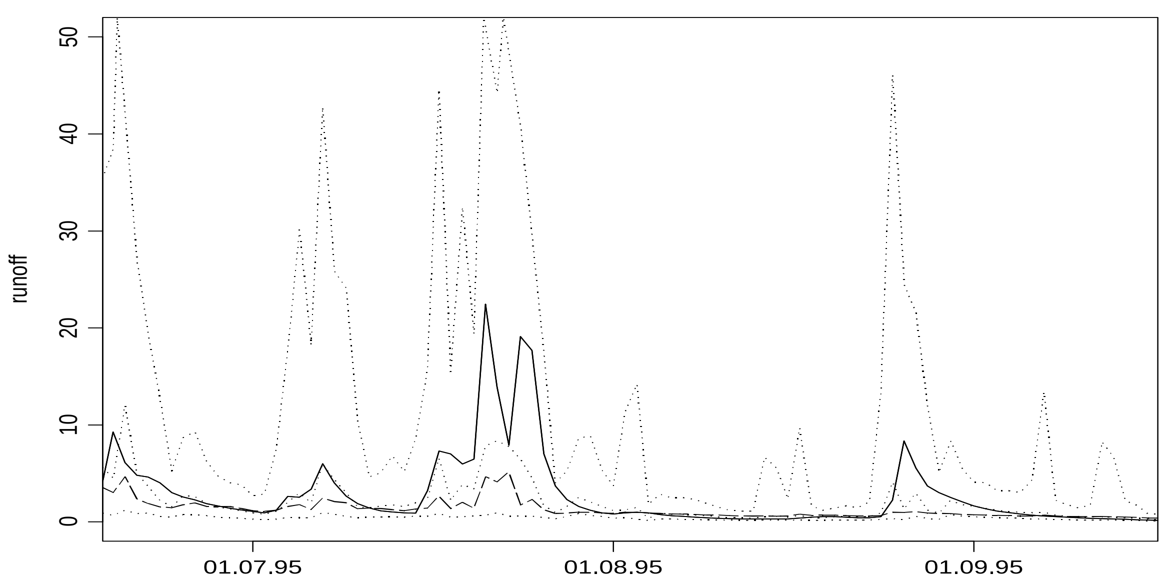
Figure 2.4 (two pages) — *Observed (Q_{OBS}) vs. HBV predicted (Q_{SIM}) and forecasted (Q_{FOR}) runoff for Knappom.*

Røykenes

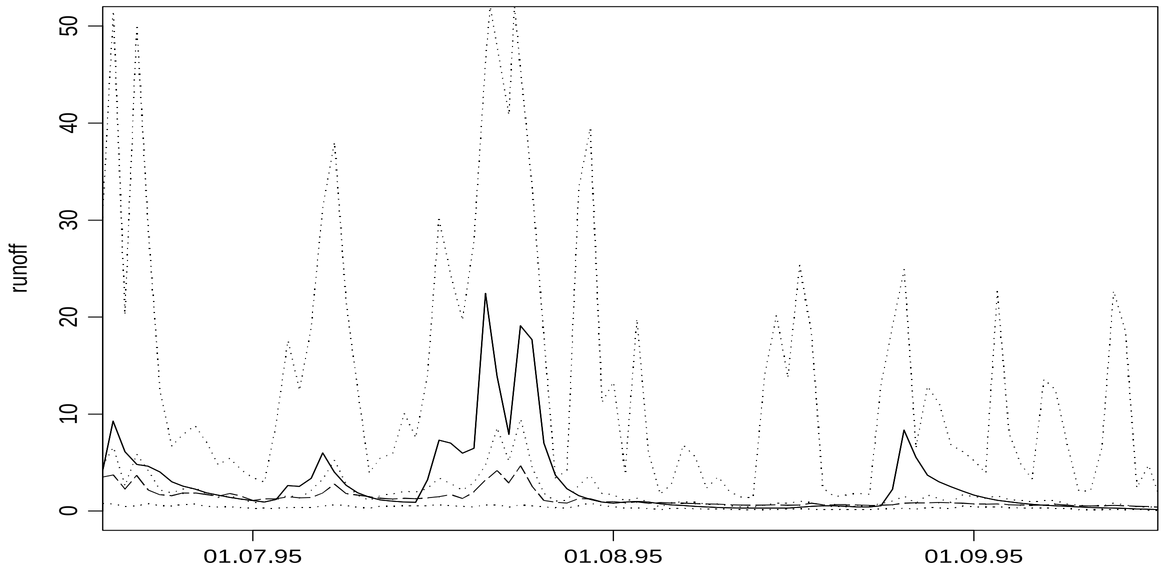
Step 1



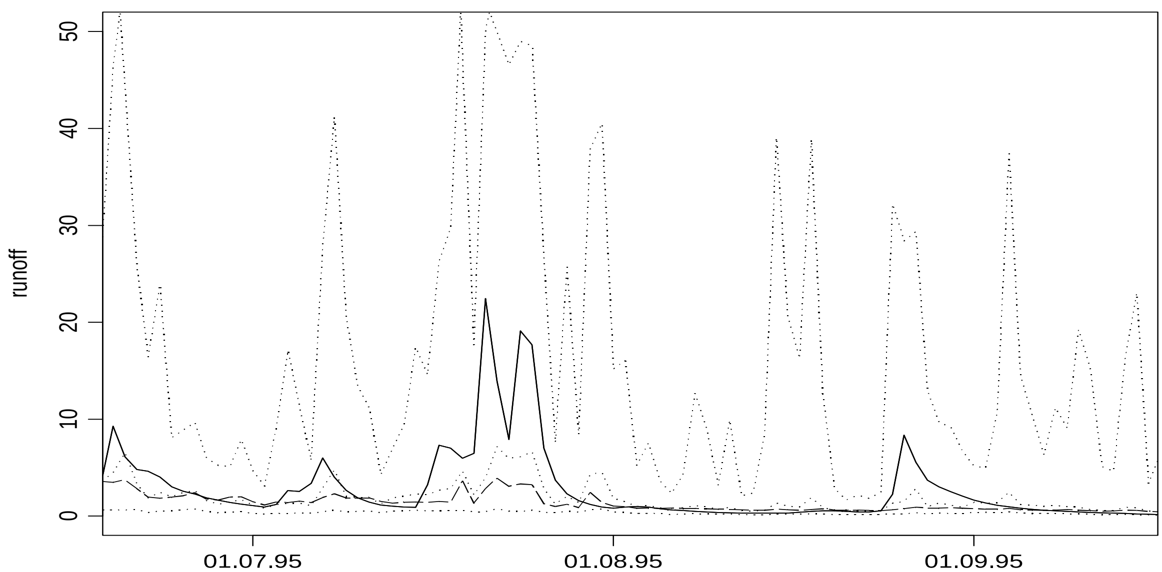
Step 2



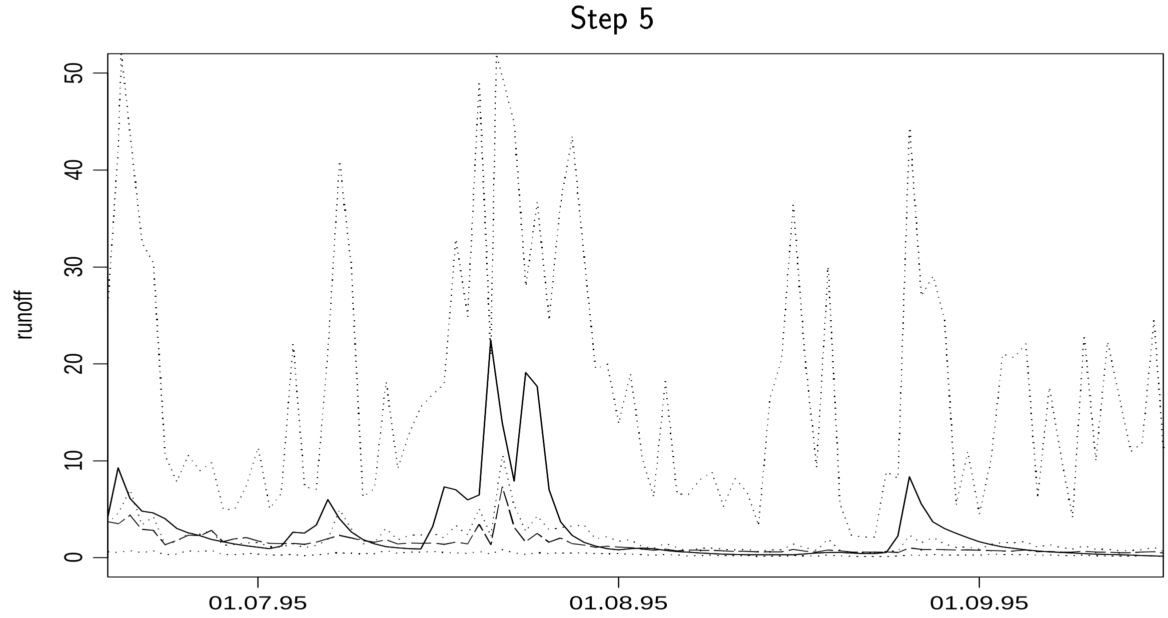
Step 3


runoff
0
10
20
30
40
50
01.07.95
01.08.95
01.09.95

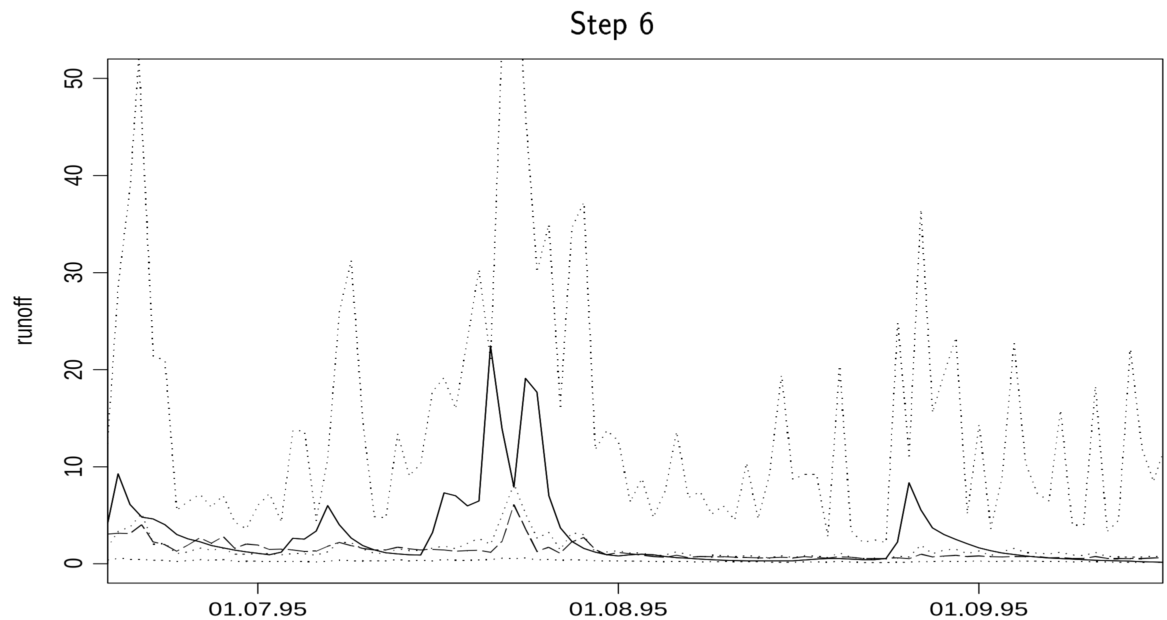
Step 4


runoff
0
10
20
30
40
50
01.07.95
01.08.95
01.09.95

Step 5



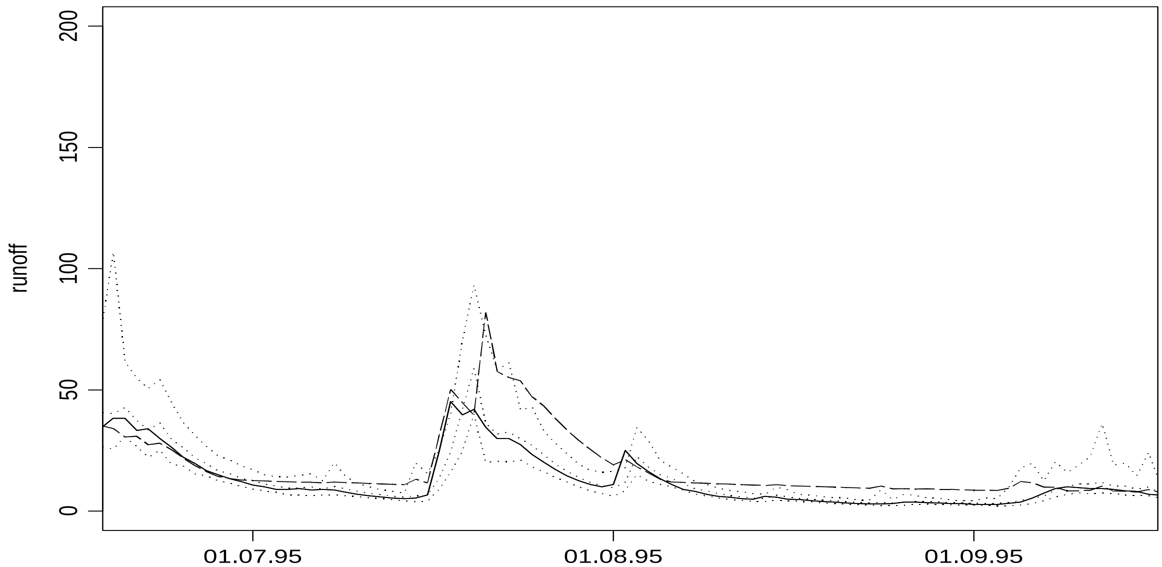
Step 6



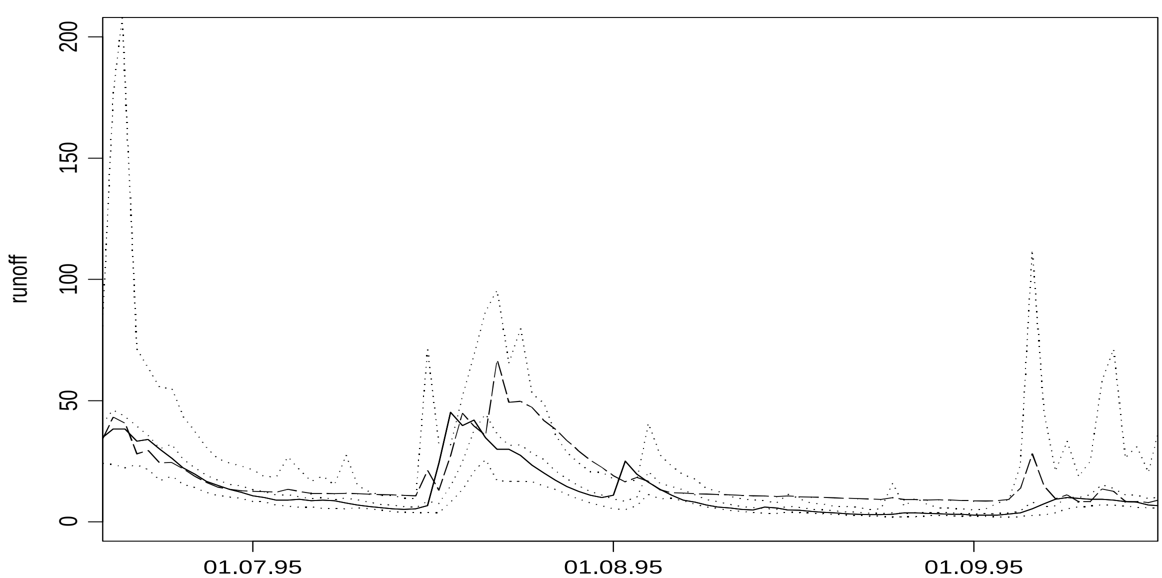
Figur 4.1 (tre sider)	*Nedre-, øvre- og median-verdier i 95% prognose-intervaller (stiplet) sammen med HBV-prognosene (lang-stiplet) og de observerte vannføringene (heltrukket) for Røykenes.*
Figure 4.1 (three pages)	*Lower, upper and median values in 95% forecasting intervals (dashed) together with the HBV forecasts (long-dashed) and the observed runoffs (solid) for Røykenes.*

Knappom

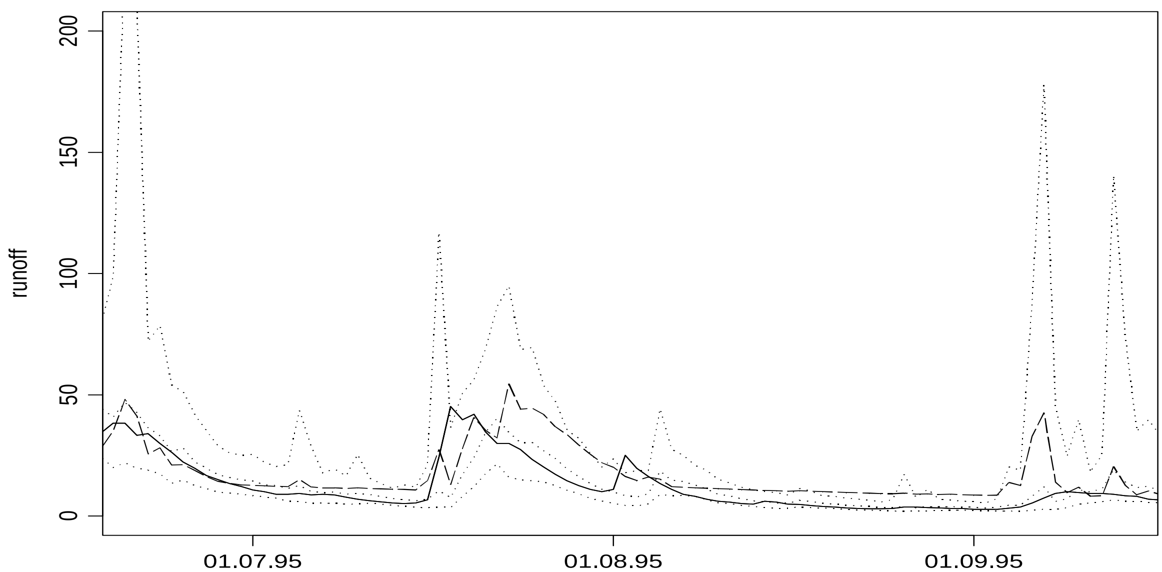
Step 1



Step 2

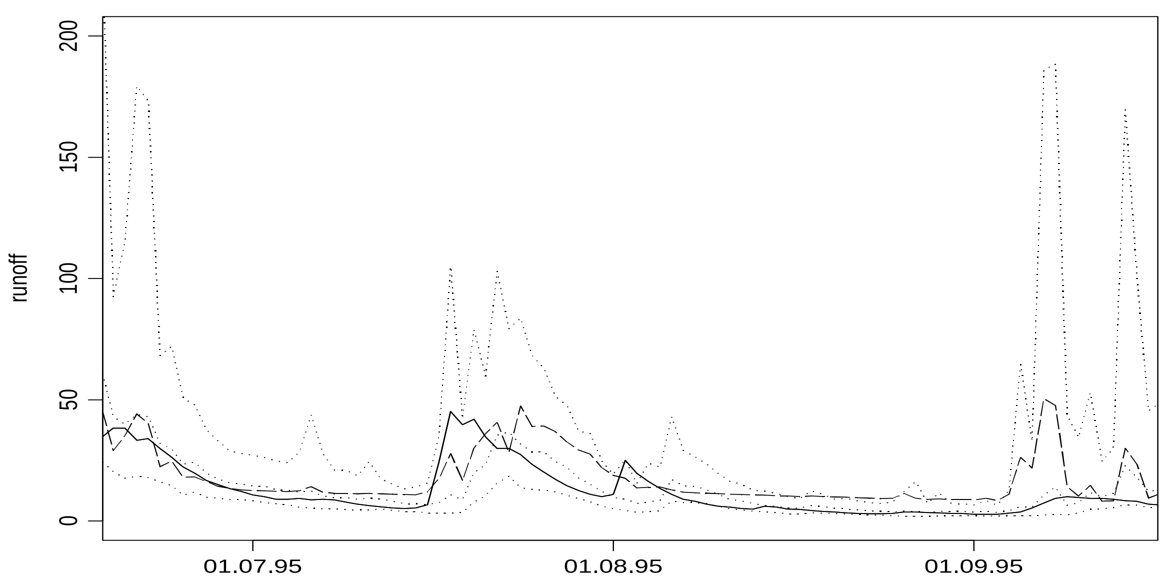


Step 3



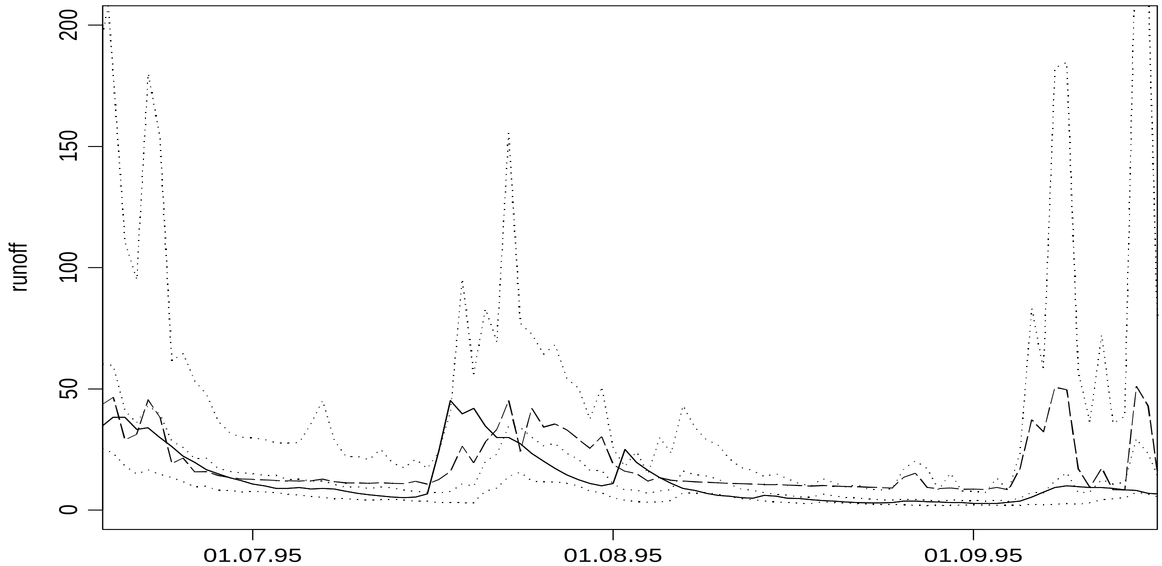
runoff
0
50
100
150
200
01.07.95
01.08.95
01.09.95

Step 4

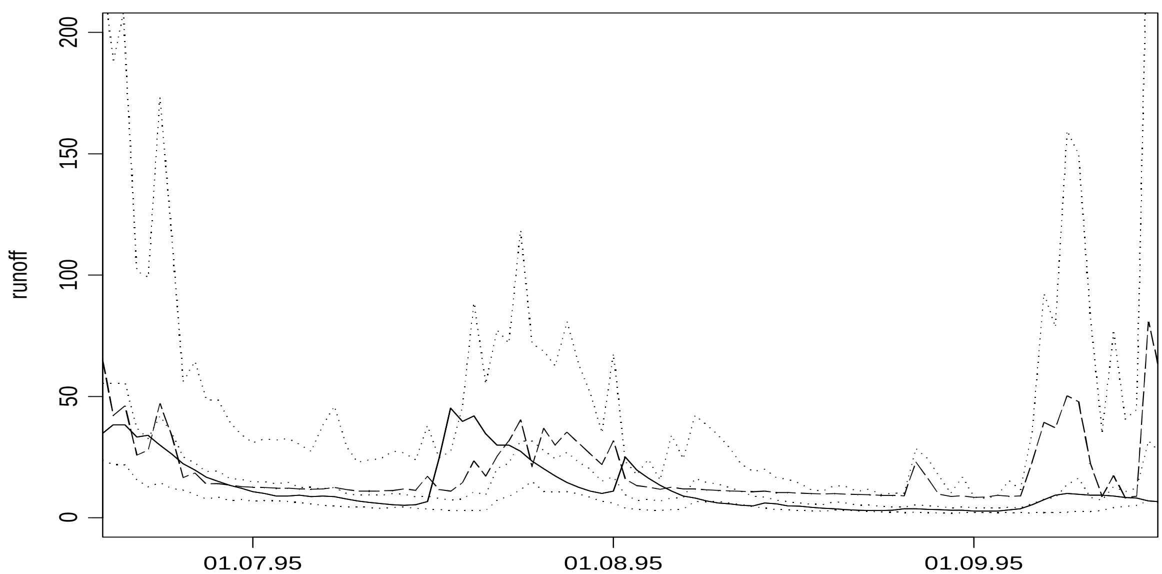


runoff
0
50
100
150
200
01.07.95
01.08.95
01.09.95

Step 5



Step 6

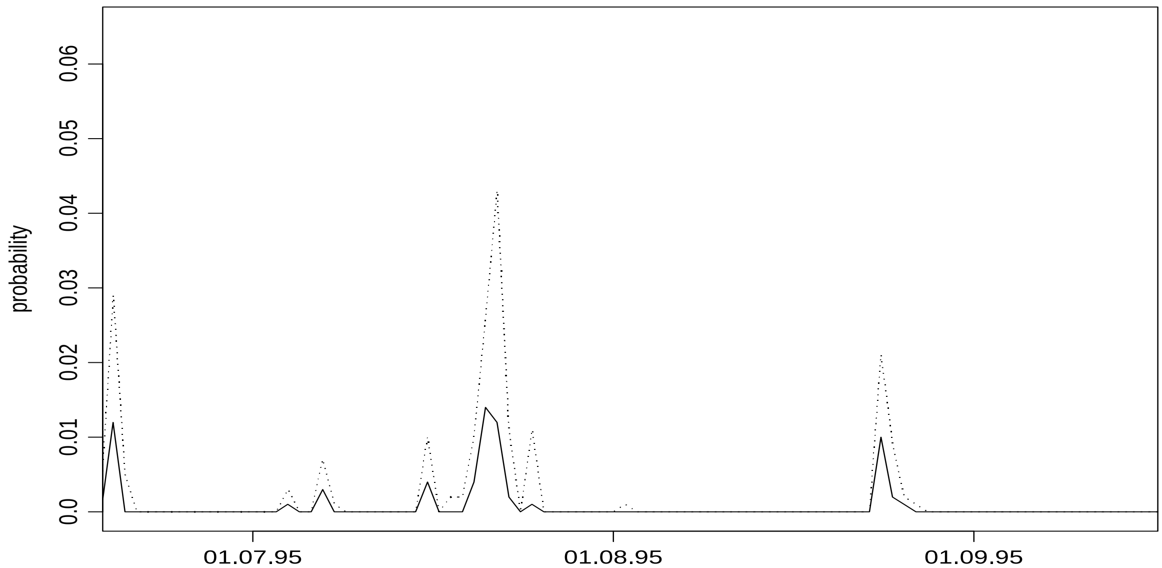


Figur 4.2 (tre sider)	*Nedre-, øvre- og median-verdier i 95% prognose-intervaller (stiplet) sammen med HBV-prognosene (lang-stiplet) og de observerte vannføringene (heltrukket) for Knappom.*

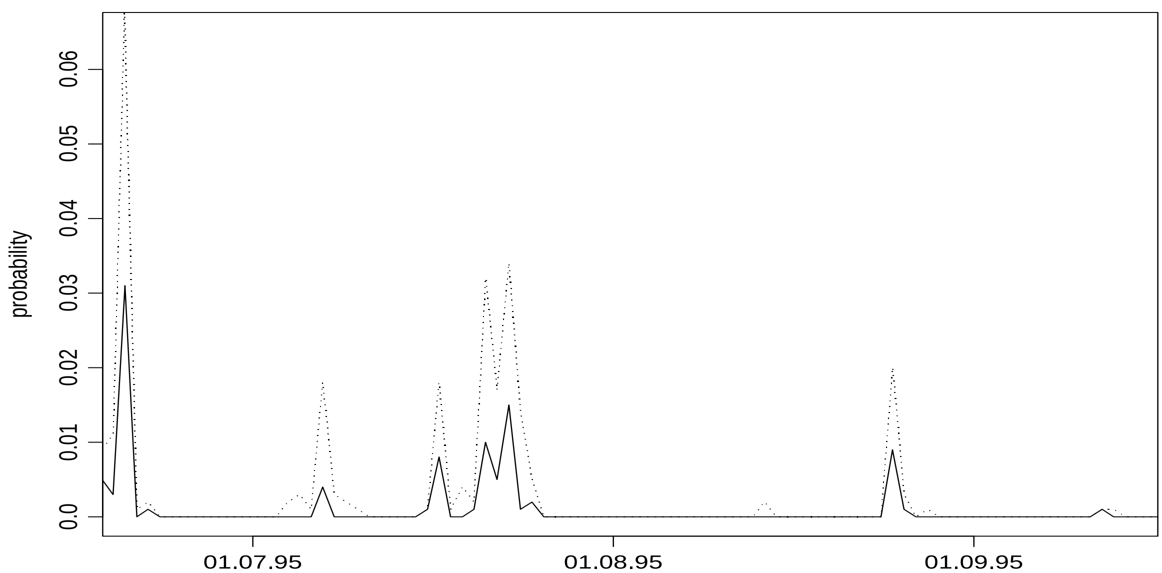
Figure 4.2 (three pages)	*Lower, upper and median values in 95% forecasting intervals (dashed) together with the HBV forecasts (long-dashed) and the observed runoffs (solid) for Knappom.*

Røykenes

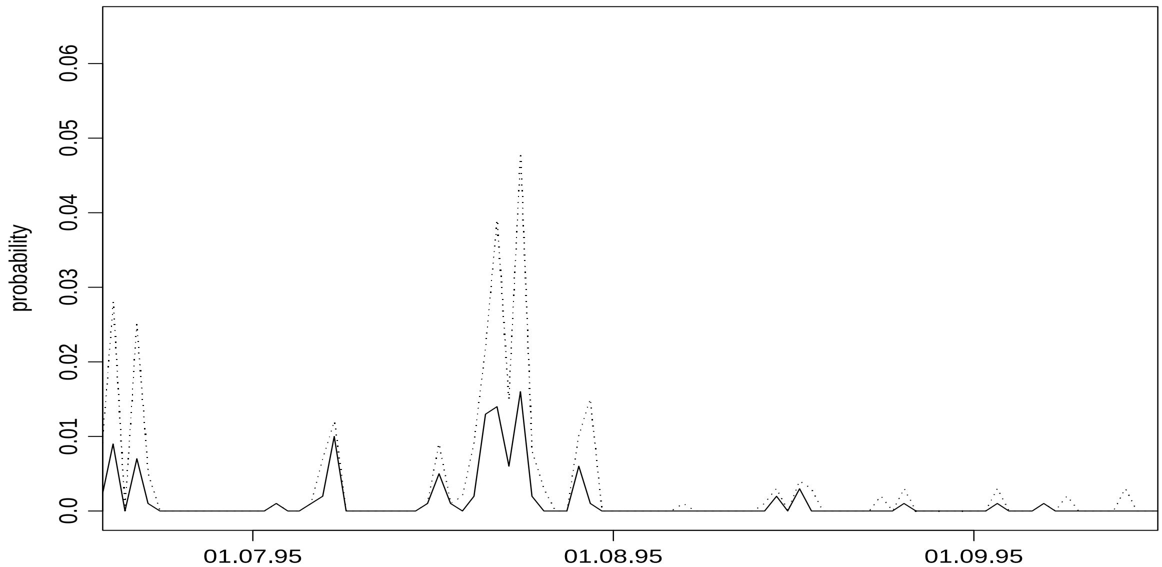
Step 1



Step 2

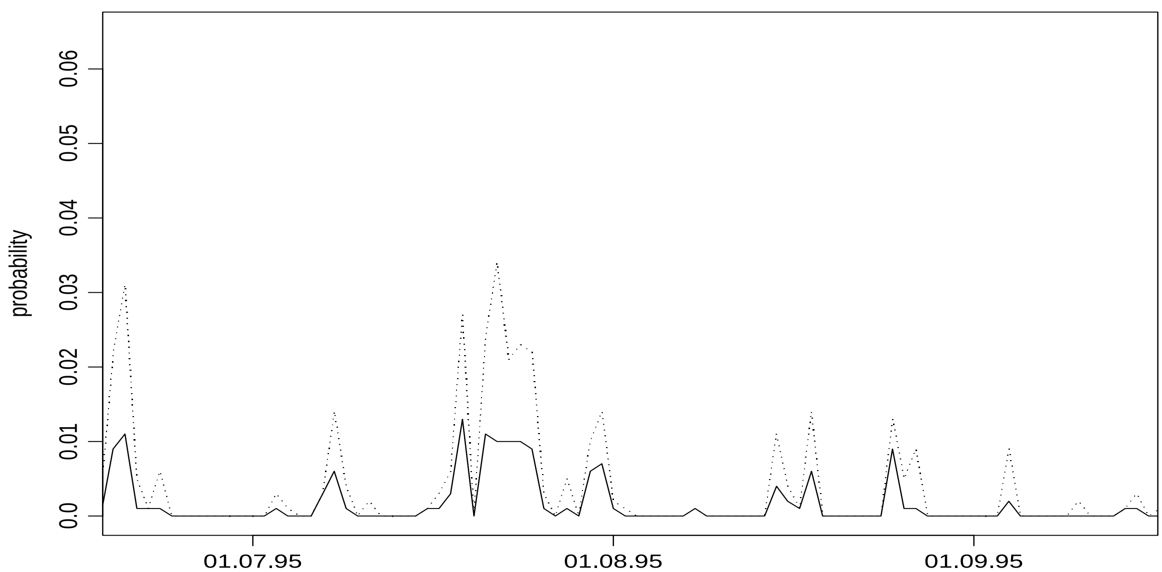


Step 3

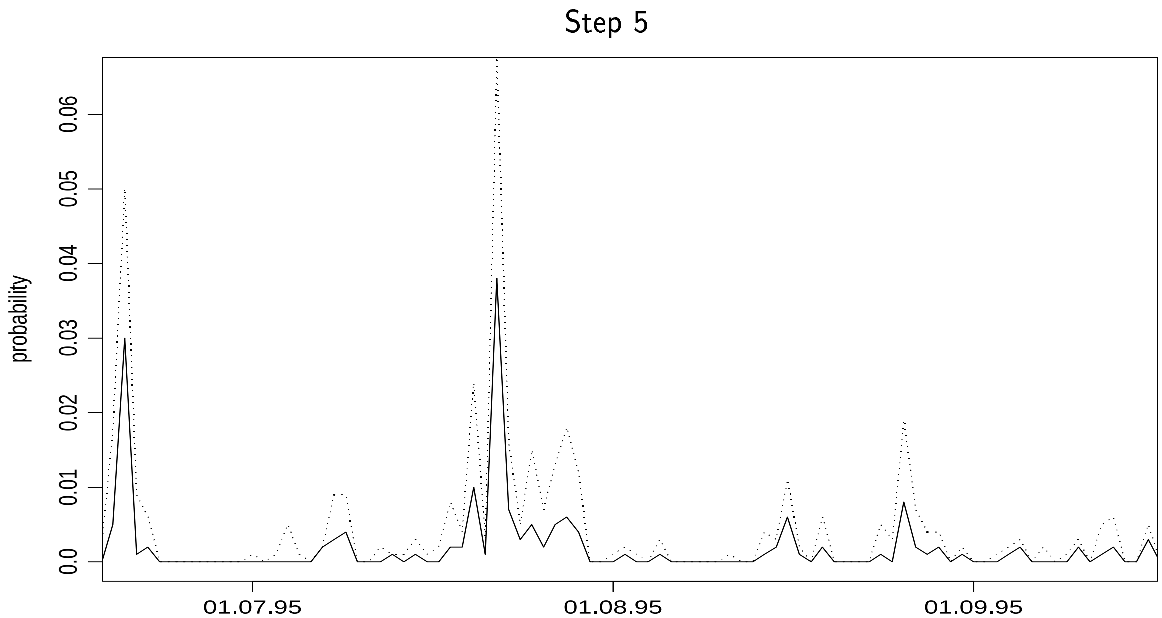


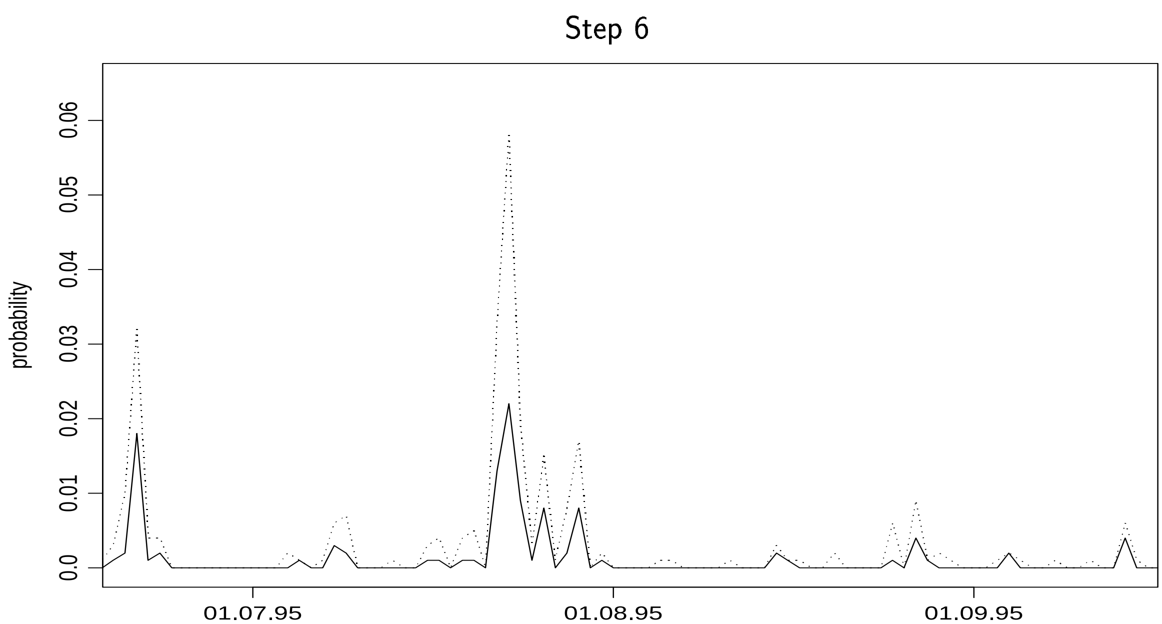
probability
0.0
0.01
0.02
0.03
0.04
0.05
0.06
01.07.95
01.08.95
01.09.95

Step 4



probability
0.0
0.01
0.02
0.03
0.04
0.05
0.06
01.07.95
01.08.95
01.09.95

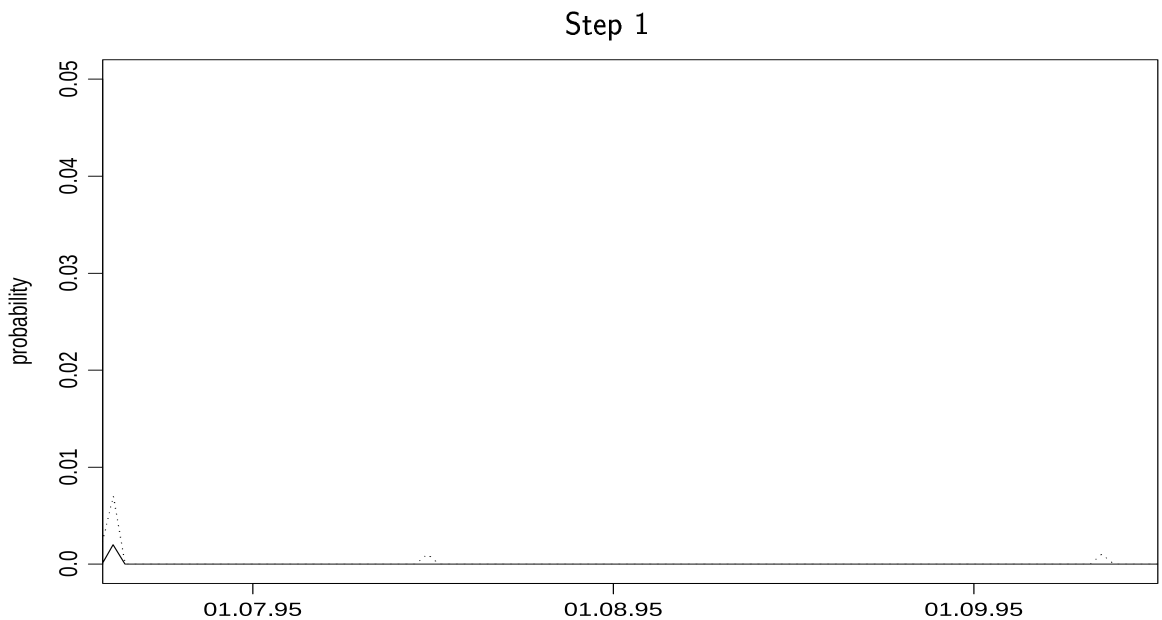




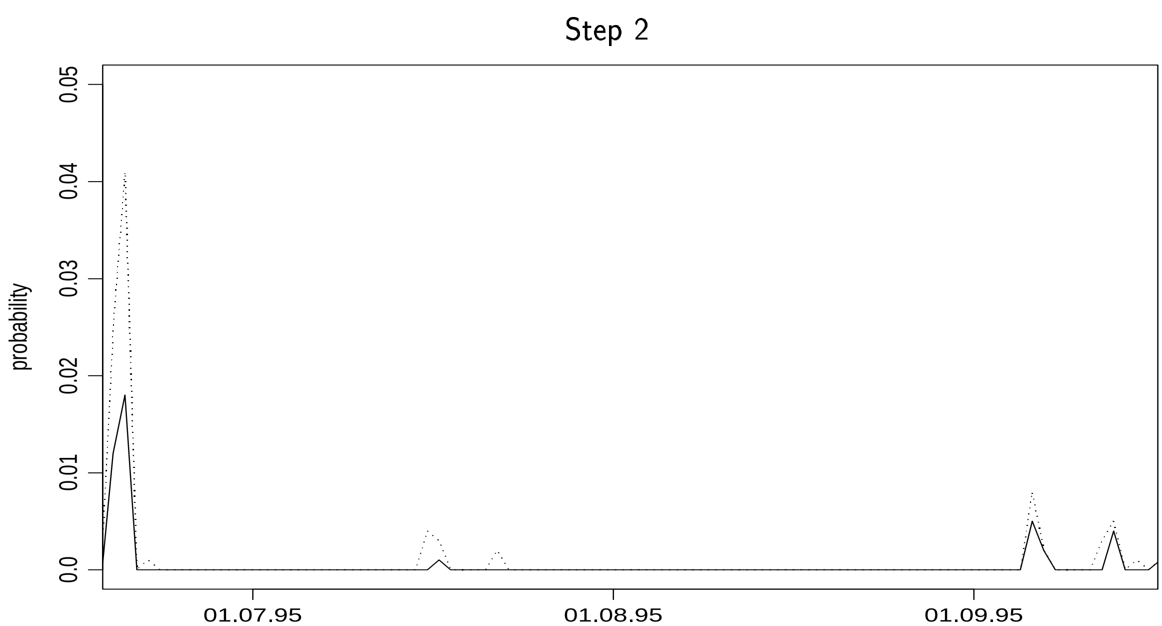
Figur 4.3 (tre sider) $P(Q_{\mathrm{OBS}} > 51)$ *(stiplet) og* $P(Q_{\mathrm{OBS}} > 73)$ *(heltrukket) for Røykenes.*

Figure 4.3 (three pages) $P(Q_{\mathrm{OBS}} > 51)$ *(dashed) and* $P(Q_{\mathrm{OBS}} > 73)$ *(solid) for Røykenes.*

Knappom

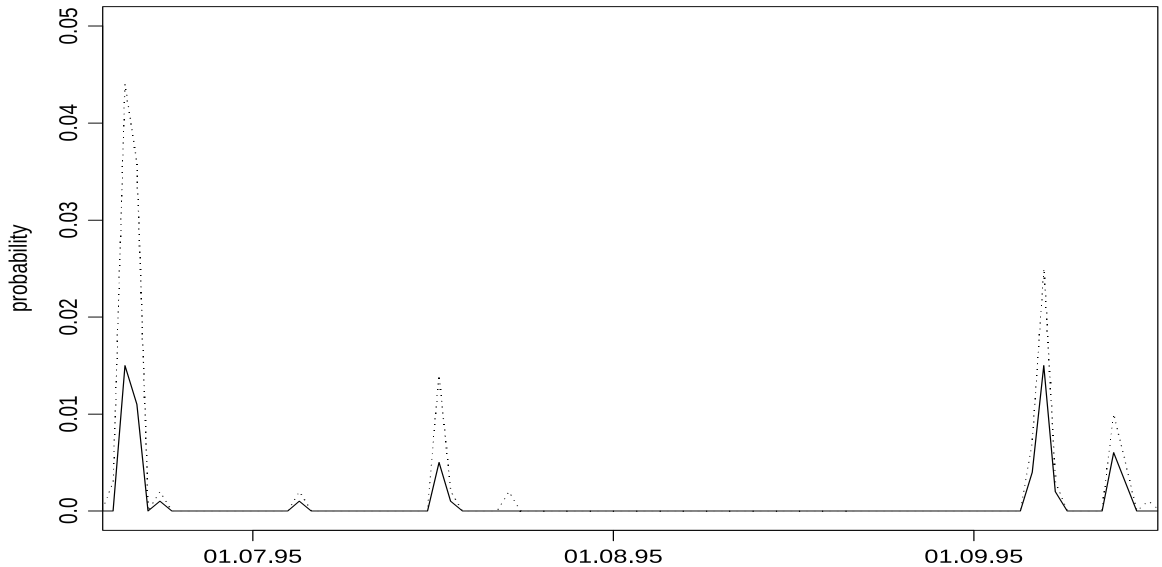


Step 1
probability
0.0
0.01
0.02
0.03
0.04
0.05
01.07.95
01.08.95
01.09.95



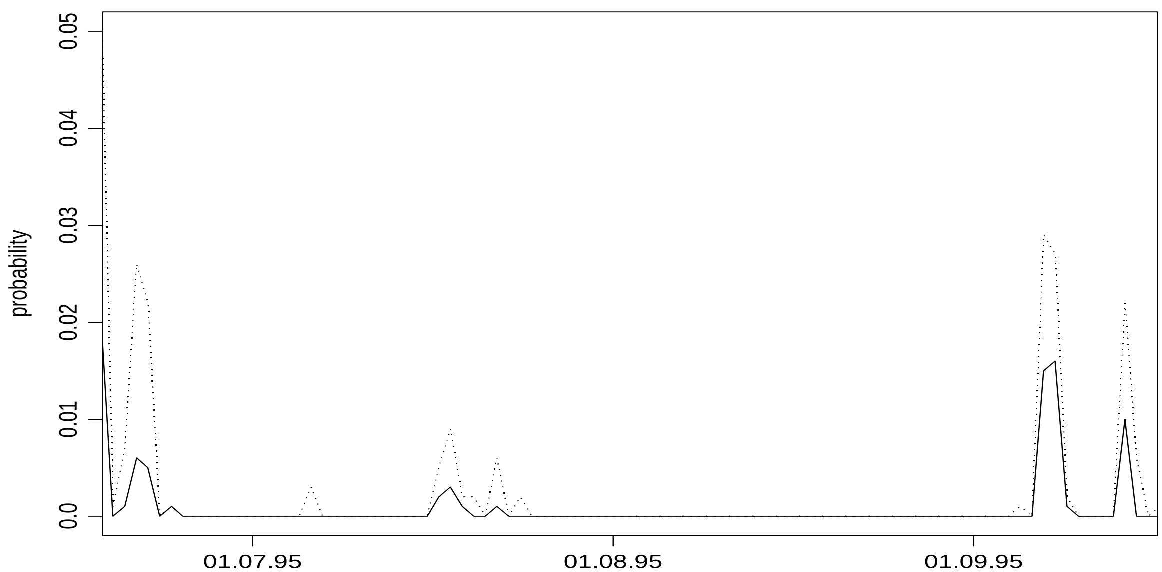
Step 2
probability
0.0
0.01
0.02
0.03
0.04
0.05
01.07.95
01.08.95
01.09.95

Step 3

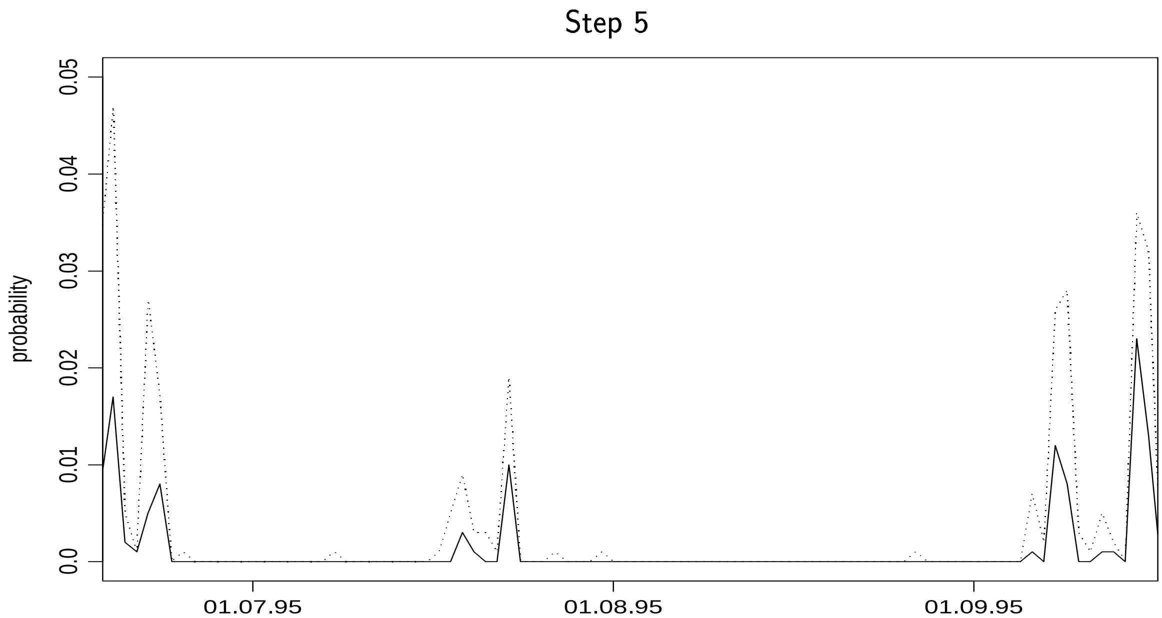


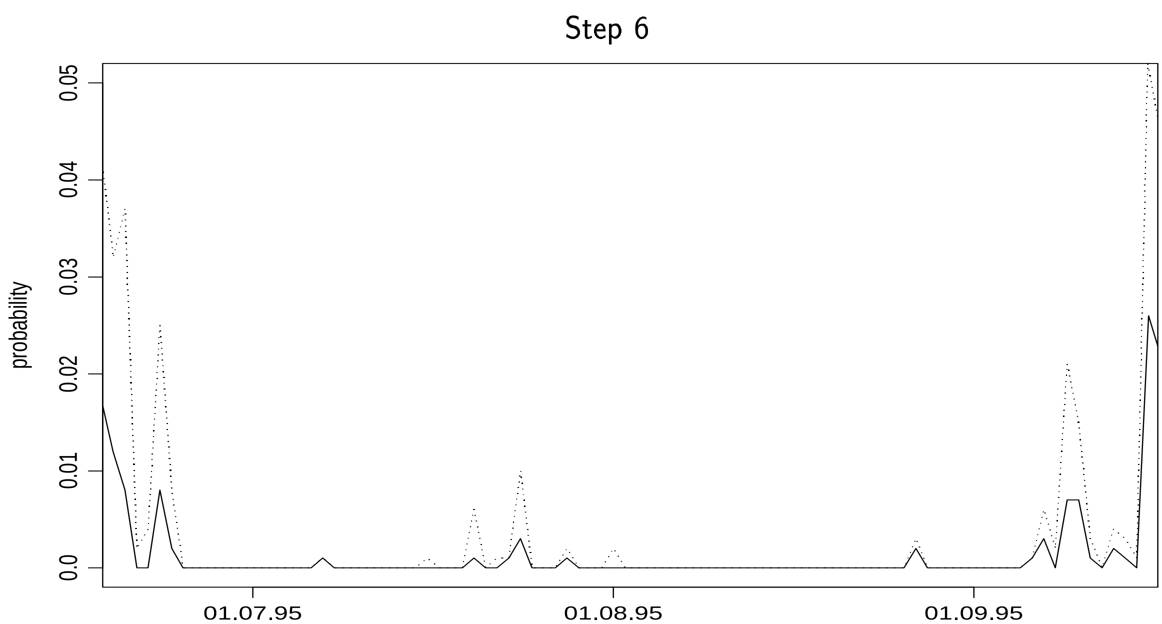
probability
0.0
0.01
0.02
0.03
0.04
0.05
01.07.95
01.08.95
01.09.95

Step 4



probability
0.0
0.01
0.02
0.03
0.04
0.05
01.07.95
01.08.95
01.09.95



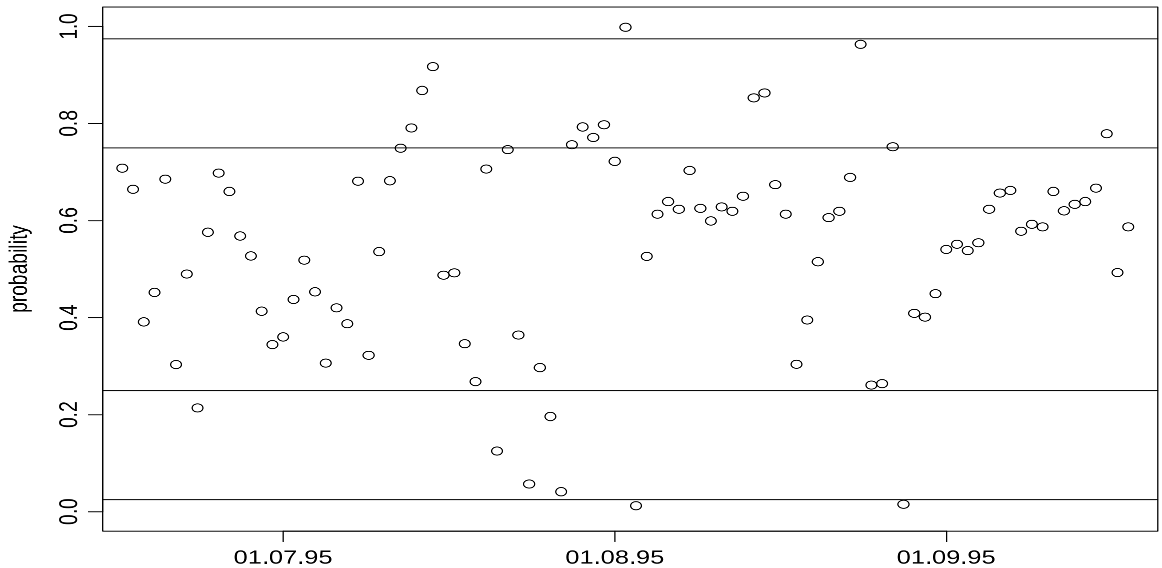


Figur 4.4 (tre sider) $P(Q_{\mathrm{OBS}} > 178)$ *(stiplet) og* $P(Q_{\mathrm{OBS}} > 264)$ *(heltrukket) for Knappom.*

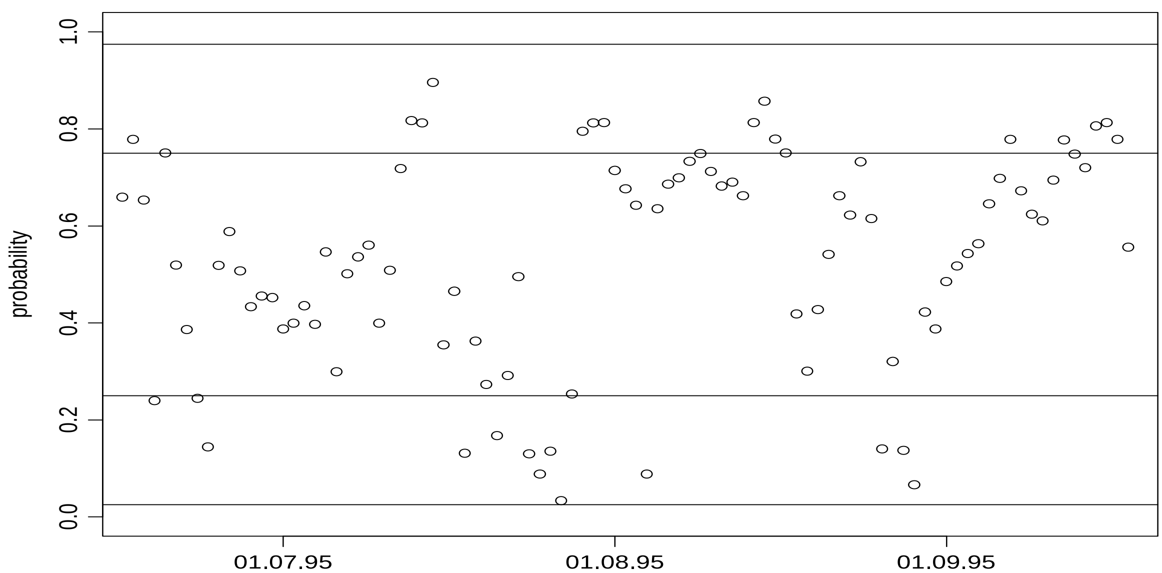
Figure 4.4 (three pages) $P(Q_{\mathrm{OBS}} > 178)$ *(dashed) and* $P(Q_{\mathrm{OBS}} > 264)$ *(solid) for Knappom.*

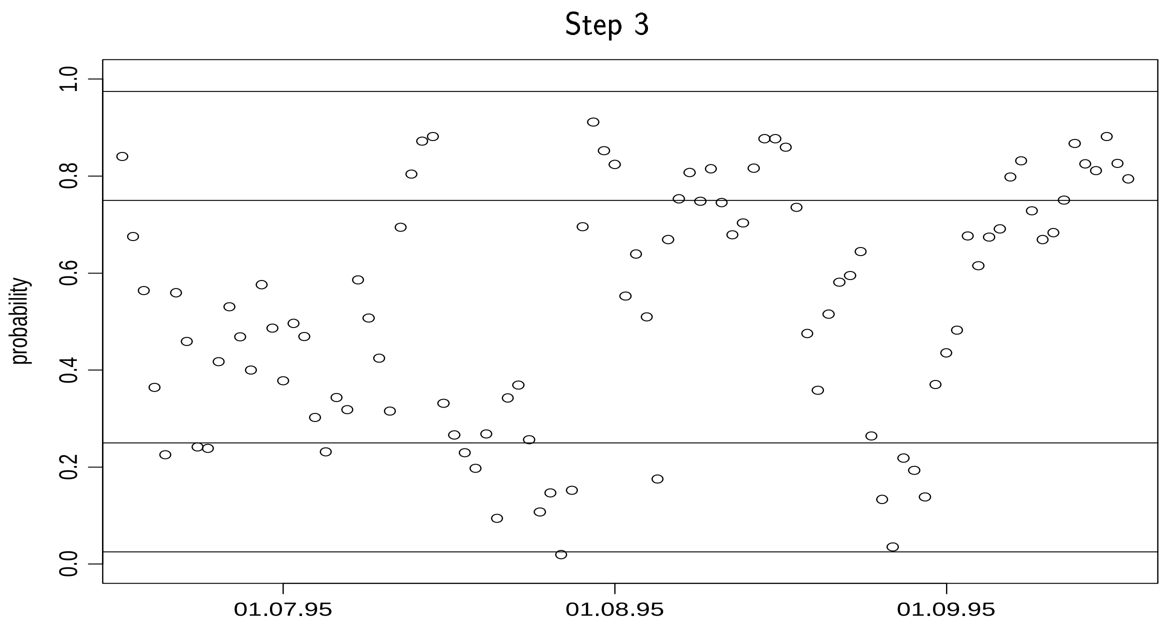
Røykenes

Step 1

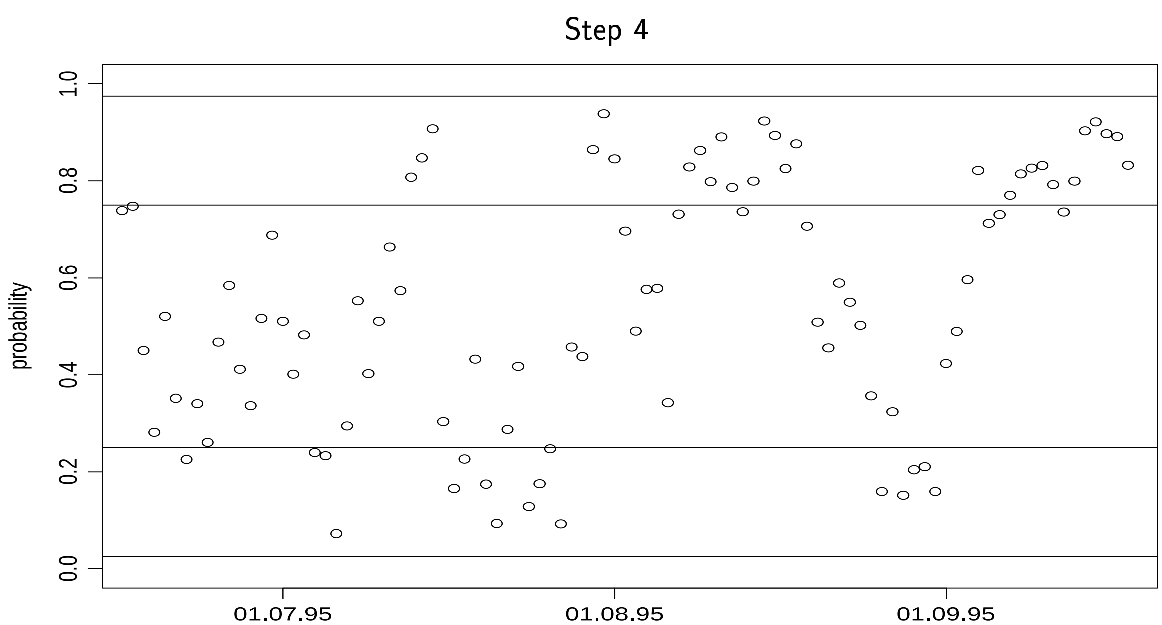


Step 2

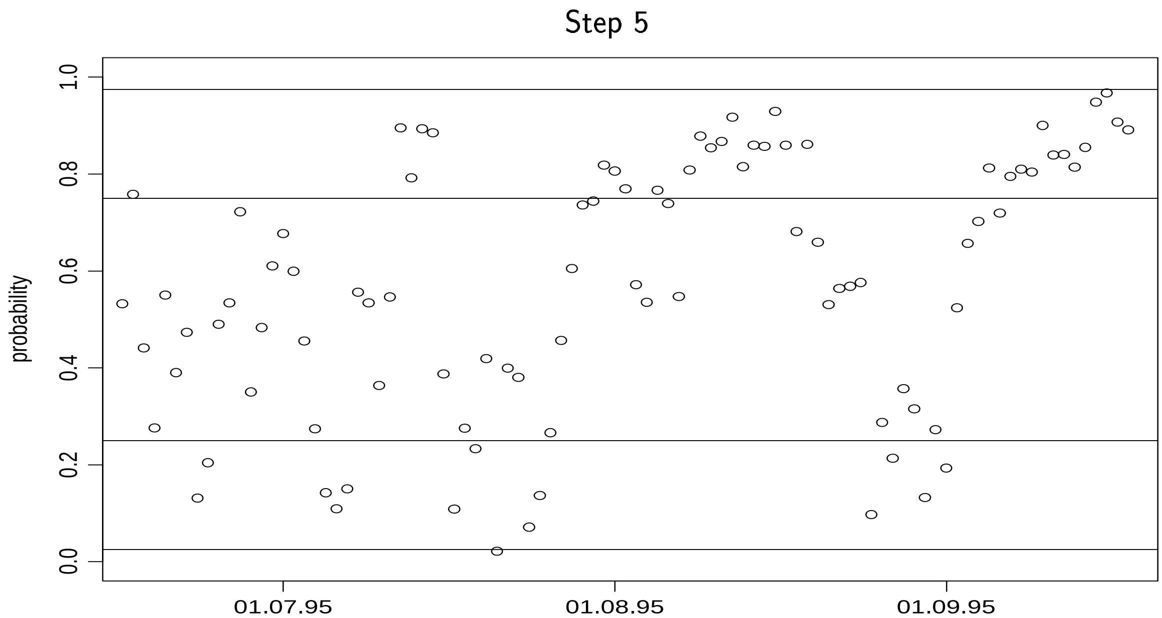


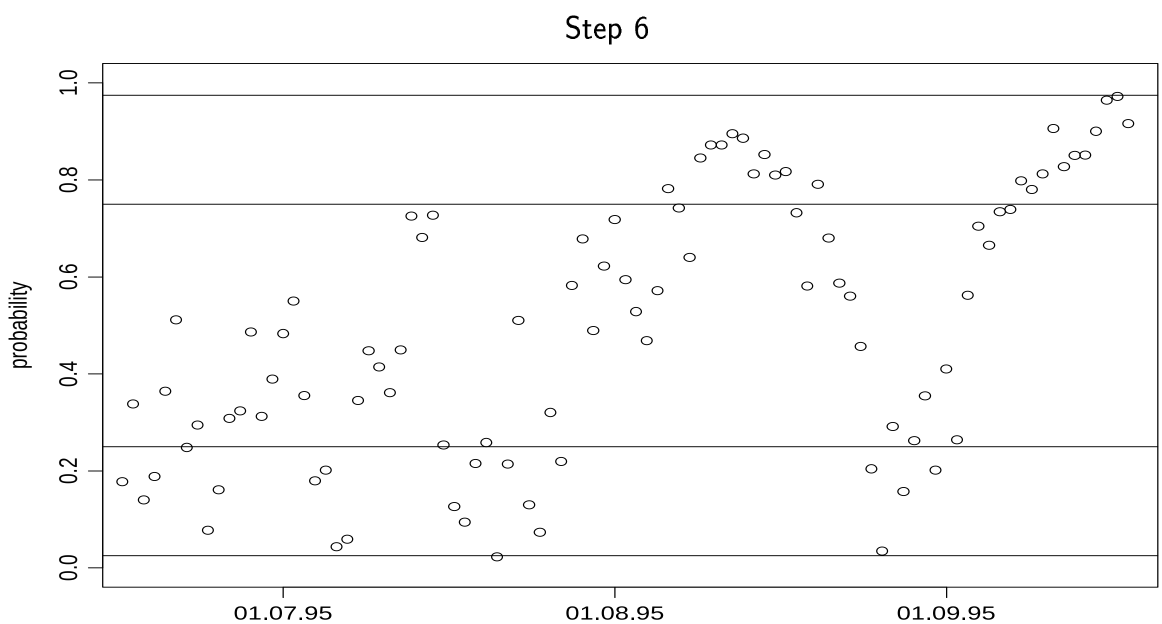


Step 3
probability
1.0
0.8
0.6
0.4
0.2
0.0
01.07.95
01.08.95
01.09.95



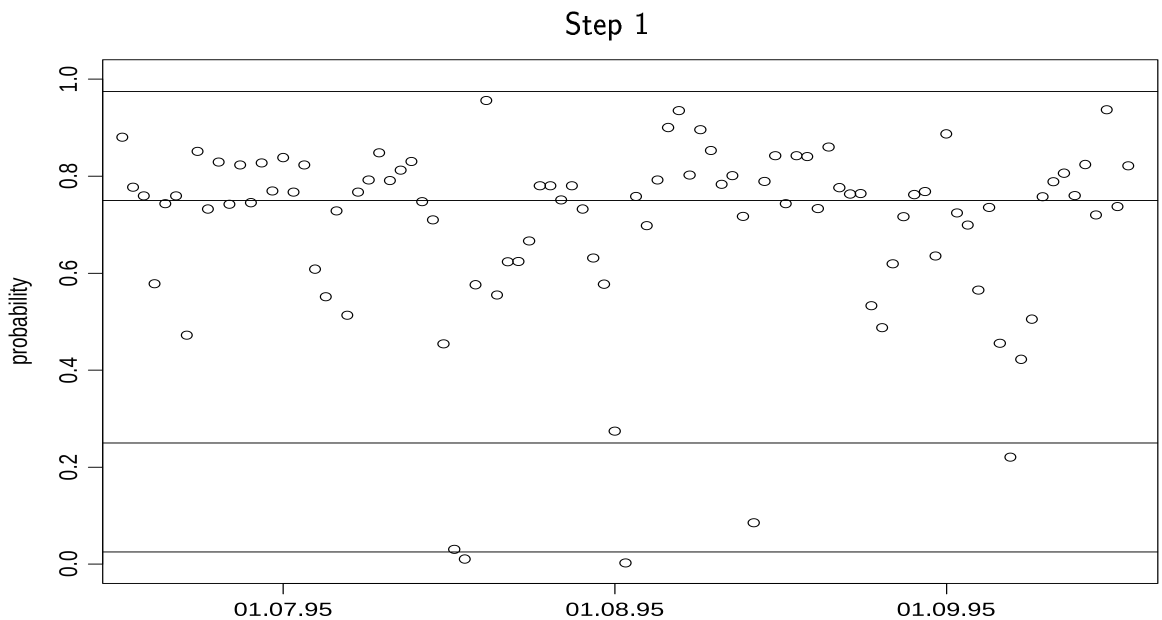
Step 4
probability
1.0
0.8
0.6
0.4
0.2
0.0
01.07.95
01.08.95
01.09.95

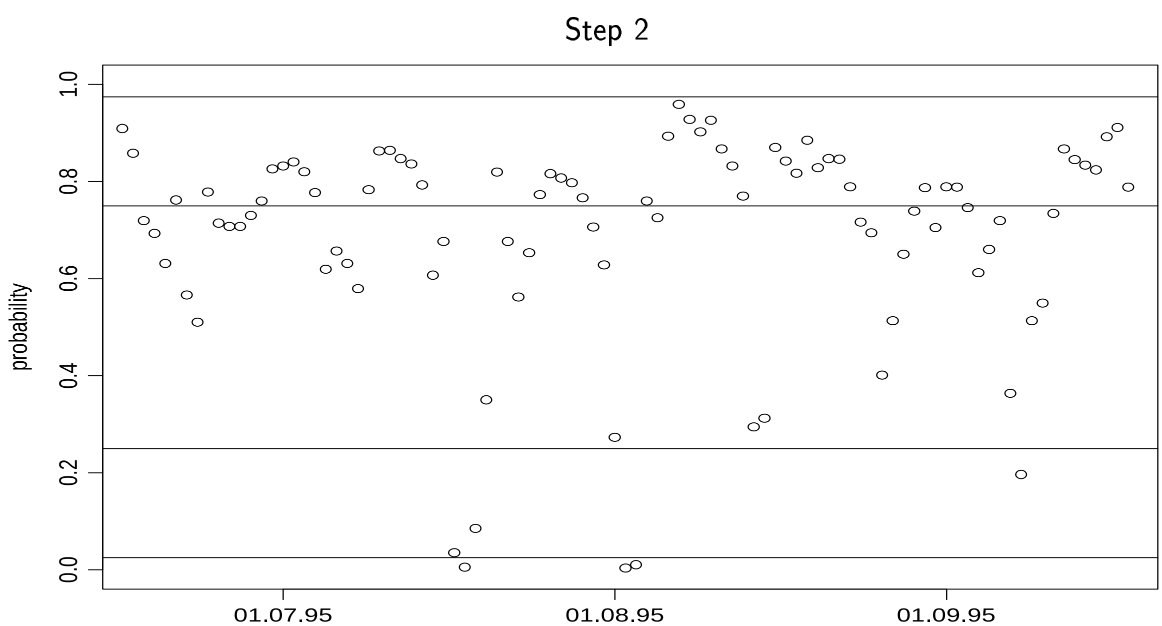


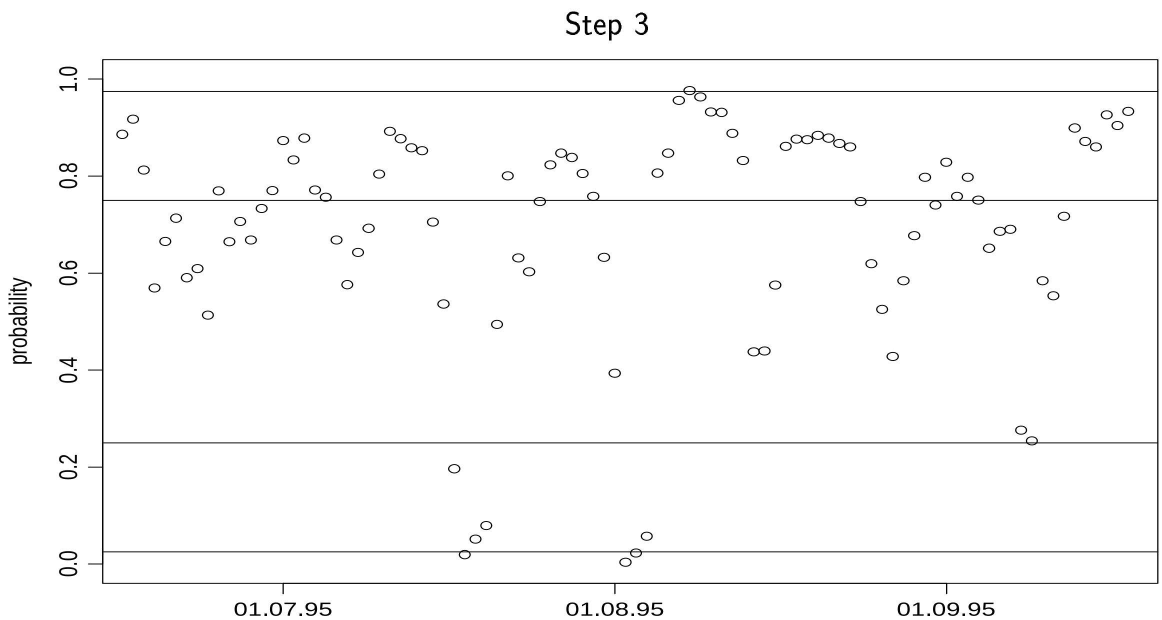


Figur 4.5 *(tre sider)*	$P(Q_{\mathrm{OBS}} >$ *"faktisk* Q_{OBS}*") for Røykenes. Linjene viser 50% og 95% intervaller.*
Figure 4.5 *(three pages)*	$P(Q_{\mathrm{OBS}} >$ *"real* Q_{OBS}*") for Røykenes. Lines illustrate 50% and 95% intervals.*

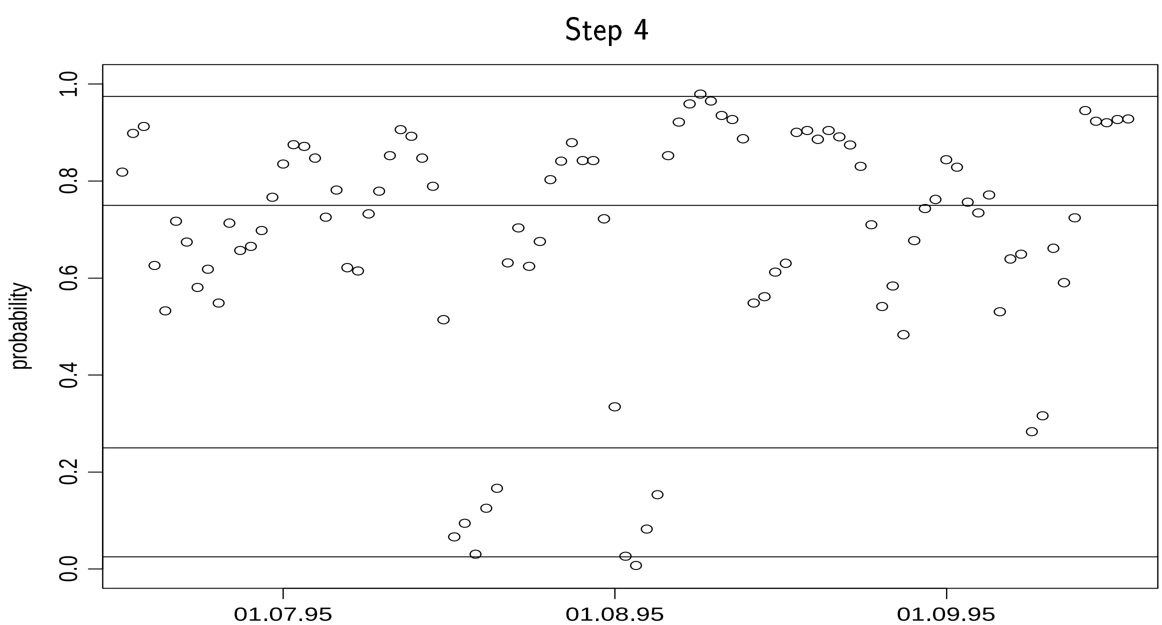
Knappom


Step 1
probability
1.0
0.8
0.6
0.4
0.2
0.0
01.07.95
01.08.95
01.09.95

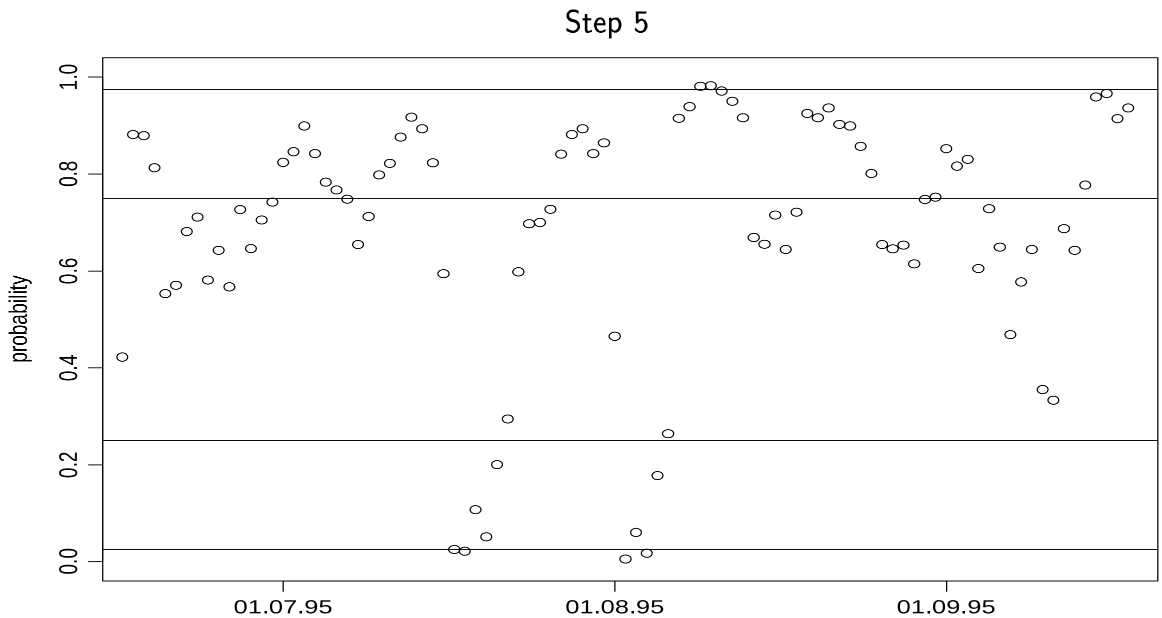

Step 2
probability
1.0
0.8
0.6
0.4
0.2
0.0
01.07.95
01.08.95
01.09.95

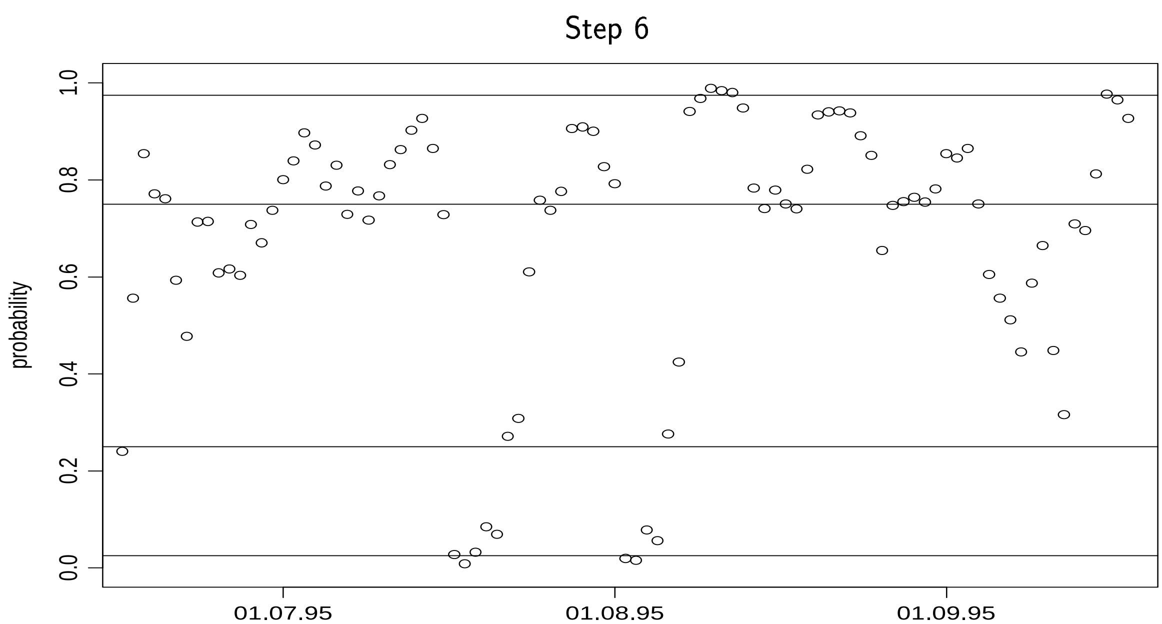


Step 3
probability
1.0
0.8
0.6
0.4
0.2
0.0
01.07.95
01.08.95
01.09.95



Step 4
probability
1.0
0.8
0.6
0.4
0.2
0.0
01.07.95
01.08.95
01.09.95





Figur 4.6 (tre sider) — $P(Q_{\mathrm{OBS}} >$ *"faktisk* Q_{OBS}*")* *for Knappom. Linjene viser 50% og 95% intervaller.*

Figure 4.6 (three pages) — $P(Q_{\mathrm{OBS}} >$ *"real* Q_{OBS}*")* *for Knappom. Lines illustrate 50% and 95% intervals.*