# 1 Introduction

The introduction of well tests combined with ordinary core and log measurements in petrophysical reservoir modelling gives data on very different scales. Well tests informs about the effective permeability averaged over a volumetric region, while the core or log data gives direct or indirect information on a small scale. In the geostatistical modelling (prediction and/or simulation) the conditioning on both types of data should be satisfied. This is in the present paper achieved by looking at the well test variable as a spatial average of the small scale variable and an appropriate filter. This convolved variable is included in the kriging equation. The covariances used in the kriging are then not only between cell values, but between regions consisting of a potentially high number of cells. This is previously addressed in e.g. Holden et.al.(1995) and Srinivasan and Journel (1998). With the number of grid cells involved in a realistic model, reduction of computation time is a crucial matter. Using Fast Fourier Transform on the covariances in place of straightforward calculations is highly efficient. In addition to presenting the methodology, a simple case study is included.

# 2 Well test variable

The petrophysical property of interest (absolute permeability) in a location $u$ in the 3D simulation box, is represented by a random function $P(u)$. In addition well tests give rise to another random function $\bar{P}(u)$ defined as an average value of $P$ taken over a region with center in $u$.

The averaging relationship between $P$ and $\bar{P}$ is defined as a convolution:

$$\bar{P}(u) = \int f(u-v)P(v)dv \equiv f(u) \star P(u), \tag{1}$$

for some appropriate filter function $f$.

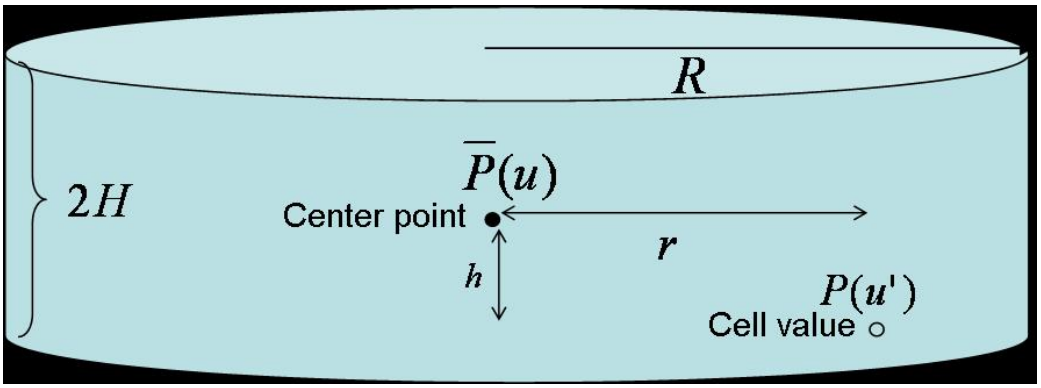
The filter function is used to define the region of influence for the well test and compute the weights relative to position inside the region. The physics from well tests suggests a cylinder as a reasonable representation of the well test region. Figure 1 describes this region with $R$ as the radius and $H$ the distance from the center to the top or bottom. The contributions from the lateral and the vertical dimensions are assumed independent. We write this as

$$f(u) = C \cdot f_{\text{lateral}}(u) \cdot f_{\text{vertical}}(u), \tag{2}$$

where $C$ is a normalization constant assuring that $\int f(u) = 1$. With $|r|$ being the radial distance from $u$ to the center of the cylinder and $|h|$ the vertical distance from $u$ to the centerplane of the cylinder, we have:

$$\begin{aligned} f_{\text{lateral}}(u) &= 1 - \frac{r}{R} && \text{for } |r| < R \\ f_{\text{vertical}}(u) &= 1 - \frac{3}{2}\left(\frac{|h|}{H}\right) + \frac{1}{2}\left(\frac{|h|}{H}\right)^3 && \text{for } |h| < H, \end{aligned}$$

and zero elsewhere. This choice of $f$ gives a damped and smooth behaviour towards the edges of the cylinder to ensure numerical stability in the Fourier transform.




Figuur 1: The cylindrical shape of the region influenced by a well test.

## 3 Inverse block kriging

Suppose the observed values of $P$ are $\mathbf{p} = \{p(u_1), \ldots, p(u_n)\}^\mathsf{T}$ and the observed values of $\bar{P}$ are $\bar{\mathbf{p}} = \{\bar{p}(u_{n+1}), \ldots, \bar{p}(u_{n+m})\}^\mathsf{T}$. The expected values of $P$ in the observation points are represented by the vector $\mathbf{m}$ and the expected values of $\bar{P}$ in the observation points by $\bar{\mathbf{m}}$. These values along with the mean value $m(u)$ are estimated using generalized least squares methods.

The kriging equation for the prediction of $P(u)$ is

$$P^*(u) = m(u) + [\mathbf{c}(u) \;\; \bar{\mathbf{c}}(u)] \begin{bmatrix} \mathbf{C} & \bar{\mathbf{C}} \\ \bar{\mathbf{C}}^\mathsf{T} & \bar{\bar{\mathbf{C}}} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{p} - \mathbf{m} \\ \bar{\mathbf{p}} - \bar{\mathbf{m}} \end{bmatrix}, \tag{3}$$

where $\mathbf{c}(u)$ is the vector of the covariances between $P(u)$ and the log observation points and $\bar{\mathbf{c}}(u)$ is the vector of the covariances between $P(u)$ and the well test observation points. $\mathbf{C}$ is the $n \times n$ matrix of covariances of the log observations, $\bar{\mathbf{C}}$ is the $n \times m$ matrix of covariances between log and well test observations and $\bar{\bar{\mathbf{C}}}$ is the $m \times m$ matrix of covariances of the well test observations. Equation (3) is an inverse block kriging problem where a point value is predicted from average values.

To solve equation (3) we need to evaluate the covariances

$$\begin{aligned} C(u, u') &= Cov\{P(u), P(u')\} \\ \bar{C}(u, u') &= Cov\{P(u), \bar{P}(u')\} = Cov\{\bar{P}(u), P(u')\} \\ \bar{\bar{C}}(u, u') &= Cov\{\bar{P}(u), \bar{P}(u')\}. \end{aligned}$$

Recalling the convolution and filter function $f$ in (1) we can write

$$\begin{aligned} \bar{C}(u_i, u_j) &= \int f(u_j - u) C(u_i, u) du \\ &= \sigma^2 \cdot [f(u_j) \star \rho(|u_i - u_j|)], \end{aligned} \tag{4}$$

where $\sigma$ is the standard deviation of $P(u)$, and $\rho(|u_i - u_j|)$ is the correlation coefficient between $P(u_i)$ and $P(u_j)$ only depending on the distance between the two locations. Similarily for the double convolved covariance we have

$$\begin{aligned} \bar{\bar{C}}(u_i, u_j) &= \int f(u_i - u_j) \bar{C}(u, u_j) du \\ &= \sigma^2 \cdot [f(u_i) \star f(u_j) \star \rho(|u_i - u_j|)]. \end{aligned} \tag{5}$$

Computing the correlations involving well test observations directly, might typically increase the number of calculations significantly depending on the size of the well test region and the grid resolution. Using Fast Fourier Transform is a highly efficient approach for reducing the computational effort.

## 4 Calculating the covariances using Fast Fourier Transform

If $\mathscr{F}$ denotes the Fourier transform, using the convolution theorem (i.e. $\mathscr{F}(f \star g) = \mathscr{F}(f) \cdot \mathscr{F}(g)$) gives that

$$\bar{C}(u_i, u_j) = \sigma^2 \cdot \mathscr{F}^{-1}[\mathscr{F}(f(u_j)) \cdot \mathscr{F}(\rho(|u_i - u_j|))] \tag{6}$$

$$\bar{\bar{C}}(u_i, u_j) = \sigma^2 \cdot \mathscr{F}^{-1}[\mathscr{F}(f(u_i)) \cdot \mathscr{F}(f(u_j)) \cdot \mathscr{F}(\rho(|u_i - u_j|))]. \tag{7}$$

These calculations can be performed using the Fast Fourier Transform (FFT). The FFTW package from MIT described in Frigo and Johnson (2005) has been used. The algorithm to find the covariances in (6) and (7) is as follows:

1. **Decide the size of the FFT-grids.** This has to be (i) big enough to avoid cyclicity and (ii) designed for execution speed. Since we use the same grid for both the single and the double convolution, the minimum size is twice the range of $P$ plus twice the radius $R$ of the well test region.

2. **Fill the grids.** The filter grid is filled with weights given by $f(u')$ in (2) and the correlation grid is filled with values of the correlation function $\rho(|u - u'|)$.

3. **Perform FFT** on each of the two grids from the above step.

4. **Multiply** the two transformed grids cell by cell. This gives a new grid which has been convolved once.

5. **Perform Inverse FFT** on the multiplied grid from the step above.

The resulting grid now contains the covariances in (6). To produce a grid containing the covariances in (7) step 5. is substituted by:

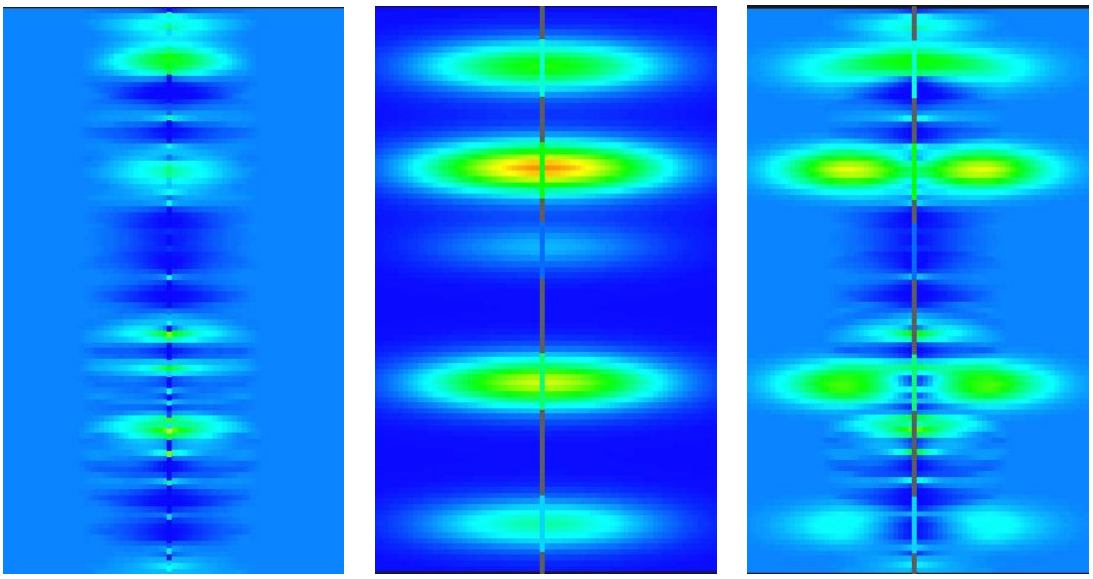
5. **Multiply** the single convolved grid once more with the transformed filter grid cell by cell. This gives a new grid which has been convolved twice.

6. **Perform Inverse FFT** on the multiplied grid from the above step.

The advantage with this method is that the convolutions in (4) and (5), which on a discrete grid would involve big triple sums, is substituted by the far more rapid operations of FFT and cell by cell multiplications.

## 5 Example

Figure (2) shows the method used in a synthetic reservoir with one well containing both log and well test observations. The figures shows the same vertical cross section penetrated by the well. To the left is the permeability conditioned on the log observations only, showing the range of the variogram. In the middle the conditioning is on well test observations only. The radius of influence is 50% longer than the horizontal range. To the right conditioning on both log and well test observations is performed showing that low or high point values forces the method to compensate over the region of influence to condition exact on the regional average value.



Figuur 2: Cross section of well test influence in well. Left: Only well logs. Middle: Only well tests. Right: Both well logs and well tests

## 6 Conclusion

Since well tests give information of effective permeabilities in a region around the wells on a scale appropriate for the actual well rates, they provide important information that should be accounted for when doing predictions or simulations of the absolute permeability field of a reservoir. Inverse block kriging ensures that these data can be included among the conditioning data. Using Fast Fourier Transform on the convolution defining the well test variable makes it possible to produce software that performs rapidly even with a great number of data points and on large grids.

## Acknowledgments

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## References

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