CONVERGENCE OF MARKOV CHAINS IN THE RELATIVE SUPREMUM NORM

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ABSTRACT It is proved that the Doeblin condition (i.e., $r^s(y|x) \geq a_s \pi(y)$ for all x, y in the state space) implies convergence in the relative supremum norm for a general Markov chain. The convergence is geometric with ratio $(1 - a_s)^{1/s}$. If the detailed balance condition is satisfied, the Doeblin condition is equivalent to convergence in the relative supremum norm. Convergence in other norms is proved under weaker assumptions. The results in the paper also give a qualitative understanding of the convergence.

KEYWORDS Markov chain, geometric convergence, relative supremum norm.