MARKOV MESH SIMULATIONS WITH DATA CONDITIONING THROUGH INDICATOR KRIGING

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ABSTRACT

We propose to use probabilities calculated from indicator kriging to modify the probabilities of unilateral stationary Markov mesh models to condition on data. The method can be applied both to soft and hard data. We also propose an iterative method based on block updates created by using the modified Markov mesh model. The iterative method can be used for doing local update on existing realizations when new data is available. Both the unilateral and the iterative method are illustrated with examples.

INTRODUCTION

Markov mesh models constitute a subclass of Markov Random Fields (Politis) that allows efficient calibration and simulation combined with good ability to reproduce the characteristics of a training image (Daly).

Markov mesh models are defined sequentially using a unilateral scan of a regular grid; the probability of a node depends only on previously simulated nodes. This construction ensures that the model is statistically well defined and easy to simulate when the unilateral path is followed. The drawback is that the sequential nature of the model makes it hard to compute the influence of data that is located along the future simulation path. Examples of data are hard data in the form of well observations, and soft data in the form of seismic probability cubes.

We propose to use probabilities computed from indicator kriging to modify the probabilities from the Markov mesh model in order to do the well conditioning. In an approximate approach we simulate directly with these modified probabilities. In an iterative approach we use the modified probabilities to construct a block update in a Metropolis Hastings algorithm. This iterative approach is made possible by the Markov random field properties of the Markov mesh model.

The proposed approximate approach will also provide a method for doing well conditioning when using multigrids or subgrids in the standard snesim algorithm (Strebelle). The quality and efficiency of snesim simulations depends much on the application of multigrid and subgrids. A problem arises in this setting since the wells are not guaranteed to be present in the grid except on the finest level. The potentials computed by indicator kriging provides a method for doing soft conditioning on wells present in a finer grid when doing simulations on a coarse grid.

The proposed iterative approach can also be used to do local updates for a Markov mesh model. Frequently it is such that more data will become available after initial simulations have been generated, e.g. a new well is drilled. The iterative approach can be used to generate models that are identical to the previous simulations except in a region around the new well.

METHODOLOGY

We consider a regular grid with N cells. The cells are indexed with a one-dimensional index i. Each cell is assigned a categorical variable z_i that can take on a finite number of values $0,1,...,K-1$. A typical application is to let z_i represent geological facies. The full configuration of the grid is denoted $\mathbf{z} = \{z_1, z_2, ..., z_N\}$. We will frequently use the notation z_i when we mean the value taken on by the variable, and hence write the joint probability for the grid to have a specific configuration $\mathbf{z}$ as

$$p(\mathbf{z}) = p(z_1)p(z_2|z_1)...p(z_N|z_{N-1}, z_{N-2}, ..., z_2, z_1). \quad (1)$$

Assume that the conditional probability for z_i depends only on a subset Γ_i of all cells $j < i$, such that

$$p(z_i|z_{i-1}, z_{i-2}, ..., z_2, z_1) = p(z_i|\mathbf{z}_{\Gamma_i}). \quad (2)$$

Here $\mathbf{z}_{\Gamma_i}$ is short hand notation for the configuration of the set of cells in Γ_i. The set Γ_i is denoted the sequential neighbourhood of cell i. The joint probability

$$p(\mathbf{z}) = \prod_{i=1,...,N} p(z_i|\mathbf{z}_{\Gamma_i}) \quad (3)$$

defines a Markov mesh model with respect to the set of sequential neighbourhoods $\{\Gamma_i\}_{i=1,2,...,N}$. For a stationary Markov mesh model the probability $p(z_i|\mathbf{z}_{\Gamma_i})$ depends only on the configuration $\mathbf{z}_{\Gamma_i}$, not the location i.

Simulation from a Markov mesh model is easily done once all conditional probabilities $p(z_i|\mathbf{z}_{\Gamma_i})$ are specified. Define a path that visits each cell once and only once, in the order $i = 1,2,...,N$. Scan the path once, and for each cell i draw its variable conditioned on the previously simulated cells $\{z_j : j \in \Gamma_i\}$. The resulting configuration $\mathbf{z}$ by construction follows the well defined probability distribution $\prod_i p(z_i|\mathbf{z}_{\Gamma_i})$. Since each full configuration is created by scanning the path only once, it is time efficient to simulate many configurations from the distribution.

A stationary Markov mesh model is, however, in general not consistent with data. The reason is that update of any cell i does not condition on data located along the future path from cell i. Hence it may happen that when the simulation hits a data point w the fixed value z_w may be inconsistent with the probability $p(z_w|\mathbf{z}_{\Gamma_w})$. In the following we propose several modifications of the original Markov mesh model to condition to data. The main focus of the discussion is on hard data, i.e. well observations, but the methodology applies also to soft conditioning. We assume throughout the paper that the sequential probabilities $p(z_i|\mathbf{z}_{\Gamma_i})$ of the prior Markov mesh model are known, and will henceforth denote them $P_S(z_i|\mathbf{z}_{\Gamma_i})$ to distinguish them from other probabilities.

Data Conditioning in Unilateral Markov Mesh Model

The challenge is to set up a conditional probability for cell i that honors both the previously visited cells and the future data. Let W_i be the set of well data along the future path from cell i; $\bigcup_i W_i$ is the set of all data cells. The general expression

$$p(z_i|\mathbf{z}_{j<i}, \mathbf{z}_{W_i}) = \frac{p(z_i|\mathbf{z}_{j<i}, \mathbf{z}_{W_i})}{p(z_i|\mathbf{z}_{j<i})} p(z_i|\mathbf{z}_{j<i}), \tag{4}$$

can be consider as a factorization of the posterior probability into well likelihood (leftmost factor) and prior probability (rightmost factor). We propose to use the unconditional Markov mesh probability as prior information, whereas the likelihood is approximated via indicator kriging. That is, for $p(z_i|\mathbf{z}_{j<i}, \mathbf{z}_{W_i})$ we use the approximation

$$P(z_i|\mathbf{z}_{j<i}, \mathbf{z}_{W_i}) = \frac{Z(z_i|\mathbf{z}_{j<i}, \mathbf{z}_{W_i})}{Z(z_i|\mathbf{z}_{j<i})} P_S(z_i|\mathbf{z}_{\Gamma_i}) \equiv \Psi(z_i|\mathbf{z}_{j<i}, \mathbf{z}_{W_i}) P_S(z_i|\mathbf{z}_{\Gamma_i}), \tag{5}$$

where $Z(z_i|\mathbf{z}_{j<i})$ is the optimal predictor for z_i found by indicator kriging (Journel) conditioned on cells in the past of i, $Z(z_i|\mathbf{z}_{j<i}, \mathbf{z}_{W_i})$ is the optimal predictor conditioned also on future data points, and P_S is the original Markov mesh probability. In the presence of soft and hard data Z denotes the indicator prediction using both types of data. Far away from any wells we expect $\Psi \approx 1$, and hence the new model gives statistics similar to the prior Markov mesh model. If cell i has a non-zero correlation with a future well, the kriging function Ψ affects the overall probability P. A positive correlation implies that $Z(z_i|\mathbf{z}_{j<i}, \mathbf{z}_{W_i})$ increases the probability for z_i to be updated to the same facies as the well, a negative correlation decreases the well's contribution to this probability. The overall effect of the well is modified by the past cells' influence on the predictors.

For the indicator kriging to conform with the prior Markov model, the correlations used by the kriging algorithm should be consistent with the correlations of the prior model. If the prior model is established by using a training image, the correlations of the training image will be suitable for use in the kriging. It is in general not necessary to include in Ψ cells beyond the variogram range. But less obvious is the question of whether *all* candidate cells ($j < i$ or $j \in W_i$) with a non-zero correlation to point i need to be taken into account. The general answer to this question is no. This problem is discussed more in the results section of this paper.

Simulation from the modified Markov model is just as straightforward as simulation from the prior model. The resulting configuration then follows the distribution

$$P(\mathbf{z}) = \prod_{i=1,\dots,N} P(z_i|\mathbf{z}_{j<i}, \mathbf{z}_{W_i}) = \prod_{i=1,\dots,N} \Psi(z_i|\mathbf{z}_{j<i}, \mathbf{z}_{W_i}) P_S(z_i|\mathbf{z}_{\Gamma_i}). \tag{6}$$

Iterative Method

The modified Markov mesh model is well suited for iterations. A main motivation for establishing an iterative method is to render possible local update in an already existing grid configuration, typically as a result of new well data. Iterations can also be used to wash away unwanted kriging effects during the initial establishment of a reservoir configuration, to ensure that it is consistent with the statistics of the prior model. We propose to use a Metropolis Hastings algorithm, where proposal configurations are established via block update based on the modified Markov model in the previous section, and the accept probability ensures that sampling is done from the prior model conditioned on data. In the following we describe one step in the Markov chain of the Monte Carlo simulation.

Let ν be a label for the existing configuration of the grid, i.e. the grid configuration is $\mathbf{z}_\nu$. The proposal configuration will be denoted μ. Pick according to some rule a connected set of cells $B \subseteq G$, where G is the full grid. If the set B includes data cells or cells that for some other reason are supposed to have fixed facies throughout the iterations, let the set of these cells be denoted B_0. Let $B_1 = B \setminus B_0$, and define ∂B_1 as the set of cells in $G \setminus B_1$ that according to the prior model may be affected by a change in the set B_1. That is, $j \in \partial B_1$ iff $\exists k \in \Gamma_j : k \in B_1$. Then $(B_0 \cup \partial B_1) \cap (j > i)$ is the union of the set of future cells within B that we want to condition on, and the set of future cells from position i that according to the prior model are affected by a change in the set B_1. The new configuration μ is established as follows: scan through the part of the path (according to the order of cell indices, $1, 2, \dots, N$) that are within B_1, and for each cell $i \in B_1$ draw its new facies value z_i for configuration μ from the probability

$$\Psi(z_i|\mathbf{z}_{j<i}, \mathbf{z}_{W_i \cup \{(B_0 \cup \partial B_1) \cap (j>i)\}}) P_S(z_i|\mathbf{z}_{\Gamma_i}); \tag{7}$$

for all cells in $G \setminus B_1$ let the facies be as in the state ν. This defines the proposal configuration μ.

Acceptance or rejection of the configuration ν is done with probability

$$\alpha = \min(\frac{q_\nu P_S(\mathbf{z}_\mu)}{q_\mu P_S(\mathbf{z}_\nu)}, 1), \tag{8}$$

where q_μ is the probability for suggesting the new configuration μ, starting from ν, q_ν is the probability for suggesting the old configuration ν, starting from μ, and $P_S(\mathbf{z}_\mu)$ and $P_S(\mathbf{z}_\nu)$ are the prior Markov mesh probabilities. Since all suggest states conform with data, this accept probability ensures that sampling is done from the prior model conditioned on data. The accept probability can be rewritten as

$$\alpha = \min\left(\left(\prod_{i \in B_1} \frac{\Psi(z_i)|_\nu}{\Psi(z_i)|_\mu}\right)\left(\prod_{i \in B_0 \bigcup \partial B_1} \frac{P_S(z_i)|_\mu}{P_S(z_i)|_\nu}\right), 1\right). \tag{9}$$

In the notation of equation 9 the conditional dependencies in the function arguments are suppressed for readability.

Evaluation Criteria

The prior Markov mesh model is assumed to provide a correct description of the unconditional reservoir. Hence evaluation of the success or failure of the modified model should be done relative to the prior model. We henceforth use the expression 'conditional marginal probabilities' to mean 'marginal probabilities in the conditional case', and present two criteria for evaluation:

- The conditional marginal probabilities of the modified models, be it the unilateral modified Markov mesh model or the iterative model, should be identical to the conditional marginal probabilities of the prior model. We obtain the latter by rejection sampling, i.e. by making many realizations from the prior, but only keeping the ones that match the data at hand;
- The size, shape and spatial distribution of objects should be the same in the modified models as in the unconditional prior Markov mesh model.

In the results and discussion section of this paper we apply the criterion on conditional marginal probabilities by computing all relevant probabilities. The criterion on object characteristics we apply by visually comparing individual grid configurations of the modified models with grid configurations of the prior Markov model.

Implementation

We have chosen to use a shell search algorithm to identify the conditional cells to use in the expression $\Psi(z_i|\mathbf{z}_{j<i}, \mathbf{z}_{W_i})$, see equation 5. Two independent parameters are defined: N_1 is the maximum number of conditional cells along the future path, and N_2 is the maximum number of conditional cells along the past path. The algorithm is:

- Search outwards from i in cubic shells;
- From each shell include in the set of future conditioning variables each cell j that satisfies $j \in W_i \cup \{(B_0 \cup \partial B_1) \cap (j > i)\}$. If unilateral simulation without iterations is carried out, the set $B_0 \cup \partial B_1 = \emptyset$, and hence the criterion reduces to $j \in W_i$;
- From each shell include in the set of past conditioning variables each cell j that satisfies $j < i$.

The procedure continues until both limits N_1 and N_2 have been reached, or until the candidate cells from shell search are outside the variogram range of cell i. If not all candidate cells in a shell can be included due to the limits N_1 and/or N_2, the cells to include are randomly chosen among the candidates.

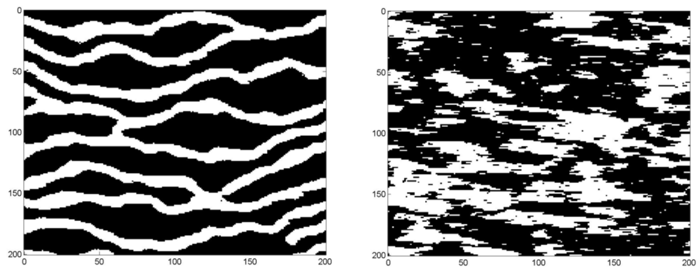

Figure 1: Random sample from prior model A (left) and B (right).

RESULTS AND DISCUSSION

We will in this section consider two different prior models. One model, denoted model A, describes a situation with channel objects embedded in a background. The channels are mainly running east-west, but with a slight mean tilt relative to the horizontal axis. Most channels are connected all the way through the grid, but can occasionally be broken. Channels can also merge and split. The other model, model B, describes a situation with more limited connectivity, characterized by irregular objects spread out against a uniform background. The objects are more elongated in the horizontal than the vertical direction. In each model there are two categorical values for z_i; we describe the background with $z_i = 0$ and the embedded objects with $z_i = 1$. Both models are 2-dimensional. Figure 1 displays random samples of each prior model.

We consider three different cases of data: isolated well data with $z_w = 1$ (object facies); isolated well data with $z_w = 0$ (background facies); two neighbouring well data points, describing a transition from $z_w = 0$ to $z_w = 1$. For selected combinations of prior model and data type we compare the results of the unilateral modified Markov mesh model to the prior model. The iterative method is illustrated and discussed at the end of the section.

Unilateral Modified Markov Mesh Model

We study the two unilateral modified models, A and B, conditioned on isolated wells and on neighbouring wells. Conditional marginal probabilities and object characteristics are discussed.

Conditional Marginal Probabilities

Figure 2 displays the conditional marginal probabilities for model A, conditioned on object well (top row) and background well (bottom row). In each case the well is located at position (126,126) in the grid. The rejection sampling (left column) illustrates, for each well type, the prior model's tilt relative to the horizontal axis,

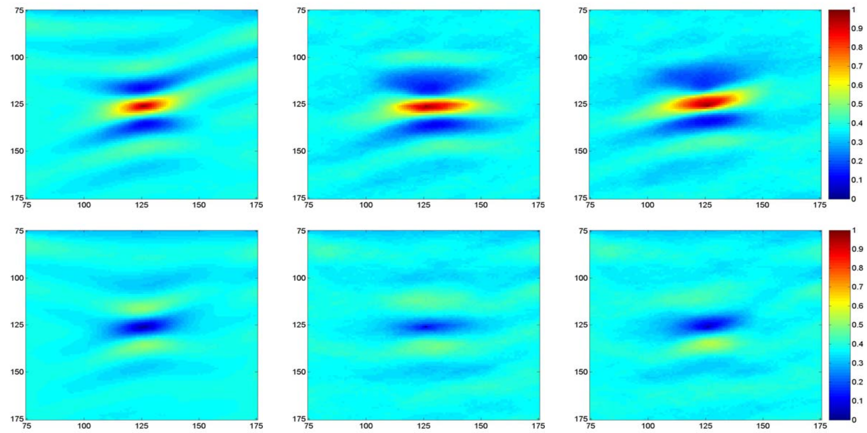

Figure 2: Model A, conditional marginal statistics, displayed as the cell-wise probability $p(z_i = 1)$. Left column: rejection sampling from prior; middle column: indicator kriging with respect to the well only; right column: indicator kriging with respect to the well and two cells in the past path. Top row: isolated object facies well; bottom row: isolated background facies well.

and displays close to a perfect point-wise reflection symmetry around the well point. Column two and three display the results of two different experiments: in column two the maximum number of past conditional cells used in the indicator kriging, N_2, was set to 0, while in column three it was set to 2. The parameter N_1 was in both columns set to 1, i.e. we always condition on the well if it is within the correlation cut-off limit. In addition, both experiments used simple kriging. The simulation path is from the top left corner of each figure, each row being simulated from left to right.

The results clearly show that the indicator kriging used to modify the original Markov mesh model successfully conditions on the data considered. When kriging is done with respect only to the well data (second column) the skewed symmetry of the prior model is broken, and the conditioning along the path leading up to the well (above the well in the figure) creates a vertical symmetry line. This discrepancy with the prior model, as judged from comparison to the rejection sampling in the leftmost column, is eliminated in the third column. Here the tilt relative to the horizontal axis is restored. This is attributed to the fact that in the simulations of the third column, also two already simulated cells were taken into account by the indicator kriging. This softens the impact of the well and allows the prior model's tilt to survive. For all four simulations from the modified Markov model (second and third column, top and bottom row), when the unilateral scan of the path is past the well, the statistics is governed by the prior model.

Figure 3 displays results for conditioning on a channel edge, i.e. a transition from a $z = 0$ well to a $z = 1$ well. Comparison to rejection sampling (left) shows that the main features are well reproduced by the modified model (right). In particular the results are good close to the wells.

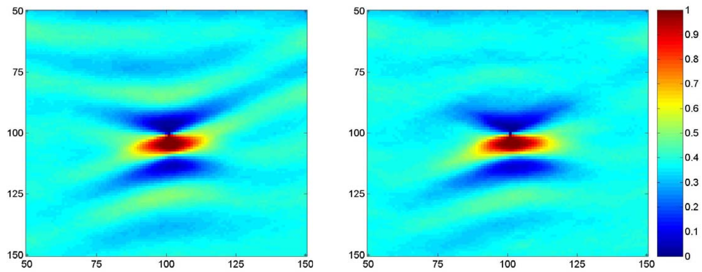

Figure 3: Model A, conditioning on two neighbouring wells, one with $z = 0$ in position (101,100), the other with $z = 1$ in position (101,101). Cell-wise probability $p(z_i = 1)$. Left: rejection sampling; Right: Modified unilateral Markov mesh model.

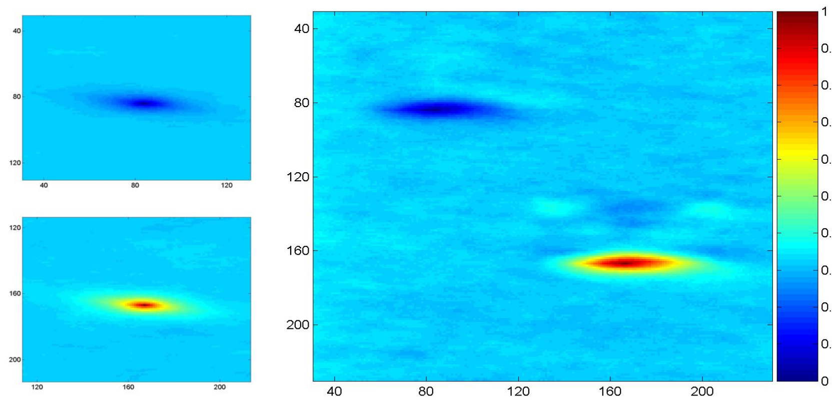

Figure 4: Model B, cell-wise probability $p(z_i = 1)$. Left: rejection sampling for isolated wells; Right: unilateral simulation conditioned on two isolated wells.

Results for model B are shown in 4. The figure illustrates conditioning on two isolated wells, and results for rejection sampling are included for comparison. Indicator kriging was performed with respect to the future wells and two cells from the past path. Apart from minute discrepancies on symmetry and extension, the results of the modified Markov mesh model are in good agreement with the rejection sampling.

Object Characteristics

Figure 5 shows two random samples from simulations using the modified unilateral Markov mesh models. Conditioning is done with respect to a well in grid cell (100,100), in the middle of each figure. Model A is illustrated to the left, model B to the right in the figure. Comparing these two samples with the samples from the unconditional Markov mesh models, see figure 1, there is no observable statistical

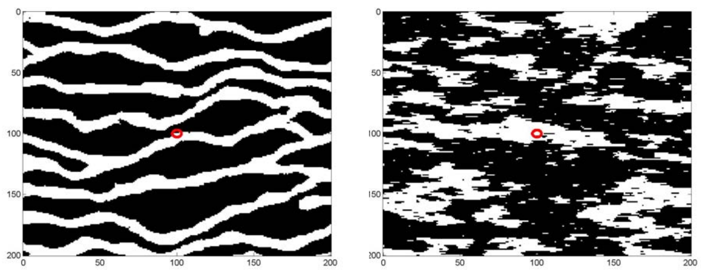

Figure 5: Random samples when conditioning on object facies well in cell (100,100), the well being indicated with a circle. Left: Modified Markov mesh model A; right: Modified Markov mesh model B.

difference between the samples of the prior models and the modified models.

Local update

The iterative model can be used to perform local update of an existing realisation. We illustrate this in figure 6. The left hand side of the figure shows an example of an initial grid configuration that was randomly drawn from prior. This realization is to be updated with respect to a new well in grid position (120, 120). The well position is indicated with a circle at the center of the figure. The value of the well is supposed to be $z = 1$, which is not in agreement with the initial grid configuration. Local update is to be carried out in an area A defined by the rectangular box shown in the initial grid. All cells outside A are fixed. The iterative method described previously is now applied on this problem. Block updates are performed using equations 7 and 9. The region B used to generate a suggestion state for the Metropolis-Hastings algorithm is for each element of the Monte Carlo chain a randomly drawn rectangle, and the cells that satisfy $i \in B \cap A$ are updated according to the algorithm.

The middle grid of figure 6 displays a snapshot of the grid during the iterative process. The channels have now been adjusted such that they are consistent with the well data. In addition the channels nicely match the fixed part of the grid outside the marked rectangle.

At the right hand side of figure 6 we display the cell-wise statistics of the grid after 3000 iterations. The figure illustrate that along the edges of the update rectangle A the probabilities follow the conditioning provided by the cells outside the rectangle. Also conditioning to the well is good. The other areas of the local update rectangle provide evidence that the mixing of the iterative process is satisfactory. This conclusion follows from the fact that the cell-wise statistics tend to smear out and match the marginal probability $p(z = 1)$, only broken by areas with a higher channel probability, areas that to a large extent obviously follow from the edge and well conditioning.

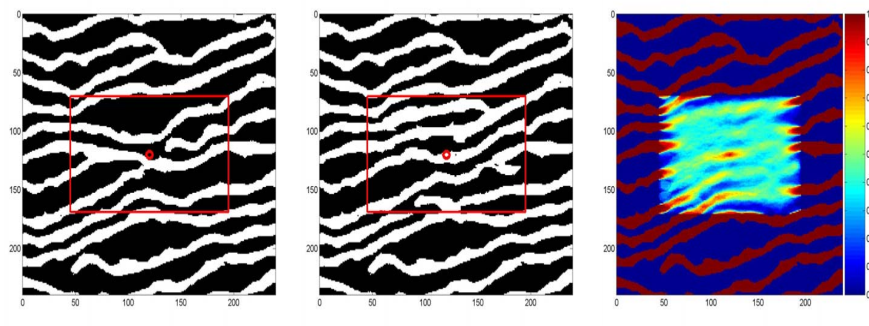

Figure 6: Left: Initial grid configuration before local update; middle: snapshot of the grid configuration during iterative process; right: cell-wise probability $p(z_i = 1)$ after 3000 iterations. The well position is indicated, but not conditioned to, in the initial grid. The iterative process conditions on the well, which has facies $z_w = 1$. The rectangle marks the area of local update.

CONCLUDING REMARKS

We have proposed two methods for extending Markov mesh models to also include data conditioning, be it hard or soft data. Both methods have been illustrated by examples, and the results evaluated with respect to rejection sampling and object characteristics. The results are good, and establish a sound basis for further utilization of the proposed methods.

ACKNOWLEDGEMENTS

We wish to thank the Research Council of Norway (NFR), StatoilHydro and ENI for financial support.

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