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Note

# Valuation of Commodity-Based Swing Options: A survey

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## Abstract

This report gives a summary of existing research on swing option valuation. The most popular valuation methods are explained and additional methods are mentioned. The report gives suggestions to possible future work on the problem, focusing on long term natural gas contracts and all their constraints.

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|---|---|
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# Contents

# 1 Introduction

Many contracts in the energy markets are designed to allow flexibility of delivery. Both the timing and the amount of energy are allowed to be flexible, within certain constraints. In the natural gas market, there are many long term contracts with such flexibility. These contracts typically last for 10 years or more and the volumes are large. Especially in continental Europe, these contracts contribute to more than half of the turnover in the gas markets. In the United Kingdom, the market share is smaller. Most of these contracts are priced according to price formulas linked to other commodities prices, such as oil and coal indexes, often with a time lag.

Neuhoff and von Hirschhausen (2006) investigate the future of long term contracts in Europe. Their conclusion is that in the short run, the role of long term contracts in the supply mix is likely to diminish. In the long run, however, they argue that long term contracts will remain an important element of the natural gas markets in Europe. Neumann and von Hirschhausen (2005) find that as the market liberalises, the long term contracts get fewer and the duration shorter.

In addition to the long term contracts, there are several other contracts that include flexibility. The duration of these contracts are shorter, typically one or some years. As opposed to the long term contracts, these can be purely financial contracts, i.e. without physical gas delivery. When the contracts are financial, they are compared to a gas index, typically NBP or Zeebrügge. Finding a fair price for these types of contracts is a demanding task, but there is an increasing amount of research in the area.

This report provides an introduction to commodity-based flexibility contracts, called swing options, and the valuation of these. We focus on long term flexible gas contracts, which may be seen as complex swing options.

Chapter 2 explains the contracts in more detail. Chapter 3 deals with valuation based on simulation, whereas Chapter 4 deals with valuation based on dynamic programming or other methods (an overview of different methods is given in Table 2.1). Future work is discussed in Chapter 5. Finally, Chapter 6 summarises.

# 2 Background

## 2.1 Long term take or pay contract

Osikilo (2005) discusses the background on the existence of long term natural gas contracts. The most important reasons for their existence are ensurance to regain what was initially invested in the gas project for the seller, and security of supply for the buyer. As the gas markets develop to more liquid market places with natural gas traded as spot, the need for long term contracts decreases and in some cases the contracts can force the parties not to operate optimally in the markets. Osikilo (2005) suggests how to improve the situation for the buyers. Many of these suggestions are in fact carried out in the market. Most of existing contracts include renegotiation clauses to adjust the contract according to the developments in the gas markets. In relation to such renegotiatons, both contract parties need thorough understanding of the impacts of new contract parameters. Hence, correct valuation of long term contracts are not important only for pricing when entering the contract and possibly hedging, but also continuously as renegotioations take place.

Turning to more technical aspects of the long term contracts, a typical price formula is, according to Asche et al. (2002),

$$P_p = P_0 + \sum_j \alpha_j (AE_j - AE_{j0}) EK_{SEj} \lambda_j, \tag{2.1}$$

where $P_p$ is the gas price paid to the producer, $P_0$ is the basis price, $\alpha_j$ is the weight for substitute $j$. Often, $\sum_j \alpha_j = 1$. $(AE_j - AE_{j0})$ is the price change for substitute $j$, (actual minus historic price). $EK_{AEj}$ is an energy conversion factor. $\lambda_j$ is the "pass through factor" for price changes in substitute $j$. $\lambda_j$ is typically 0.85 or 0.90. Hence, the producers are carrying a large part of the price risk. Current prices are computed as averages of periods of three to nine months.

From our experience, however, we have seen that the price formulas can be more complicated than (2.1). The contract price can typically depend on several different commodity indexes. This dependence can also be non-linear, and there might be limits on to what levels the indexes can reach (like in floor and roof options). This makes the future prices difficult to foresee, and both seller (often a producer) and buyer are exposed to several commodity indexes.

One general example of such a formula can be

$$P_p = P_0 + k_0(\alpha_1 \mathrm{OIL}_1 \lambda_1 + \alpha_2 \mathrm{OIL}_2 \lambda_2 + \alpha_3 \mathrm{COAL}_1 \lambda_3), \tag{2.2}$$

where $P_0$ and $k_0$ are constants. $\alpha_i$, $i = 1, 2, 3$, are weights that add up to one, and $\lambda_i$, $i = 1, 2, 3$, are energy conversion factors. The commodity indexes (here $\text{OIL}_1$, $\text{OIL}_2$ and $\text{COAL}_1$) are typically some indexes that the buyer already is exposed to, such as high or low sulphur oil delivered in Germany for a contract with delivery in the same country. In some cases, there can be limits on the indexes. For example, $\text{OIL}_1$ is replaced with $\text{OIL}_1^* = \max(\text{OIL}_1, K)$, where $K$ is a pre-defined value. To be able to valuate long term contracts correctly, all commodity indexes should be treated with the appropriate dependencies. Without strong simplifications, this puts high demands on the complexity of the underlying price processes in the valuation algorithms.

In addition to the value of swing options, finding an optimal exercise strategy is of interest. Knowing the optimal exercise strategy, the buyer can follow this when nominating volumes and the seller can predict the buyers nominations.

## 2.2 Constraints and penalties

A flexible commodity contract can be flexible in many ways (see e.g. (Asche et al., 2002)). In the following, we describe the most common constraints. One contract may include some, but not necessarily all of the following constraints.

**DCQ (Daily Contract Quantity):** A minimum (min DCQ) and maximum (max DCQ) volume that can be taken out of the contract each day. Typical values of min DCQ and max DCQ are 40 % and 110 %, respectively.

**ACQ (Annual Contract Quantity):** A minimum and maximum volume that can be taken out of the contract each year. Typical values of min ACQ and max ACQ are 90 % and 110 %, respectively.

**CF (Carry Forward) and MU (Make Up):** A carry forward right gives the holder of the option the possibility to build up unused flexibility for following years. Typically, volumes taken above minimum ACQ reduces the minimum ACQ level the following year, but up to a certain amount. A make up right is the possibility to use more flexibility in the current year by reducing the flexibility in following years, being the opposite of a carry forward right. For both carry forward and make up, the original min ACQ often holds on the average over some years.

Typical values for CF or MU are 3%, i.e. min ACQ can be temporarily adjusted with 3%. The period over which the total min ACQ should not be violated is typically 3 or 5 years long.

**TA (Temperature Adjustment):** If the temperature in certain months is higher or lower than the historical temperature (index), the min ACQ may be changed.

**HM (Hardship Month):** If the next month's forward price is below next month's contract price multiplied by a constant, the next month is declared as a "hardship month". No **penalty** will be given for violating minimum contract volumes during the next month.

**P (Penalty):** If, e.g., less than min ACQ is taken out during a year, the difference is subject to a penalty.

**R (Rebate):** If the contract holder has taken out more than a specified volume this year, the contract price is reduced by a certain percentage or a fixed sum.

In some sense, a rebate is the opposite of a penalty.

**Ramping:** These constraints limit the slope of the load pattern corresponding to a given exercise strategy. In other words, you cannot increase or decrease the volume taken out too fast. This is more common in electricity swing options (Haarbrücker and Kuhn, 2006).

**Other constraints:** In addition to daily and yearly volume restrictions, there can be, e.g., weekly, monthly or quarterly volume constraints as well. Swing options, at least for electricity prices, can be rights to exercise only in base, peak or off-peak hours. Furthermore, there can be a restriction on the number of swings (Dörr, 2003).

The contracts usually have a renegotiation clause. Renegotiation takes place when the contract terms are far from the current market conditions.

A price adjustment clause allows the price to be renegotiated when the market price of gas is substantially different from the contract price.

Large volumes may affect prices (it may not be possible to exercise an option fully). This reduces the flexibility and the value of flexibility. If there is a portfolio of contracts, ideally, all contracts should be optimised simultaneously.

## 2.3 Swing option valuation

Generally, a swing option is equal to $N$ nested American-style call options ($N$ being the number of exercise rights), similar to a Bermudan option. But while the Bermudan option has predetermined exercise dates, the swing option has further optionality. An upper bound of the value of the swing option with $N$ exercise rights is given by $N$ identical American options. A lower bound is given by the maximum value of $N$ European options, with predetermined exercise dates. When $N = n$ ($n$ being the number of exercise dates), the value of the swing option is equal to a series of European options.

| Author(s) | Method | Price process | Commodity studied | Constraints handled |
|---|---|---|---|---|
| Lari-Lavassani et al. (2001) | Dynamic programming (trees/ forest) | One-factor/ two-factor/ mean-reverting | Natural gas | ACQ DCQ P |
| Davison and Anderson (2003) | Monte Carlo | Mixture of Poissons | Electricity | ACQ DCQ |
| Dörr (2003) | Least-squares Monte Carlo | Two-factor mean-reverting | Electricity | ACQ DCQ (number of swings) P |
| Jaillet et al. (2004) | Dynamic programming (trinomial trees) | One-factor mean-reverting with seasonality | Natural gas (Henry Hub) | ACQ DCQ P |
| Meinshausen and Hambly (2004) | Least-squares Monte Carlo | Mean-reverting | Natural gas | ACQ DCQ |
| Ibáñez (2004) | Monte Carlo with optimal exercise frontier | Lognormal/ mean-reverting/ general multi-factor | General energy | ACQ DCQ P |
| Gravås (2004) | Least-squares Monte Carlo | Two-factor mean-reverting | Natural gas | ACQ DCQ |
| Thanawalla (2005) | Least-squares Monte Carlo | Mean-reverting plus Brownian motion | Natural gas/ electricity | ACQ DCQ P |
| Barrera-Esteve et al. (2006) | Parametric approximation | Multi-factor | Natural gas | ACQ DCQ P |
| Haarbrücker and Kuhn (2006) | Stochastic programming | Forward price model | Electricity | ACQ DCQ Ramping |
| Bender and Schoenmakers (2006) | Monte Carlo | No example | No example | ACQ DCQ |
| Baldick et al. (2006) | Dynamic programming | Demand/ supply model | Electricity | ACQ DCQ |
| de Jong and Walet (2003) and Boogert and de Jong (2008) (Maycroft) | Least-squares Monte Carlo | Mean-reverting with jumps | Natural gas | ACQ DCQ P Storage constraints |
| Bardou et al. (2007a,b) | Optimal (vector) quantisation | One- and two-factor | – | ACQ DCQ |
| Wilhelm and Winter (2006) | Finite elements methods | One-factor | Electricity | – |

Table 2.1. Overview of methods for pricing swing options.

# 3 Simulation based swing option valuation

In this chapter, we give a short description of the simulation based valuation methods and comment on their advantages or disadvantages.

## 3.1 Least-squares Monte Carlo (LSM)

Longstaff and Schwartz (2001) describe a method for valuing American options using least-squares Monte Carlo simulations (LSM). The method uses scenarios from any price process, giving full flexibility on the underlying price process. We will first describe the method for pricing American options, and the extend it to swing options (Section 3.1.1).

The idea is to work backwards in time. At the last time step (i.e. the last day of the contract period), the option is exercised if the option is in the money and expired if not. At the time step prior to the last time step, the holder has two possibilities. If the option is not in the money, the option is not exercised. If the option is in the money, the option is exercised if the instant payoff is higher than the expected value of waiting, which is referred to as the continuation value. The idea behind the least-squares Monte Carlo method is to use least-squares regression to find the continuation values.

The method starts with defining a set of basis functions and use a regression to find the parameter vector $\beta_i$ for each time step $i$ in (3.1) below. The idea is based on following equality

$$E[V_{i+1}(S_{i+1})|S_i = x] = \sum_{r=1}^{M} \beta_{ir}\psi_r(x). \tag{3.1}$$

Here, $V_i$ is the value of the option (which is known at the final time $T$). $S_i$ is the price of the underlying, $\psi_r(x)$ $r = 1, \dots, M$ are the basis functions and $\beta_{ir}$ are the parameter vectors to be estimated. It can be shown that (3.1) is true when $M \to \infty$. The method assumes that for a finite $M$, we have an appropriate approximation. $\beta_{ir}$ are estimated by least-squares regression, hence the name of the method.

The basis functions can be functions of the underlying asset, or different states of factors included in the price process. Typically they are polynomials of degree $M - 1$. The regression is done with price paths that are in the money only and each continuation value, $C_i(x)$, is computed as

$$C_i(x) = \psi(x)^{\mathrm{T}} \cdot \beta_i. \tag{3.2}$$

The equation for the regression to estimate $\beta_i$ is

$$\psi(S_i)^{\mathrm{T}} \cdot \beta_i = e^{-r\Delta t} \cdot V_{i+1}, \tag{3.3}$$

where $r\Delta_t$ is the discounting factor with interest rate $r$.

The simulation algorithm is as follows:

- Simulate b price paths
- Set $\hat{V}_{Tj} = h(S_{Tj})$ at the end points of each price path, $j = 1, ..., b$, where $h(S_{Tj})$ is the payoff at time $T$
- For each time step, $i = T - 1, ..., 1$, work backwards in time
  - Calculate $\hat{\beta}_i$ with estimated values $\hat{V}_{i+1,j}$ from (3.3)
  - Calculate continuation values $\hat{C}_i(S_{ij})$ using $\hat{\beta}_i$ as in (3.2)
  - Set
$$\hat{V}_{ij} = \begin{cases} h(S_{ij}), & \text{if } h(S_{ij}) \geq \hat{C}_i(S_{ij}) \\ e^{-r\Delta t}\hat{V}_{i+1,j}, & \text{if } h(S_{ij}) < \hat{C}_i(S_{ij}) \end{cases}$$
- The option value is given as $\hat{V}_0 = (\hat{V}_{11} + \cdots + \hat{V}_{1b})/b$

Stentoft (2004) shows that the least-squares Monte Carlo method gives results that converge to the true expectation functions for American option valuation as the number of scenarios gets large. Clément et al. (2002) also treat the convergence of the least-squares method. They contribute with information on the rate of convergence.

### 3.1.1 LSM for swing options

Meinshausen and Hambly (2004) use the least-squares Monte Carlo method and extend the algorithm to swing options. The method is based on the Longstaff-Schwartz method described above, but it is modified for multiple exercise rights. The modified algorithm for $n$ exercise rights is as follows:

- Simulate b price paths
- Set $\hat{V}^{(k)}_{Tj} = h(S_{Tj})$ for $k = 1, ..., n$ at the end points of each price path, $j = 1, ..., b$, where $h(S_{Tj})$ is the payoff at time $T$ and $k$ is the number of remaining of exercise rights
- Set $\hat{C}^{(k)}_{Tj} = 0$ for $k = 0, ..., n$ and all price paths, $j = 1, ..., b$

- For each time step, $i = T-1, ..., 1$, work backwards in time
  - Calculate $\hat{\beta}_i^{(k)}$ with estimated values $\hat{V}_{i+1,j}^{(k)}$ from (3.3) for $k = 0, ..., n$
  - Calculate continuation values $\hat{C}_i^{(k)}(S_{ij})$ using $\hat{\beta}_i^{(k)}$ as in (3.2) for $k = 0, ..., n$
  - Set
$$\hat{V}_{ij}^{(k)} = \max\{h(S_{ij}) + \hat{C}_{ij}^{(k-1)}, \hat{C}_{ij}^{(k)}\}$$
    for all price paths, $j = 1, ..., b$ and $k = 1, ..., n$ with $\hat{C}_{ij}^{(0)} = 0$
- Simulate b new price paths
- For each time step $i = 1, ..., T$, each price path $j = 1, ..., b$ and each exercise right, work backwards
  - Compute $\hat{C}_{ij}^{(k-1)}$ and $\hat{C}_{ij}^{(k)}$ until $h(S_{ij}) + \hat{C}_{ij}^{(k-1)} > \hat{C}_{ij}^{(k)}$ and set $\hat{V}_j^{(k)} = e^{-ri\Delta t}h(S_{ij})$
  - Continue the loop until $h(S_{ij}) > \hat{C}_{ij}^{(1)}$ and set $\hat{V}_j^{(1)} = e^{-ri\Delta t}h(S_{ij})$
- The option value is given as $\hat{V_0}^n = \frac{1}{b}\sum_{k=1}^{n}\sum_{j=1}^{b}\hat{V}_j^{(k)}$

The estimation of the value of a swing option needs two sets of price scenarios from the same price process. The algorithm finds a negative-biased value, but the accuracy is increased with the number of price scenarios. Further, a positive-biased value can be estimated with a new set of price scenarios. Gravås (2004) uses the method described above to estimate values of short term swing options using a more realistic gas price process than the one used by Meinshausen and Hambly (2004).

In the literature, only a few constraints from Section 2.2 are treated or discussed. Meinshausen and Hambly (2004) treat a fixed number of maximum exercise rights. Dörr (2003) uses the same methods and treats constraints with respect to the number of swings, but not with respect to volumes. He also shows how an exercise strategy can be derived from the least-squares Monte Carlo simulation. Figueroa (2006) follows up on the work of Dörr (2003), and provides pricing under a mean-reverting jump-diffusion (MRJD) model with seasonality. For the computation of the lower bound for the price, he provides a formula, which greatly reduces the computational burden.

Thanawalla (2005) comments on the least-squares Monte Carlo method. He argues that, although flexible with many basis functions, the standard deviation of the estimated coefficients in the regressions gets very high when many basis functions are used. He proposes the use of non-parametric regression using splines. However, he seems to have problems when the number of exercise rights gets high. Already with 25 exercise rights, his method needs more than eight hours to run. Thanawalla (2005) also describes how penalties for validating volume constraints can be treated using the least-squares Monte Carlo approach.

Boogert and de Jong (2008) develop a least-squares Monte Carlo valuation method for storage contracts, which incorporates "realistic gas price dynamics and complex physical constraints". The price process is a one-factor model with drift. The constraints are given as limits on maximum injection and withdrawal rates, cost on injection, profit of withdrawal (both transaction and bid-ask spreads) and, finally, minimum and maximum volume for the gas storage. In the example studied, the pricing algorithm converges very fast; "as few as 50 simulations often suffice", which is extraordinary few. The authors also discuss computational issues, and claim that it is sometimes sub-optimal to choose between maximum possible injection, maximum possible withdrawal and no action, hence that the bang-bang property (the optimal daily decision is either min DCQ or max DCQ) is not always valid.

## 3.2 Monte Carlo valuation through computation of the optimal exercise frontier

Ibanez and Zapatero (2004) and Ibáñez (2004) have developed another method based on simulation to value swing options. The first paper introduces the method and values Bermudan options and the second paper shows how to modify the algorithm to price swing options.

The idea behind their method is to find an optimal exercise price at every time step. Knowing the optimal exercise price at all times, one can easily compute the option price for each price scenario. Here, we give a short description of the method they developed.

Denote with $P_t(V_t, K)$ the price of the option, where $V_t$ is a vector of stochastic variables (stock price, interest rate, etc.) and $K$ is the strike price. The method relies on the fact that the optimal exercise frontier is a set of points $V_t^*$ for which the value of the un-exercised American option and the intrinsic value of the option are the same, i.e.

$$P_t(V_t^*, K) = I(V_t^*, K).$$

Let $M$ be the dimension of the vector of state variables $V$. One of the state variables are often $S$, the price of the underlying commodity, so we write $V = (B, S)$, where $B$ is the vector of values on which the option depends.

The initial time is set to $t_0$ and maturity at $T$ and exercise dates $t_1, t_2, ..., t_N$, $t_N = T$. Suppose there exists an optimal exercise frontier that divides the state space $V_{t_n}$ into an exercise and a wait region. Any point on this frontier, $V_{t_n}^* = (B_{t_n}, S_{t_n}^*)$ can be characterised by a function $F_{t_n}^*$ such that $S_{t_n}^* = F_{t_n}^*(B_{t_n})$. If $r$ is the interest rate and $Q$ is the risk neutral probability, then at any time $t_n$, the price of the Bermudan option is given by

$$P_{t_n}(V_{t_n}, K) = E^Q_{t_n}[e^{-\int_{t_n}^{\tau^*} r_s ds} I(V_{\tau^*}, K)|V_{t_n}], \tag{3.4}$$

where $\tau^* \in \{t_{n+1}, ..., T\}$ is the optimal stopping time. (3.4) can be computed by Monte Carlo simulation if the optimal exercise frontier is known.

At every time step, $t_n = t_1, ..., t_{N-1}$, $j$ points of $V_{t_n}^{j,*}$ are computed. Out of these points, the optimal exercise frontier, $V_{t_n}^*$, is estimated. If the dimension of the state vector is higher than one, several grid points of $\{B_{t_n}^1, ..., B_{t_n}^J\}$ are needed to compute the corresponding set $\{S_{t_n}^{1,*}, ..., S_{t_n}^{J,*}\}$. Having $M > 1$ state vectors increases the computation costs significantly.

The value of the option using this method is negatively biased, since $\hat{F}_{t_n}(B_{t_n})$ is estimated with error. The bias reduces when the number of simulations increases.

The computational costs are increasing with the number of state parameters. With $M > 1$, the computational costs are much higher than with the least-squares Monte Carlo method. The grid points, $B_{t_n}^j$, and the starting points $S_{t_n}^{1,(1)}$, can be chosen such that the convergence is fast in each iteration.

To find the value of a swing option, the algorithm is slightly modified in a similar way as in the least-squares Monte Carlo method, where the decrease in option value due to an exercise has to be included in the point on the optimal exercise frontier. The authors also discuss how the optimisation is adjusted when penalties are given for violating minimum or maximum quantities.

## 3.3 Commercial alternatives

A company called Maycroft[1] is selling software for both swing option valuation and storage valuation, using least-squares Monte Carlo methods, based on (Boogert and de Jong, 2008; de Jong and Walet, 2003). The underlying price process seem to be a fairly simple mean-reversion model;

$$\log S_t = \log S_{t-1} + \alpha(\mu - \log S_{t-1}) + \sigma\varepsilon_t, \tag{3.5}$$

where $\varepsilon_t \sim \mathcal{N}(0, 1)$, but more general price processes can be used, since the pricing is based on least-squares Monte Carlo methods. In addition, other commercial alternatives may exist.

1. http://www.maycroft.com/

# 4 Non-simulation based swing option valuation

In this chapter, we describe different valuation methods based on dynamic programming (and similar methods) found in the literature. We give a short description of the methods and comment on their advantages and disadvantages.

## 4.1 Dynamic programming

Jaillet et al. (2004) develop a framework for valuing swing options based on dynamic programming with a trinomial tree (see Figure 4.1). They use a one-factor, seasonal, mean-reverting price process for the underlying commodity price. The dynamic programming procedure starts from the option's expiry date and works backwards in time to value the option using "backward induction" in three dimensions; price, number of exercise rights left and volume. The price can be represented with more than one state variable if necessary. At each discrete point in time, the exercise possibility is evaluated by finding the most profitable of staying in the current tree and jumping down to the tree with one less exercise right left.

The method requires discretisation in both volumes and prices. The computational burden increases according to $nJN^2L^2/2$, where $n$ is the number of exercise dates, $N$ is the number of exercise rights, $J$ is the number of nodes associated with the underlying spot price and $L$ is the number of exercise amounts. Typical values ($n = 365$, $J = 5?$, $N = 200$, $L = 5$) would give approximately $10^9$ computations.

Regarding the constraints, the authors point out that restrictions in the contract can be captured by specification of a general penalty function. Assuming the volume has to be in the interval $[V_{\min}, V_{\max}]$, a penalty function can be

$$\phi(V) = \begin{cases} \infty & \text{if } V < V_{\min} \\ 0 & \text{if } V_{\min} < V < V_{\max} \\ \infty & \text{if } V > V_{\max}. \end{cases}$$

With such penalty functions, volume constraints can be taken into account, both daily and periodical.

Lari-Lavassani et al. (2001) use dynamic programming to value swing options. They use both one-factor and multi-factor models and are one of the first to use mean-reverting processes for pricing swing options. Baldick et al. (2006) also use dynamic programming to value swing type contracts. They investigate interrupt-

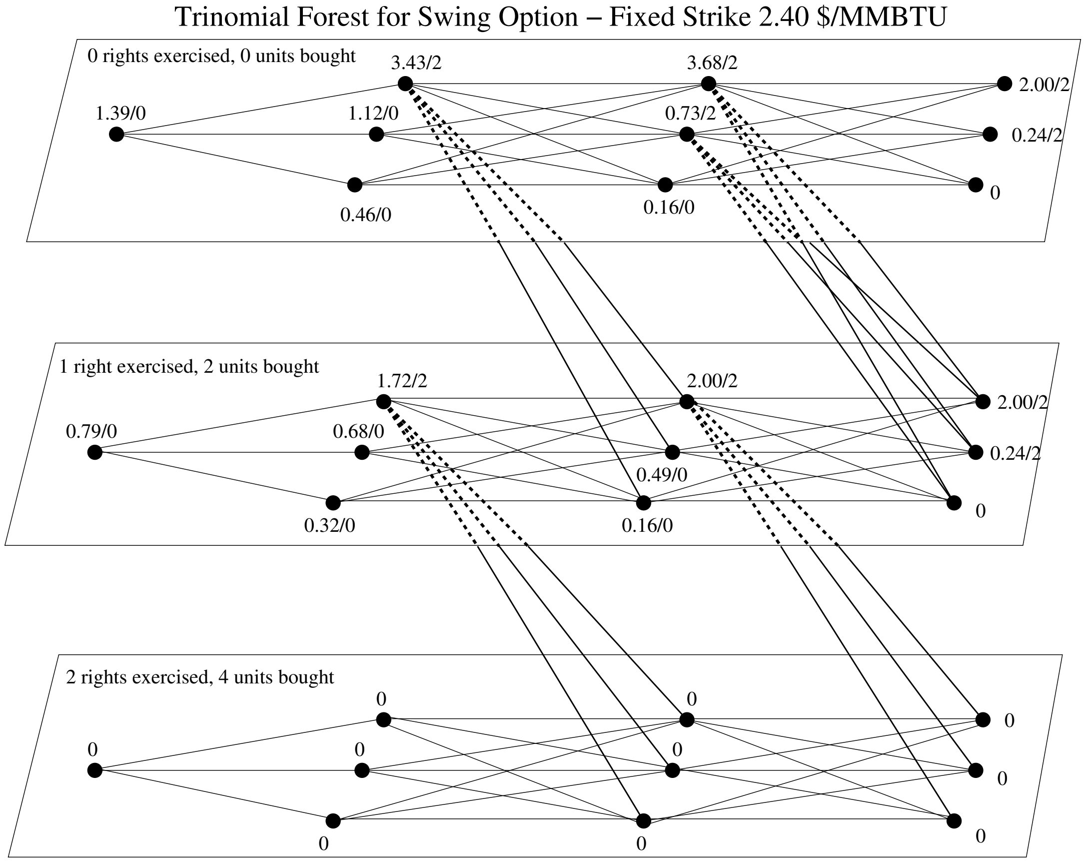


Figure 4.1. Figure from (Jaillet et al., 2004). A numerical example of pricing a swing option with two exercise rights left using a trinomial tree.

ible electricity contracts using a structural demand/supply model. They conclude that, in a de-regulated market, interruptible contracts may alleviate supply problems due to price spikes.

## 4.2 Parametric approximation

Barrera-Esteve et al. (2006) present several numerical methods for valuing swing options. They summarise the methods using least squares Monte Carlo simulation and the dynamic programming approach.

In addition, they introduce alternative algorithms based on parameterisations of the gas consumption. Observations from these methods lead them to an interesting theoretical result; under some assumptions (that are valid for all relevant practical cases) the optimal strategy is of bang-bang type, i.e. the optimal daily decision is either min DCQ or max DCQ, which is an intuitively reasonable result.

Let $q$ be a given consumption strategy, $\Phi(\cdot)$ instantaneous profit and $\mathcal{P}_T$ some terminal penalty. Then they formulate the swing option price as

$$\mathcal{J}(q) = \mathrm{E}\left(\sum_{i=0}^{N-1} \Phi_{t_i}(q_{t_i}, F_{t_i}, Q_{t_i}) + \mathcal{P}_T(F_T, Q_T)\right),$$

where the expectation is performed under the risk-neutral probability measure $\boldsymbol{Q}$. $t_i$ is a fixed date, $t_N = T$, $F$ is the gas price and $Q$ is the volume constraint. The authors seem to have DCQ and ACQ constraints in mind, but almost any penalty function should work.

To value the option, they assume that consumption decisions are based on the underlying asset price, and not on the real needs of gas. Thus, the price is given by

$$\sup_q \mathcal{J}(q).$$

Barrera-Esteve et al. propose two methods for parameterising the consumption function $q$. With a neural network approach and using the bang-bang property, they introduce a faster algorithm with a direct parameterisation of the purchase threshold. This parameterisation does not give a smooth option price function $\mathcal{J}(\theta)$. Hence, they introduce a specific optimisation procedure.

The new methods can, like the least-squares Monte Carlo methods, handle multi-factor models. The authors argue that these methods are advantageous, since they give intermediate prices throughout the optimisation stage. Results are shown from all methods, using a numerical example. The results are similar. However, the dynamic programming approach seems to give slightly higher values of the swing options. They do not comment further on why this is the case.

## 4.3 Numerical integration

The integrals involved in pricing a swing option may be solved using all sorts of numerical integration.

Andricopoulos et al. (2007) do not consider swing options, but path-dependent options with one or more underlying. They compare their method with, amongst others, Longstaff and Schwartz (2001). The method is based on numerical quadrature methods (QUAD), which are very fast. The authors claim that QUAD outperforms grid and lattice methods, and even the Monte Carlo method when there are five underlying assets or when there are early exercise features. It remains to see if this method is flexible enough to handle swing options with constraints and realistic price processes.

Bardou et al. (2007a) investigate a numerical integration method, the so-called optimal (vector) quantisation, for the pricing of swing options. The method seems promising, since "the optimal quantisation method shares the good properties of the so-called tree method but is not limited by the dimension of the underlying". The method is compared with the LSM method, which is comparably slower and less precise. The authors seem to have DCQ and ACQ constraints in mind, and demonstrate use on both one- and two-factor models. In a corresponding article (Bardou et al., 2007b) the same authors show that the optimal strategy is of bang-bang type when the constraints are global (which the ACQ constraints are). Apparently, this is a stronger result than the bang-bang result obtained by Barrera-Esteve et al. (2006).

By transforming the multiple stopping problem, which a swing option really is, into a series of single stopping time problems, Wilhelm and Winter (2006) applies finite element methods for the pricing of swing options. The speed and accuracy is "superior" to Monte Carlo methods.

## 4.4 Further methods

Haarbrücker and Kuhn (2006) investigate pricing of electricity swing options. They use stochastic programming, which they argue is more flexible than dynamic programming. The underlying price process is a forward price model. They include one more constraint on the swing option, ramping. Ramping means that the volume can not be instantly changed. As an example, it takes several hours to increase the volume from minimum to maximum. This is relevant for physical situations like power plants with limited start-up speed.

Stochastic programming should not be mixed with dynamic programming. Stochastic programming is still attractable when the price processes have several factors. For such cases, the dynamic programming algorithms often require high computational efforts. The stochastic programming algorithms are fairly complex and demands higher implementation efforts.

Bender and Schoenmakers (2006) introduce a new iterative method, not based on backward dynamic programming. They argue that their method is theoretically as good as methods based on backward dynamic programming, but may be "superior from a practical point of view".

# 5 Future work

We have found no literature on swing option pricing that include all constraints described in Section 2.2. For long term gas contracts, some of these constraints could add significant value to the contracts. In the following we discuss hardship, temperature adjustment, carry forward and make-up rights.

## 5.1 Hardship and temperature adjustment

To consider hardship months, one possible solution would be to simulate or compute forward prices in addition to spot prices. If the forward prices are known, the buyer can consider the impact due to hardship months. Temperature adjustment might be included by simulating temperature, preferably correlated with the gas prices. However, both hardship months and temperature adjustments are assumed to have relatively low impact on the contract values. In addition, the methods for including these constraints in the pricing algorithms would be fairly straight-forward, yet computationally demanding.

## 5.2 Carry forward

In cases when the forward curve is not flat, it is obvious that carry forward flexibility is valuable for the option holder. As the gas prices are volatile, carry forward may add significant values to the contracts.

If the contract includes carry forward, the option holder has the possibility to postpone unused flexibility into the future. All volumes nominated above min ACQ are saved in a carry forward account. These volumes can be reduced from min ACQ in future years. Imagine, for example, that an option holder nominates 95% of ACQ one year in a contract with min ACQ equal to 90%. The option holder now has 5% in the carry forward account. These 5% can be reduced from min ACQ in a following year, such that 90-5%=85% is enough to fulfil the contract constraints. If 85% is nominated in a following year, the carry forward account is reduced by 5% again, such that there are no carry forward rights left in this example. However, carry forward rights can be built up again.

Both carry forward and make-up have a limited duration, such that after a certain period of years, unused rights are deleted. If, for example, the duration is five years and the buyer nominates 95, 90, 90, 90, 90, 90% in a contract with min ACQ 90%, the 5% carry forward rights that are built up the first year are deleted from the carry forward account after five years. Contrary to the make-up right, carry forward volumes are paid for when taken.

We believe that including carry forward constraints in the pricing algorithms can contribute to the existing theory of swing options pricing. A natural way to include these constraints is to define more penalty functions. Below, we introduce penalty functions ((5.1) and (5.2)) to consider DCQ, ACQ and carry forward. See Table 5.1 for notation.

| Symbol | Description |
|---|---|
| $V_t$ | Volume nominated at day $t = 1, \dots, N$ |
| $N$ | Number of days in contract period |
| $V^{(y)}$ | $\sum_{t=1+(y-1)\cdot 365}^{y\cdot 365} V_t$, volume nominated for year $y$ |
| $\text{CF}_{\text{acc}}^{(y)}$ | Accumulated carry forward in year $y$ |
| $n_Y$ | Number of duration years for carry forward |
| $V_{\text{max CF}}$ | Maximum reduction of volumes due to carry forward rights |

Table 5.1. Symbols with description.

**Daily penalty function:**

$$\phi_t(V_t) = \begin{cases} \infty & \text{if } V_t < \text{min DCQ} \\ 0 & \text{if min DCQ} < V_t < \text{max DCQ} \\ \infty & \text{if } V > \text{max DCQ}. \end{cases} \tag{5.1}$$

**Yearly penalty function:**

$$\phi_y(V^{(y)}) = \begin{cases} \infty & \text{if } V^{(y)} < \text{min ACQ} - \max\left(V_{\text{max CF}}, \text{CF}_{\text{acc}}^{(y)}\right) \\ 0 & \text{if min ACQ} < V^{(y)} < \text{max ACQ} \\ \infty & \text{if } V > \text{max ACQ}. \end{cases} \tag{5.2}$$

An infinite penalty (so-called “firm constraints”) implies that the contract holder is not allowed to violate the constraints. In some cases, if, e.g., less than min ACQ is taken out during a year, the difference is subject to a (finite) penalty. However, max ACQ is always a firm constraint.

## 5.3 Make-up right

Often, the reduction of volumes due to carry forward rights is limited, typically to 10%, such that no less than min ACQ-10% can be nominated in a year, regardless of the amount in the carry forward account. If the option holder nominates volumes below minimum permissible volumes, the volumes that are not nominated have to be paid for (hence these contracts are often called take-or-pay contracts). If the contract includes make-up rights, gas that is not nominated, but paid for, can be nominated in a following year. In some sense, this can be seen as the opposite of carry forward. Examples of nominations in contracts with carry forward and make-up rights are illustrated in Figures 5.1 to 5.4.

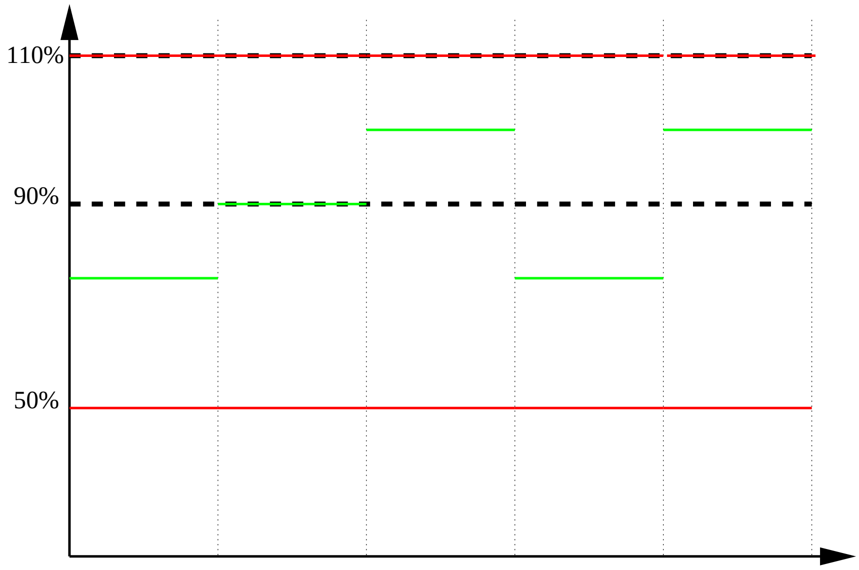


Figure 5.1. Example of make-up with five years duration. Black lines denote the contract min and max ACQ, which are 90% and 110%, respectively. The red lines are the min and max permissible volumes including make-up rights. The lower red line, at 50%, represents the min DCQ, which actually is the lower limit with make-up right. The green lines are the nominations made by the option holder. In the first year, 80% is nominated and the make-up account is built up to 10%. In the second year, 90% is nominated and the make-up account is still 10%. In the third year, 100% is nominated and the make-up account is reduced to 0%. In the fourth year, 80% is nominated and the make-up account is built up to 10%. During the fifth year, 100% is nominated and the make-up account is again reduced to 0%.

When nominated volumes are below permissible minimum, the volumes not nominated are paid for with 85 to 100% of the average yearly gas price during that year[1]. If the contract includes make-up, these volumes can be nominated in a following year after the minimum permissible volume has been reached. When this make-up gas is nominated, 0 to 15% of the average yearly gas price is paid (total payment for the gas is summed up to 100%). If the price for gas is higher when the make-up gas is nominated, the gain made by the buyer is added to the price. Hence, the buyer has no possible gains in speculating on make-up. Merely, the buyer looses interest rates on the (make-up) volumes not nominated previously.

1. Note that the contract price typically is computed three months in advance, so before the end of the year, the average contract price is known.

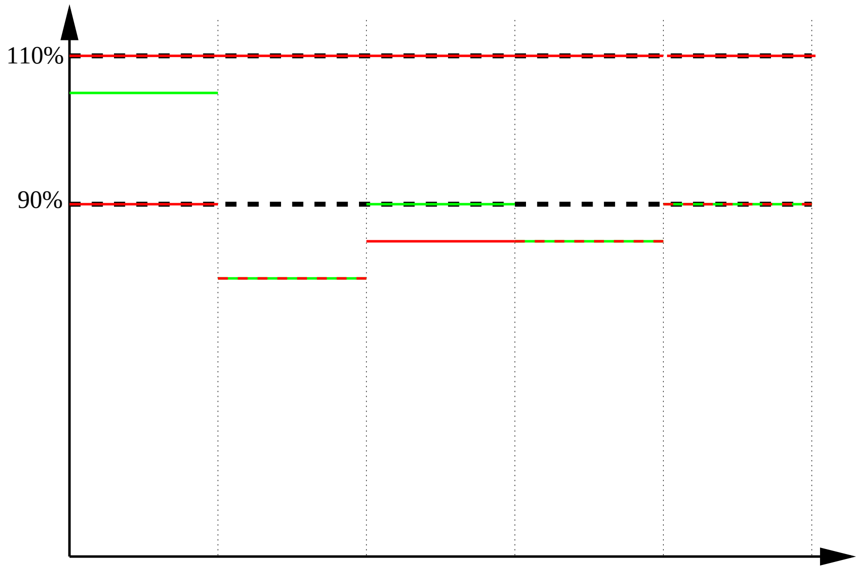


Figure 5.2. Example of carry forward with five years duration. Black lines denote the contract min and max ACQ, which are 90% and 110%, respectively. The red lines are the min and max permissible volumes including carry forward rights. The green lines are the nominations made by the option holder. In the first year, 105% is nominated and the carry forward account is built up to 15%. In the second year, the limits are [80%,110%] due to the carry forward account and the 10% limit on carry forward. 80% is nominated and the carry forward account is reduced to 5%. In the third year, the limits are [85%,110%] due to the carry forward account. 90% is nominated and the carry forward account remains on 5%. In the fourth year, the limits are [85%,110%] due to the carry forward account. 85% is nominated and the carry forward contract is reduced to zero. The fifth year, the limits are [90%,110%] and 90% is nominated.

To illustrate this point, assume that the average prices for the first and second year are $P^{(1)}$ and $P^{(2)}$, respectively. Furthermore, the annual interest rate is set to $r$. The net present value of the contract price is then

$$\frac{0.85P^{(1)}}{1+r} + \frac{0.15P^{(2)} + \max\left(0, 0.85(P^{(2)} - P^{(1)})\right)}{(1+r)^2}. \tag{5.3}$$

If $P^{(1)} > P^{(2)}$, (5.3) gives a net present value of

$$\frac{0.85P^{(1)}(1+r)0.15P^{(2)}}{(1+r)^2} > \frac{P^{(2)}}{(1+r)^2}.$$

If $P = P^{(1)} = P^{(2)}$, (5.3) reduces to

$$\frac{P(1+0.85r)}{(1+r)^2} > \frac{P}{(1+r)^2}.$$

In any case, the make-up volume is much more expensive than ordinary volumes. For the margin (contract price–market price of gas), make-up may, however, be

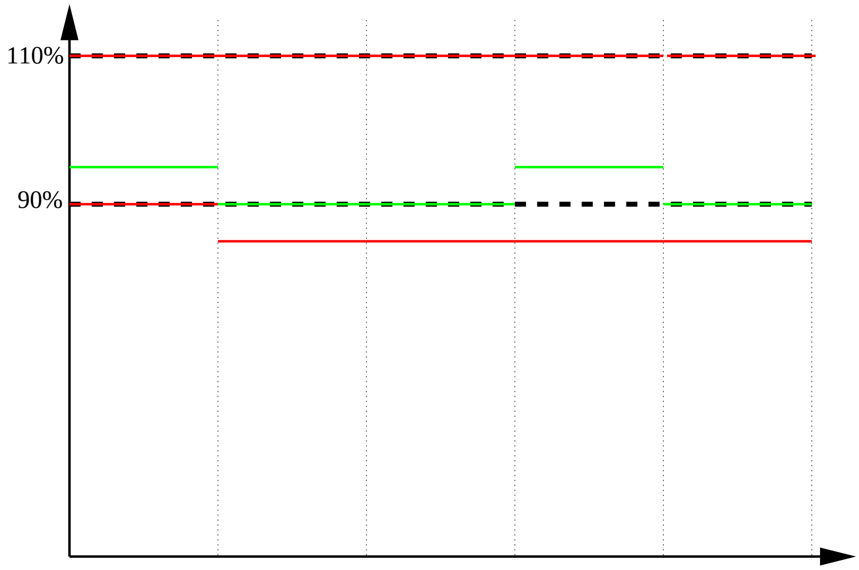


Figure 5.3. Example of carry forward with three years duration. Black lines denote the contract min and max ACQ, which are 90% and 110%, respectively. The red lines are the min and max permissible volumes including carry forward rights. The green lines are the nominations made by the option holder. In the first year, 95% is nominated and the carry forward account is built up to 5%. In the second year, the limits are [85%,110%] due to the carry forward account. 90% is nominated and the carry forward account remains at 5%. In the third year, the limits are [85%,110%] due to carry forward account. 90% is nominated and the carry forward account remains on 5%. For the fourth year, 95% is nominated. 5% is added to the carry forward account, but the 5% from the first year are deleted from the carry forward account as the duration period has exceeded. The carry forward account therefore remains at 5%. The fifth year, the limits are [85%,110%] due to the carry forward account and 90% is nominated.

beneficial, although it is quite unlikely, at least in the long run, since this would imply that yearly oil and gas prices are decoupling.

Thus, make-up is avoided by the rational buyer and make-up rights do not add value to the contract. Hence, make-up rights do not have to be considered in the pricing algorithms.

Physical reasons for using make-up rights may exist. Most buyers of these gas contracts enter them to provide large amounts of gas, not to speculate. Given that make-up has been built up, it has to be taken into account properly.

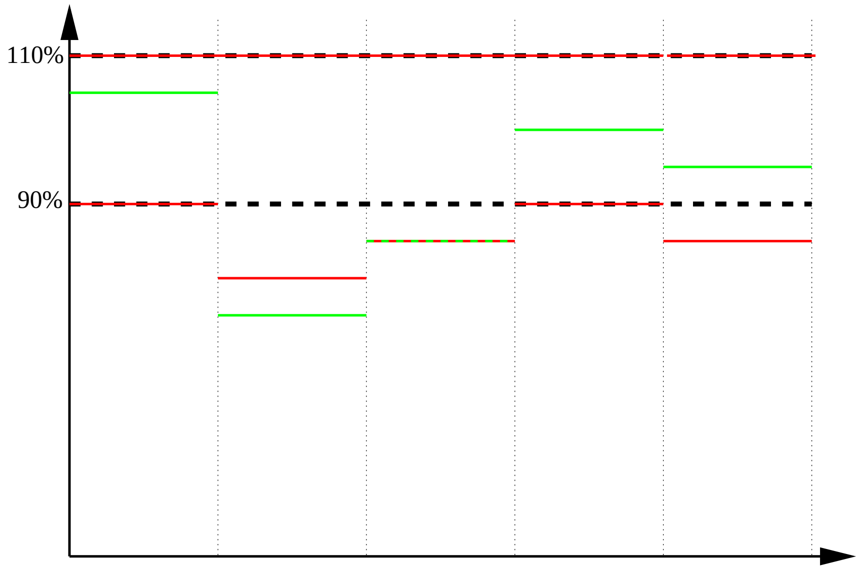


Figure 5.4. Example of carry forward and make-up with five years duration. Black lines denote the contract min and max ACQ, which are 90% and 110%, respectively. The red lines are the min and max permissible volumes including carry forward rights. The green lines are the nominations made by the option holder. In the first year, 105% is nominated and the carry forward account is built up to 15%. In the second year, the limits are [80%,110%] due to the carry forward account and the 10% limit on carry forward. Only 75% is nominated. The carry forward account is reduced by 10% and in addition make-up gas is built up with 5%. The make-up gas is paid for (85-100%) in this year. In the third year, the limits are [85%,110%] due to the carry forward account. 85% is nominated and the carry forward account is reduced to zero. The fourth year, 100% is nominated, of which 5% are make-up gas (where the remaining costs are paid) and 5% are added to the carry forward account. In the fifth year, the limits are [85%,110%] due to the carry forward account. 95% is nominated.

# 6 Summary

Correct valuation of long term contracts is important for many reasons. There are continuous renegotiations of such contracts. It is therefore important to understand the values of the contracts. In addition, long term contract volumes are often very large with significant shares of the total volume in the market players portfolios. Furthermore, exercise strategies are needed for optimising the nominations for the buyer and estimating future deliveries for the seller.

The most popular methods in the literature for valuing swing options are the least-squares Monte Carlo based methods and the dynamic programming methods. In addition, we have described some methods that are more or less similar to the two popular methods. Long term contracts can be described by complex price process depending on different commodities prices. This makes it preferable to use multi-factor models. The least-squares Monte Carlo method is very flexible in terms of choice of price process for the underlying commodity.

Most methods found consider volume constraints on ACQ and DCQ in some form. Some of them also include penalties for violating the constraints.

Further work in this area should include more of the contract constraints in the valuation methods. Carry Forward/Make Up Right, and possibly Hardship Month, should be the primary focus, since these are important for valuation of real gas contracts.

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